A Self-Similar Solution of Shock Propagation in a Mixture of Non-ideal Gas and Small Solid Particles in Magnetogasdynamics

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Abstract

Similarity solutions are obtained for unsteady, one-dimensional, self-similar flow of a perfectly conducting mixture of a non-ideal gas and small solid particles, behind a strong shock (cylindrical or spherical) driven by a piston moving according to power law in the presence of an azimuthal magnetic field. The small solid particles are considered as pseudo-fluid and assumed to be continuously distributed in the mixture. Effects of change in the values of parameters K_p, G_a (dust parameters), $\bar{b}$ (non-idealness parameter of the gas) and M_A^{-2} (magnetic parameter) on the shock strength, piston position and on the flow-variables in the flow-field behind the shock front are obtained. It is found that there is a decrease in the shock strength and the value of piston position due to the non-idealness of gas as well as due to the presence of dust-particles and the magnetic field. This decrease in the shock strength and the value of piston position is interpreted as a result of decrease in the compressibility of the mixture. Mutual effects of parameters are also obtained to investigate the deviations in the effects of parameters K_p, G_a and $\bar{b}$ due to the presence of magnetic field. It is observed that effects of parameters K_p, G_a and $\bar{b}$ on the shock strength and on the piston position are reduced due to the presence of magnetic field while the effects of these parameters on the flow-variables are enhanced due to the presence of magnetic-field.

Keywords : Shock waves, piston problem, self-similar solutions, perfectly conducting mixture of non-ideal gas and small-solid particles, magnetic-field.
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1. Introduction

The study of shock wave propagation in a mixture of a gas and small solid particles is of great importance due to its application in nozzle flow, lunar ash flow, bomb blast, coal-mine blast, underground, volcanic and cosmic explosions, metallized propellent rocket, supersonic flight in polluted air, collision of coma with planet and many other engineering problems. Also, there are many natural phenomena such as explosion of supernova, sand storms, aerodynamic ablation, cosmic dusts etc. which include gas particle two-phase mixture in which shock wave appears. So, its study in gas-particle mixture becomes more important. So far, a number of papers have been reported on the shock wave propagation in a mixture of gas and dust particles.

Miura and Glass [1] obtained an analytic solution of a planer dusty gas with constant velocities of the shock and the piston moving behind it. As they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust virtually has a mass fraction but no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the shock propagation. Vishwakarma and Pandey [2] obtained a self-similar solution for a one-dimensional, unsteady, adiabatic flow of a mixture of a perfect gas and small solid particles behind a strong spherical shock wave with time dependent energy input. They analysed that the presence of solid particles affects the medium in two ways: first, the volume fraction of solid particles lowers the compressibility of the mixture and second, the particle load increases the inertia of the mixture (Pai et al. [3], Narasimhula Naidu, Venkatanandam and Ranga Rao [4], Steiner and Hirschler [5]).

In extreme conditions that prevail in most of the problems associated with shock wave, the assumption that the gas is ideal is no longer vaild. Therefore, several authors studied the problem of shock wave propagation in non-ideal gases, for example, Anisimov and Spiner [6], Ranga Rao and Purohit [7], Ojha and Tiwari [8], Vishwakarma, Patel and Chaube [9], Vishwakarma and Nath [10]. Anisimov and Spiner [7] have taken an equation of state for non-ideal gases in simplified form, and investigated the effect of parameter for non-ideal gas on the problem of a strong point explosion. Vishwakarma and Nath [10] obtained the similarity solution for a strong shock driven by a piston moving according to power law in a medium which is assumed to be a mixture of non-ideal gas and small solid particles, in both the cases when the flow is isothermal and adiabatic.

Since at high temperature that prevail in most of the problems associated with shock waves a gas is ionized, electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of the study of gas-dynamic flow and the electromagnetic field simultaneously. The study of shock wave propagation in a medium under the influence of magnetic field always creates a great interest since its study is very useful in many space research and astrophysical phenomena like formation of magnetor during supernova explosion, flare produced in solar wind, central part of burst of galaxies, nuclear explosion etc. Many authors studied the problem of shock wave propagation in presence of magnetic field.

Parker [11] pointed out that the hydrodynamic blast wave theory can usefully describe the large-scale regime to which flow due to a sudden expansion of solar corona asymptotically converge. Using similarity assumptions, he presented a number of solutions for his idealized adiabatic "solar wind" model. These solutions correspond to the flow driven by a spherical piston in power law motion whose surface is the contact discontinuity enveloping the fresh flare corona in the centre core. Rosenau and Frankenthal [12] extended the work of Parker [11] to the hydromagnetic case. Lee and Chen [13] and Summers [14] have studied an idealized model of a magnetohydrodynamic spherical blast wave applied to a flare produced in magnetic-field. Christer and Helliwell [15] studied the cylindrical shock and detonation waves in magneto-hydrodynamics. In recent years the propagation of shock wave in the presence of magnetic field has been described by many authors, particularly, by Vishwakarma and Yadav [16], Nath [17], Vishwakarma, Maurya and Singh [18], Nath [19] and Vishwakarma, Nath and Srivastava [20]. Vishwakarma and Srivastava [20] obtained the self-similar solution for a cylindrical shock wave in a weakly conducting dusty gas.

In all the works, mentioned above, where the effect of magnetic field is considered on the shock wave propagation, the medium has been taken to be either a non-ideal gas, a perfect gas or a dusty gas (mixture of perfect gas and small solid particles). No author has studied the effects of magnetic field on the propagation of shock wave in a mixture of a non-ideal gas and dust particles. Therefore, in the present work, we extend the work of Vishwakarma and Nath [10] by taking a magnetic field in the perfectly conducting mixture of non-ideal gas and small solid particles through which shock wave propagates.

To get some essential features of shock wave propagation, small solid particles are considered as pseudo-fluid, and it is assumed that the equilibrium

flow condition is maintained in the whole flow-field and viscous stress and heat conduction of the mixture are negligible (Pai et al. [3], Higashino and Suzuki [21], Steiner and Hirschler [5]). Also, it is assumed that the medium is perfectly conducting and it is permeated by a variable azimuthal magnetic field.

Because of high temperature in the flow, transfer of heat takes place behind a strong shock by the mode of radiation. So, for such flows the assumption of adiabaticity may not be valid. Therefore, an alternative assumption of zero temperature gradient throughout the flow (flows which satisfy this condition are also known as isothermal flow) may approximately be taken (Korobeinikov [22], Laumbach and Probstein [23], Sachdev and Ashraf [24]). With these assumptions, we therefore derive a similarity solution for both the isothermal and adiabatic flows behind a strong cylindrical or spherical shock propagating in a medium (mixture of non-ideal gas and small solid particles) driven by a piston moving according to power law in presence of a variable azimuthal magnetic field. Effects of change in the values of different parameters (dust parameter K_p, G_a, non-idealness parameter of the gas $\bar{b}$, magnetic parameter M_A^{-2}) are obtained. A comparison is also made between the solutions of isothermal and adiabatic flows. Through this study our main purpose is to find out that how the effects of the non-idealness of the gas and the dust particles on the shock strength, on the piston position and on the flow variables in the flow field behind the shock deviate due to the presence of the azimuthal magnetic field. For this purpose, we obtain the mutual effects of parameters K_p, G_a, $\bar{b}$ and M_A^{-2} on the shock strength, piston position and on the different flow variables.

2. Fundamental Assumptions

We take the medium to be a perfectly conducting mixture of a non-ideal gas and small solid particles. Here the small solid particles are taken as pseudo fluid. The equation of state for non-ideal gas is taken to be (Anisimov and Spiner [6], Ranga Rao and Purohit [7], Vishwakarma and Nath [10])

$$p_g = R^* \rho'_g (1 + b\rho'_g) T, \tag{1}$$

where p_g and ρ'_g are the partial pressure and the partial density of the gas in the mixture, T is the temperature of the gas (and of the solid particles as the equilibrium flow condition is maintained), R^* is the specific gas constant and b is the internal volume of the molecules of the gas. Because of intermolecular force of interaction present among the component molecules of the gas, the deviations of an actual gas from the ideal state results in this equation. The density of the non-ideal gas is assumed to be so small that the triple, quadruple and higher

order collisions among the molecules of the gas are negligible and therefore the gas molecules interact through binary collisions only.

The specific volume of solid particle is assumed to remain unchanged by variation in the temperature and pressure. Thus, the equation of state of solid particles in the mixture is, simply,

$$\rho_{sp} = \text{constant}, \tag{2}$$

where ρ_{sp} is the specific density of the solid particles. Proceeding on the same line as Pai [25], we obtain the equation of state of the mixture as

$$p = \frac{1 - K_p}{1 - Z}[1 + b\rho(1 - K_p)]\rho R^* T, \tag{3}$$

where p and ρ are the pressure and density of the mixture, $Z = \frac{V_{sp}}{V}$ is the volume fraction and $K_p = \frac{M_{sp}}{M}$ is the mass fraction (concentration) of the solid particles in the mixture. Here M_{sp} and V_{sp} are the total mass and volumetric extension of the solid particles and V and M are the total volume and total mass of the mixture.

The relation between K_p and Z is given by (Pai [25])

$$K_p = \frac{Z\rho_{sp}}{\rho}. \tag{4}$$

In equilibrium flow, K_p is constant in whole flow field. Therefore from (4)

$$\frac{Z}{\rho} = \text{constant}. \tag{5}$$

Also, we have the relation (Pai [25])

$$Z = \frac{K_p}{G(1 - K_p) + K_p}, \tag{6}$$

where $G = \frac{\rho_{sp}}{\rho_g}$ is the ratio of the density of the solid particles to the species density of the gas. The internal energy per unit mass of the mixture may be written as

$$E_m = [K_p C_{sp} + (1 - K_p)C_v]T = C_{vm}T, \tag{7}$$

where C_{sp} is the specific heat of the solid particles, C_v is the specific heat of the gas at constant volume and C_{vm} is the specific heat of the mixture at constant volume. The specific heat of the mixture at constant pressure is

$$C_{pm} = K_p C_{sp} + (1 - K_p)C_p, \tag{8}$$

where C_p is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (Pai et al. [3], Pai [25], Marble [26]),

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma \frac{1 + \delta\beta'/\gamma}{1 + \delta\beta'},$$

(9)

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{K_p}{1-K_p}$ and $\beta' = \frac{C_{sp}}{C_v}$.

Now

$$C_{pm} - C_{vm} = (1 - K_p)(C_p - C_v) = (1 - K_p)R^*,$$

(10)

where $R^* = (C_p - C_v)$, neglecting the term $b^2\rho^2$ (Anisimov and spiner [7], Singh [27]). The internal energy per unit mass of the mixture is, therefore, given by

$$E_m = \frac{p(1 - Z)}{\rho(\Gamma - 1)[1 + b\rho(1 - K_p)]}.$$

(11)

3. Equations of Motion and Shock Conditions- Isothermal Flow

The equations of motion for a one-dimensional, unsteady, isothermal flow of a perfectly conducting mixture of a non-ideal gas and small solid particles in the presence of an azimuthal magnetic field, may in Eulerian co-ordinates be written as-

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{j\rho u}{r} = 0,$$

(12)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] = 0,$$

(13)

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (j - 1)\frac{hu}{r} = 0,$$

(14)

$$\frac{\partial T}{\partial r} = 0,$$

(15)

where u is the flow velocity, h is the azimuthal magnetic-field, μ is the magnetic permeability, r and t are the space and time co-ordinates and $j = 1$ or 2 for cylindrical or spherical symmetry.

From equation (3), we have

$$\left(\frac{\partial p}{\partial \rho}\right)_T = \frac{p\{1 + b\rho(1 - K_p)(2 - Z)\}}{\rho(1 - Z)\{1 + b\rho(1 - K_p)\}}.$$

(16)

Thus, the isothermal sound speed a_{is} is given by

$$a_{is}^2 = \frac{(1 - K_p)R^*T\{1 + b\rho(1 - K_p)(2 - Z)\}}{(1 - Z)^2}.$$

(17)

Equation (15) together with (3) give

$$\frac{p}{p_n} = \frac{\rho(1 - Z_n)[1 + b\rho(1 - K_p)]}{\rho_n(1 - Z)[1 + b\rho_n(1 - K_p)]}.$$ (18)

We consider that a strong shock (cylindrical or spherical) driven by a moving piston is propagated into the medium (a perfectly conducting mixture of a non-ideal gas and small solid particles) of constant density at rest, in the presence of an azimuthal magnetic field which is varying as $h = Ar^{-m}$, where A and m are constants.

The flow variables immediately ahead of the shock are

$$u = 0,$$
$$\rho = \rho_a = \text{constant},$$
$$h = h_a = Ar_s^{-m},$$
$$p = p_a = \frac{(1 - m)\mu A^2}{2mr_s^{2m}}, \qquad 0 < m < 1,$$ (19)

where r_s is the radius of the shock and variables with subscript 'a' denote their values immediately ahead of the shock.

From relation (6) we have

$$Z_a = \frac{K_p}{G_a(1 - K_p) + K_p},$$ (20)

where $G_a = \frac{\rho_{sp}}{\rho_{ga}}$ is the ratio of the density of the solid particles to the initial density of gas.

Jump conditions across the strong shock front are as follows:

$$\rho_n(W_s - u_n) = \rho_a W_s,$$
$$h_n(W_s - u_n) = h_a W_s,$$
$$p_n + \rho_n(W_s - u_n)^2 + \frac{\mu h_n^2}{2} = \rho_a W_s^2 + \frac{\mu h_a^2}{2},$$
$$E_{m_n} + \frac{p_n}{\rho_n} + \frac{1}{2}(W_s - u_n)^2 + \frac{\mu h_n^2}{\rho_n} = \frac{W_s^2}{2} + \frac{\mu h_a^2}{\rho_a},$$
$$\frac{Z_n}{\rho_n} = \frac{Z_a}{\rho_a},$$ (21)

where subscript 'n' refers to the values immediately behind the shock and $W_s = \frac{dr_s}{dt}$ is the velocity of the shock front.

From shock conditions (21), we have

$$u_n = (1 - \beta)W_s,$$

$$p_n = \left\{ (1 - \beta) + \frac{M_A^{-2}}{2}\left(1 - \frac{1}{\beta^2} \right) \right\}\rho_a W_s^2,$$

$$\rho_n = \frac{1}{\beta}\rho_a,$$

$$h_n = (1 - \beta)W_s,$$

$$(22)$$

where $\beta(0 < \beta < 1)$ is given by the relation

$$
\begin{aligned}
(\Gamma + 1)\beta^3 &+ [\{\bar{b}(1 - K_p) - 1\}(\Gamma - 1) - 2Z_a - \Gamma M_A^{-2}]\beta^2 \\
&+ [\{(Z_a - 2) + \Gamma - \bar{b}(1 - K_p)(\Gamma - 1)\}M_A^{-2} - \bar{b}(1 - K_p)(\Gamma - 1)]\beta \\
&+ \{\bar{b}(1 - K_p)(\Gamma - 1) + Z_a\}M_A^{-2} = 0,
\end{aligned}
\qquad (23)
$$

and $\bar{b} = b\rho_a$. Also, the Alfven Mach number M_A is defined by

$$M_A^2 = \frac{\rho_a W_s^2}{\mu h_a^2}. \qquad (24)$$

4. Self-similarity Transformations

The whole flow-field is bounded by the shock front and the piston. In the frame work of self -similarity (Sedeov [28]) the velocity $W_p = \frac{dr_p}{dt}$ of the piston is assumed to obey a power law given by

$$W_p = \frac{dr_p}{dt} = W_o\left(\frac{t}{t_o}\right)^n, \qquad (25)$$

where r_p is the radius of piston and t_o denotes the time at reference state, W_o is the piston speed at time t_o and n is a constant.

Due to the ambient magnetic field there is a restriction on n, $\frac{-1}{2} < n < 0$. Thus the piston velocity jumps almost instantaneously from zero to infinity, leading to the formation of a shock of high strength in the initial phase. Self-similarity requires that the velocity of shock should be proportional to the velocity of piston. Therefore

$$W_s = \frac{dr_s}{dt} = CW_o\left(\frac{t}{t_o}\right)^n, \qquad (26)$$

where C is dimensionless constant.

The time and space coordinates can be transformed into a dimensionless self-similarity variable η as

$$\eta = \frac{r}{r_s} = \frac{(n+1)t_o^n}{CW_O}\left(\frac{r}{t^{n+1}}\right). \tag{27}$$

The variable η possess the value '1' at the shock front and $\eta_p = \frac{r_p}{r_s}$ at the piston.

To obtain the self-similar solutions, we write the unknown variables in the following form (Steiner and Hirschler [5])

$$u = \frac{r}{t}U(\eta), \quad \rho = \rho_a D(\eta), \quad p = \rho_a \frac{r^2}{t^2}P(\eta), \quad Z = Z_a D(\eta), \quad \mu^{\frac{1}{2}}h = \rho_{a^{\frac{1}{2}}}\frac{r}{t}H(\eta), \tag{28}$$

where U, P, H and D all are functions of η only.

For existence of similarity solutions M_A should be constant. Thus using (19) and (26) in (24), we have $n + (n+1)m = 0$.

Now, since

$$0 < m < 1, \qquad -\frac{1}{2} < n < 0. \tag{29}$$

From equation (18) with aid of equation (28) and (22), we get

$$P(\eta) = \frac{(n+1)^2(1-\beta)(\beta - Z_a)D[1 + \bar{b}D(1 - K_p)][\beta^2 - \frac{1}{2}M_A^{-2}(1+\beta)]}{\eta^2(1 - Z_a D)\beta[\beta + \bar{b}(1 - K_p)]}. \tag{30}$$

Using similarity transformations (28) in the set of partial differential equation (12)-(14), we get the following system of ordinary differential equations with respect to η

$$[U - (n+1)]\frac{dD}{d\eta} + D\frac{dU}{d\eta} + (j+1)\frac{DU}{\eta} = 0, \tag{31}$$

$$[U - (n+1)]\frac{dH}{d\eta} + H\frac{dU}{d\eta} + (j+1)\frac{HU}{\eta} - \frac{H}{\eta} = 0, \tag{32}$$

$$[U - (n+1)]\frac{dU}{d\eta} + \frac{H}{D}\frac{dH}{d\eta} + Q\frac{dD}{d\eta} + \frac{U(U-1)}{\eta} + \frac{2H^2}{D\eta} = 0, \tag{33}$$

where

$$Q = Q(\eta)$$
$$= \frac{(n+1)^2(1-\beta)(\beta - Z_a)[1 + \bar{b}D(1 - K_p)(2 - Z_a D)][\beta^2 - \frac{1}{2}M_A^{-2}(1+\beta)]}{\eta^2(1 - Z_a D)^2\beta D[\beta + \bar{b}(1 - K_p)]}.$$

Solving equations (31) to (33), we get

$$\frac{dD}{d\eta} = \frac{XD}{\eta}, \tag{34}$$

$$\frac{dD}{d\eta} = \frac{-[U - (n + 1)]X - (j + 1)U}{\eta}, \tag{35}$$

$$\frac{dH}{d\eta} = \frac{H + HX\{U - (n + 1)\}}{\eta\{U - (n + 1)\}}, \tag{36}$$

where X is a function of similarity variable η given by

$$X(\eta) = \frac{DU[\{U - (n + 1)\}j - n] - 2H^2 - \frac{H^2}{\{U-(n+1)\}}}{[(H^2 + MD^2) - D\{U - (n + 1)\}^2]}.$$

After using similarity transformations (28), the shock boundary conditions (22) take the form

$$U(1) = (1 - \beta)(n + 1),$$

$$D(1) = \frac{1}{\beta},$$

$$P(1) = \left[(1 - \beta) + \frac{M_A^{-2}}{2}\left(1 - \frac{1}{\beta^2}\right)\right](n + 1)^2, \tag{37}$$

$$H(1) = (n + 1)\frac{M_A^{-1}}{\beta}.$$

The piston path coincides at $\eta_p = \frac{r_p}{r_s}$ with a particle path. Using (28) and (25) the relation

$$U(\eta_p) = (n + 1) \tag{38}$$

can be derived. In addition to the shock boundary condition (37) the kinematic condition (38) must be satisfied at the piston surface.

The ordinary differential equations (34) to (36) can be numerically integrated with shock boundary conditions (37) to obtain the solution of the problem.

5. Adiabatic Flow

In this section, we present the similarity solution for the adiabatic flow of a perfectly conducting mixture of a non-ideal gas and small solid particles behind a strong shock driven by a piston (cylindrical or spherical) moving according to power law in presence of an azimuthal magnetic field.

Here, the jump conditions are the same as the jump conditions of isothermal flow given in (21).

For adiabatic flow, equation (15) is replaced by (Vishwakarma [29], Steiner and Hirschler [5], Vishwakarma and Nath [10])

$$\frac{\partial E_m}{\partial t} + u\frac{\partial E_m}{\partial r} - \frac{p}{\rho^2}\left\{\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right\} = 0. \tag{39}$$

For isentropic change of state of gas-particle mixture, under the thermodynamic equilibrium condition, we may calculate the equilibrium sound speed of mixture from

$$a_m = \left(\frac{\partial p}{\partial \rho}\right)_s^{\frac{1}{2}} = \frac{[\{\Gamma + (2\Gamma - Z)b\rho(1 - K_p)\}(1 - K_p)R^*T]^{\frac{1}{2}}}{(1 - Z)^2}. \tag{40}$$

With the help of similarity transformations (28) equations (12), (13), (14) and (39) can be transformed as

$$[U - (n + 1)]\frac{dD}{d\eta} + D\frac{dU}{d\eta} + (j + 1)\frac{DU}{\eta} = 0, \tag{41}$$

$$[U - (n + 1)]\frac{dH}{d\eta} + H\frac{dU}{d\eta} + (j + 1)\frac{HU}{\eta} - \frac{H}{\eta} = 0, \tag{42}$$

$$[U - (n + 1)]\frac{dU}{d\eta} + \frac{H}{D}\frac{dH}{d\eta} + \frac{1}{D}\frac{dP}{d\eta} + \frac{U(U - 1)}{\eta} + \frac{2(P + H^2)}{D\eta} = 0, \tag{43}$$

$$\frac{dP}{d\eta} + N\frac{dD}{d\eta} + \frac{2(U - 1)P}{\eta[U - (n + 1)]} = 0, \tag{44}$$

where $N = N(\eta) = \frac{P\bar{b}D(1-K_p)(z_a D-2)-P-P(\Gamma-1)\{1+\bar{b}D(1-K_p)\}^2}{D(1-Z_a D)\{1+\bar{b}D(1-K_p)\}}$.

Solving equations (41) to (44) for $\frac{dU}{d\eta}$, $\frac{dD}{d\eta}$, $\frac{dH}{d\eta}$ and $\frac{dP}{d\eta}$, we have

$$\frac{dD}{d\eta} = \frac{YD}{\eta}, \tag{45}$$

$$\frac{dU}{d\eta} = \frac{-[U - (n + 1)]Y - (j + 1)U}{\eta}, \tag{46}$$

$$\frac{dH}{d\eta} = \frac{H + HY\{U - (n + 1)\}}{\eta\{U - (n + 1)\}}, \tag{47}$$

$$\frac{dP}{d\eta} = \frac{-NYD}{\eta} - \frac{2(U - 1)}{\eta\{U - (n + 1)\}}, \tag{48}$$

where $Y = Y(\eta) = \frac{DU[\{U-(n+1)\}j-n]-2(P+H^2)+\frac{2(U-1)P-H^2}{\{U-(n+1)\}}}{(H^2-ND)-D\{U-(n+1)\}^2}$.

In the adiabatic flow, the shock boundary conditions and kinematic condition at the piston will be same as in the case of isothermal flow.

To obtain the solution of problem behind the shock surface for adiabatic flow we can numerically integrate the ordinary differential equations (45) to (48)

with the shock boundary conditions (37). Normalising the variables u, p, ρ and h with their respective values at the shock, we obtain,

$$
\begin{aligned}
\frac{u}{u_n} &= \eta \frac{U(\eta)}{U(1)}, \\
\frac{p}{p_n} &= \eta^2 \frac{P(\eta)}{P(1)}, \\
\frac{h}{h_n} &= \eta \frac{H(\eta)}{H(1)}, \\
\frac{\rho}{\rho_n} &= \frac{D(\eta)}{D(1)}.
\end{aligned}
\tag{49}
$$

6. Results and Discussion

To obtain the solution of differential equations (34) to (36) in the isothermal flow and (45) to (48) in adiabatic flow, we use numerical integration by Runge - Kutta method of order four along with the shock boundary conditions (37). We start numerical integration from shock front ($\eta = 1$) and continue it until a value η_p (piston position) is approached, where

$$
U(\eta_p) = (n+1).
$$

For the purpose of numerical integration the values of constant parameters are taken to be (Pai et al. [3], Miura and Glass [1], Vishwakarma [29], Steiner and Hirschler [5]) $j = 2$; $\gamma = \frac{5}{3}$; $\beta' = 0.25$; $K_p = 0, 0.2$; $G_a = 1, 100$; $\bar{b} = 0, 0.1$; $M_A^{-2} = 0, 0.005, 0.01$ and $n = -0.15$. The value $j = 2$ corrensponds to spherical shock, $K_p = 0$ to the dust free case, $K_p = 0$, $\bar{b} = 0$ to the perfect gas case, $\bar{b} = 0.1$ to the non-ideal gas, $M_A^{-2} = 0$ to the non-magnetic case. $\beta' = 0.25$ is the typical value of ratio of specific heat of small solid particles and specific heat of gas at constant volume.

Table-1 shows the values of density ratio β across the shock front and the value of piston position η_p in both the cases, when the flow is isothermal and adiabatic, for different values of the parameters K_p, G_a, $\bar{b}$ and M_A^{-2}. Variation of flow variables in the flow-field behind the shock are shown in figures 1(a, b, c, d) for isothermal flow and in figures 2 (a, b, c, d) for adiabatic flow, with respect to dimensionless variable η. It is clear from Table-1 that the distance of piston is significantly affected due to the presence of dust particles.

Figures 1(a) and 2(a) show that the fluid velocity $\frac{u}{u_n}$ increases as we move from the shock front towards the piston. Figures 1(b, c) and 2(c) show that the

density $\frac{\rho}{\rho_n}$ and pressure $\frac{p}{p_n}$ in isothermal flow and pressure $\frac{p}{p_n}$ in the adiabatic flow remain almost constant in the region near to the shock front but decrease rapidly near the piston. Figure 2(b) shows that density $\frac{\rho}{\rho_n}$ decreases everywhere. Figures 1(d) and 2(d) show that as we move inward from the shock front, the magnetic field $\frac{h}{h_n}$ first increases slowly and then increases rapidly. It is also clear from figures 1(a, b, c, d) and 2(a, b, c, d) that the effects of parameters K_p, G_a and $\bar{b}$ on the flow profile of the variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ are enhanced due to the presence of magnetic field.

Effects of an increase in the value of M_A^{-2} (strength of ambient magnetic field) are

(i) to increase the value of β i.e. to decrease the shock strength (see Table 1);

(ii) to decrease η_p i.e. to increase the distance of piston from the shock front (see Table 1);

(iii) to decrease the fluid velocity $\frac{u}{u_n}$ and magnetic field $\frac{h}{h_n}$ at any point in the flow field behind the shock (see Figures 1(a, d) and 2(a, d)); and

(iv) to decrease the density and pressure in the region near to the shock front but to increase them near the piston (see Figures 1(b, c) and 2(b, c));

The above effects (i) and (i) can be physically interpreted as:

Since whole gas-particle mixture is perfectly conducting, all the constituent particles of the mixture interact through Lorentz force. Therefore in this case shock needs more energy to compress the mixture, resulting an increase in the distance between the shock and piston, hence a decrease in shock strength.

Effects of an increase in the value of G_a are

(i) to decrease the value of β i.e. to increase the shock strength, (see Table 1);

(ii) to increase η_p (see Table 1);

(iii) to decrease the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ (see Figures 1(a, b, c) and 2(a, b, c)); and

(iv) to increase the magnetic field $\frac{h}{h_n}$ (see Figures 1(d) and 2(d)).

Effects of an increase in the value of mass concentration of solid particles K_p are

(i) to increase the value of β at $G_a = 1$, but to decrease at $G_a = 100$ i.e. to decrease the shock strength at $G_a = 1$, but to increase at $G_a = 100$ (see Table 1);

(ii) to decrease η_p at $G_a = 1$, but to increase at $G_a = 100$ (see Table 1);

(iii) to increase the flow variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ at $G_a = 1$ but to decrease at $G_a = 100$ at any point in the flow-field behind the shock (see Figures 1(a, b, c) and 2(a, b, c)); and

(iv) to decrease the magnetic field $\frac{h}{h_n}$ at $G_a = 1$, but to increase at $G_a = 100$ (see Figures 1(d) and 2(d)).

The above effect (iii) is more significant in the presence of magnetic field, i.e. magnetic field enhances the effect of K_p on the profile of the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$.

The above effects (i) and (ii) can be physically interpreted as follows. In the case of $G_a = 1$, small solid particles of density equal to that of the gas in the mixture occupy a significant portion of the volume which lowers the compressibility of the mixture. Also, when $G_a = 1$, the mass concertation of solid particles K_p equal to their volumetric extension Z_a. Thus, by an increase in K_p there is an increase in the distance between the shock and piston, and a decrease in the shock strength. While for $G_a = 100$ (at constant K_p), there is high decrease in Z_a, which causes comparatively more compression in the mixture and decreases the distance between the shock and the piston, and therefore there is an increase in the shock strength.

Effects of an increase in the value of non-idealness parameter $\bar{b}$ are

(i) to increase the value of β i.e. to decrease the shock strength, (see Table 1).

(ii) to decrease η_p (see Table 1);

(iii) to increase the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ (see Figures1(a, b, c) and 2(a, b, c)); and

(iv) to decrease the magnetic field $\frac{h}{h_n}$ (see Figures 1(d) and 2(d)).

The above effects (i) and (ii) can be physical interpreted as:

An increase in the non-idealness parameter $\bar{b}$, the volume occupied by the gas molecules increases, which causes a decrease in the compressibility of the mixture and so an increase in the distance between the shock and piston. Hence a decrease in the shock strength.

The above effect (iii) is more significant in the presence of magnetic field, i.e. magnetic field enhances the effect of non-idealness of gas on the profile of the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$.

Isothermal Flow
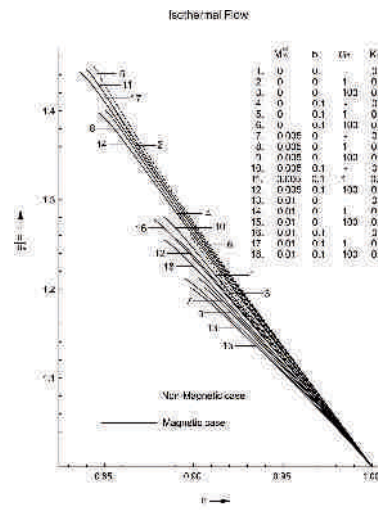
Figure 1(a): Variation of fluid velocity u/u. in the region behind the shock front.

Isothermal Flow
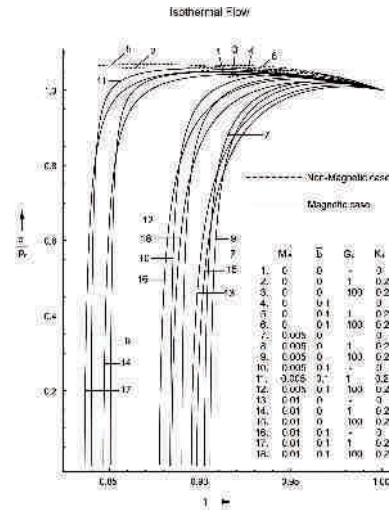
Figure 1(b): Variation of density ρ/ρ_s in the region behind the shock front.

Isothermal Flow
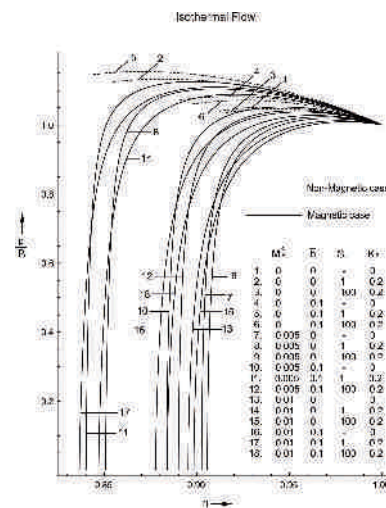
Figure 1(c): Variation of pressure p/p_s in the region behind the shock front.

Isothermal Flow
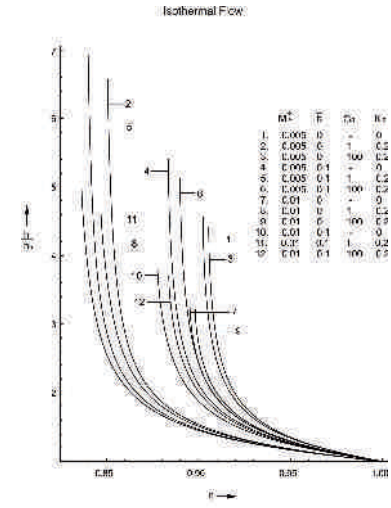
Figure 1(d): Variation of magnetic field h/h_s in the region behind the shock front.
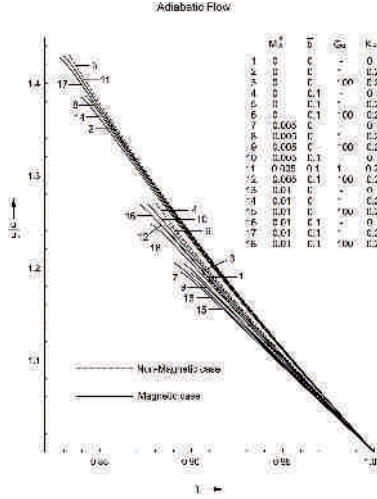
Figure 2(c): Variation of fluid velocity u/u_n in the region behind the shock front
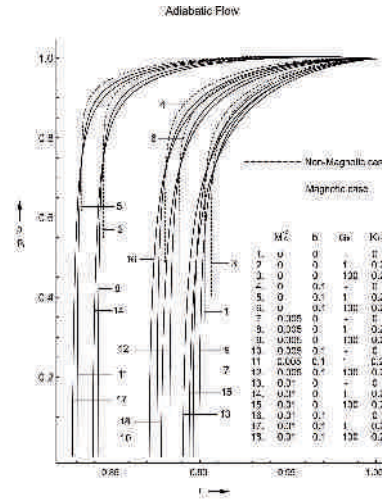
Figure 2(b): Variation of density ρ/ρ_n in the region behind the shock front
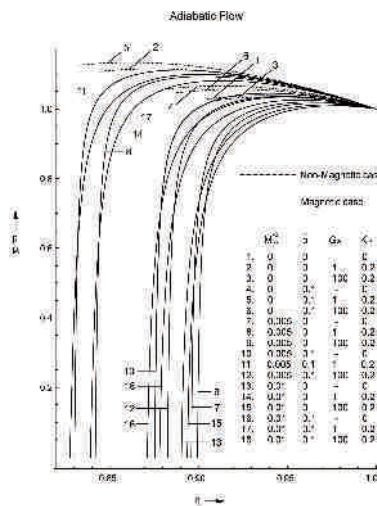
Figure 2(a): Variation of pressure p/p_n in the region behind the shock front
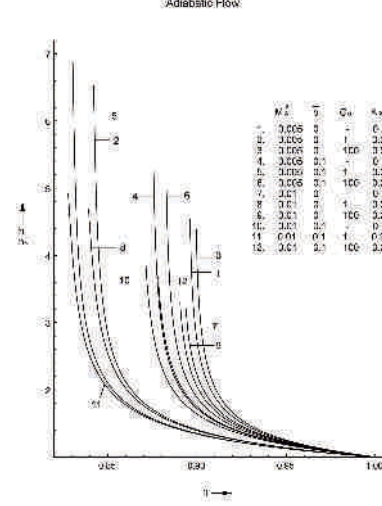
Figure 2(d): Variation of magnetic field h/h_n in the region behind the shock front

Mutual effects of the parameters K_p, G_a, $\bar{b}$ and M_A^{-2} on the shock strength $(\frac{1}{\beta})$, on the piston position (η_p) and on the profiles of the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$

(i) $\frac{1}{\beta}$ and η_p decrease by increasing $\bar{b}$, $\frac{1}{\beta}$ and η_p decrease by increasing K_p when $G_a = 1$; $\frac{1}{\beta}$ and η_p increase by increasing G_a. These effects of K_p, G_a and $\bar{b}$ on $\frac{1}{\beta}$ and η_p are reduced by increasing the value of M_A^{-2}, i.e. the effects of dust particles and the effects of non-idealness of gas on

the shock strength and on the piston position are reduced due to the presence of azimuthal magnetic field.

(ii) Flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ increase at any point in the flow-field by increasing $\bar{b}$, these flow variables increase by increasing K_p when $G_a = 1$; also these flow variables decrease by increasing G_a. These effects of K_p, G_a and $\bar{b}$ on the flow-variables become more impressive by an increase in the value of M_A^{-2} i.e. the effect of parameters K_p, G_a and $\bar{b}$ on the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ are enhanced due to the presence of magnetic-field.

(iii) $\frac{1}{\beta}$ and η_p decrease by increasing M_A^{-2}, but this effect of M_A^{-2} is reduced by by increasing the value of K_p or the value of $\bar{b}$, i.e. the effect of magnetic-field on $\frac{1}{\beta}$ and on η_p is reduced due to the presence of dust particles and the non-idealness of gas.

(iv) $\bar{b}$ has decaying effect on $\frac{1}{\beta}$ and η_p, but this effect is lessimpressive when medium is dusty ($K_p = 0.2$), i.e. the effect of non-idealness of gas on $\frac{1}{\beta}$ and on η_p is reduced due to the presence of dust particles.

(v) K_p has decaying effect on $\frac{1}{\beta}$ and η_p when $G_a = 1$, whereas G_a has increasing effect on both $\frac{1}{\beta}$ and on η_p. These effects of K_p and G_a are reduced when the gas is non-ideal in the mixture ($\bar{b} = 0.1$), i.e. the effects of dust particles on $\frac{1}{\beta}$ and η_p is reduced due to the non-idealness of the gas.

Comparison between isothermal and adiabatic flows

(i) The density remains almost constant in the flow-field behind the shock when there is no magnetic field, in the case of isothermal flow, whereas in the case of adiabatic flow it decreases everywhere (see Figures 1(b) and 2(b)).

(ii) The piston position η_p is greater in the case of isothermal flow in comparison with that in the case of adiabatic flow, i.e. distance of piston from the shock front is less in the case of isothermal flow in comparison with that in the case of adiabatic flow (see table 1).

(iii) The strength of ambient magnetic field M_A^{-2} enhances the effects of K_p, G_a and $\bar{b}$ on the density profile more significantly in the case of isothermal flow in comparison with that in the case of adiabatic flow.

Table 1: Values of the density ratio β across the shock front and the position of piston η_p at different values of parameters K_p, G_a, $\bar{b}$ and M_A^{-2} for $j = 2$, $\gamma = \frac{5}{3}$, $\beta' = 0.25$ and $n = -0.15$.

M_A^{-2}	$\bar{b}$	G_a	K_p	β	η_p	
					Isothermal flow	Adiabatic flow
0	0	-	0	0.25	0.91236	0.9026775
0	0	1	0.2	0.391045	0.85498	0.8450835
0	0	100	0.2	0.2407042	0.915568	0.9064225
0	0.1	-	0	0.3065522	0.89106	0.8803975
0	0.1	1	0.2	0.4176798	0.84414	0.8330445
0	0.1	100	0.2	0.2879469	0.89785	0.8880115
0.005	0	-	0	0.255571	0.9024595	0.8961045
0.005	0	1	0.2	0.3925079	0.8506335	0.8420475
0.005	0	100	0.2	0.2465791	0.9055355	0.8996055
0.005	0.1	-	0	0.309912	0.8835385	0.875553
0.005	0.1	1	0.2	0.4189384	0.8400425	0.8302835
0.005	0.1	100	0.2	0.2917437	0.8899585	0.8827765
0.01	0	-	0	0.2610386	0.8952085	0.8905195
0.01	0	1	0.2	0.3940022	0.8468465	0.8392345
0.01	0	100	0.2	0.252331	0.8982025	0.8938455
0.01	0.1	-	0	0.3132893	0.8776525	0.8712675
0.01	0.1	1	0.2	0.4202221	0.8364665	0.8277095
0.01	0.1	100	0.2	0.2955502	0.8838325	0.8781825

7. Conclusion

In the present work, similarity solutions are obtained for the isothermal and adiabatic flows of a perfectly conducting mixture of a non-ideal gas and small solid particles, behind a strong shock driven out by a piston moving according to power law in the presence of an azimuthal magnetic field. It is observed that the parameters of K_p, G_a, $\bar{b}$ and M_A^{-2} have significant effects on the shock strength ($\frac{1}{\beta}$), piston position (η_p) and on the flow profiles of the variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$. On the basis of this work one may draw the following conclusions:

(i) The shock strength ($\frac{1}{\beta}$) and the value of piston position (η_p) decay due to the presence of magnetic-field.

(ii) The effects of dust parameters K_p, G_a and the non-idealness parameter of the gas $\bar{b}$ on the shock strength and on the piston position are reduced

due to the presence of magnetic field, while the effects of these parame-
ters on the profiles of the flow-variables $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$ and $\frac{p}{p_n}$ are enhanced due
to the presence of magnetic-field.

(iii) The density and pressure abruptly decrease to zero near the piston due
to presence of the magnetic field.

(iv) Effects of magnetic field on the shock strength and on the piston position
η_p are reduced due to the presence of the dust particles as well as the
non-idealness of gas.

(v) The effects of non-idealness of gas on the shock strength and piston
position are reduced due to the presence of dust particles.

(vi) Due to the presence of the non-idealness of the gas the shock strength
$\frac{1}{\beta}$ and the piston position η_p are decreased. Also, the non-idealness of
the gas reduces the effect of the presence of dust particles on $\frac{1}{\beta}$ and η_p.

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