J. T. S.
Vol. 10 (2016), pp.69-77
https://doi.org/10.56424/jts.v10i01.10573

A Study on Atomic Hausdorff and Atomic Regular Measure Manifolds

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(Received: February 9, 2017)

Abstract

In this paper atomic Hausdorff and atomic regular measure manifolds are generated by using the tools of inverse function theorem for measure manifold and pullback function on a measure manifold. Also, we investigate the possibility of generating a broader class of atomic measure manifolds.

Keywords: Atomic measure space, Measure atlas, Measure chart, Measure invariant transformation, Atomic Hausdorff space, Atomic regular space, Atomic measure manifold.
MSC(2010):28-XX,49Q15,54-XX,57NXX,58-XX,54H15,57SXX

1. Introduction

It is observed that, the topological structure on (R^n, τ) generates an algebraic structure σ-algebra Σ, that transforms the topological space[5][8] into a measurable space (R^n, τ, Σ)[4][6]. By admitting a suitable measure function μ, the measurable space is designated as a measure space (R^n, τ, Σ, μ). As a particular case, by introducing an atomic measure μ_A [7], a measure space is re-designated as atomic measure space $(R^n, \tau, \Sigma, \mu_A)$. In this paper we study the atomic Hausdorff and atomic regular measure manifolds by using the tools of inverse function theorem for measure manifold and pullback function on a measure manifold[2]. Also, we investigate the possibility of generating a broader class of atomic measure manifolds.

2. Preliminaries

Definition 2.1. ***:Measure Space*** (R^n, τ, Σ, μ)
A measure μ on a measurable space (R^n, τ, Σ) is a function

$\mu : \Sigma \to [0, \infty]$

such that
a) $\mu(\emptyset)$=0
b) If $\{A_i \in \Sigma$ for i $\in N\}$ is a countable disjoint collection of sets in Σ, then $\mu(\cup_{i=1}^{\infty} A_i)$=$\Sigma_{i=1}^{\infty}$ $\mu(A_i)$........(Countable additivity).
Therefore the space (R^n, τ, Σ, μ) is called a measure space.

Definition 2.2. *: Sets of Measure Zero*
A set of measure zero or a null set is a measurable set N such that $\mu(N)$=0.
A property which holds for all x $\in R^n \backslash N$ where N is a set of measure zero is said to hold almost everywhere (or μ-a.e.).
In general, a subset of a measure zero need not be measurable but if it is, it must have measure zero.

Definition 2.3. *: Complete Measure Space*
A measure space (R^n, τ, Σ, μ) is complete if every subset of a set N of measure zero is measurable.
that is, A $\subseteq$ N: $\mu(N)$=0,then A is measurable and $\mu(A)$=0.

Definition 2.4. *:Measure Preserving Transformation/Invariant Measure*
Let $(R^n, \tau_1, \Sigma_1, \mu_1)$ and $(R^m, \tau_2, \Sigma_2, \mu_2)$ be measure spaces and $T : (R^n, \tau_1, \Sigma_1, \mu_1) \to$

$(R^m, \tau_2, \Sigma_2, \mu_2)$ be a measurable transformation. The transformation T is said to be measure preserving if for all $A \in \Sigma_2$ we have that
$\mu_1(T^{-1}(A))$=$\mu_2(A)$

Note:
1) In this paper the smallest open sets G_δ and closed sets F_σ are Borel sets belonging to the σ-algebra Σ and the Borel open sets are denoted by the letters A, B and Borel closed sets by E,F.
2) Now onwards we denote μ for general measure space (R^n, τ, Σ, μ) and μ_A for Atomic measure space $(R^n, \tau, \Sigma, \mu_A)$. Now we shall introduce the concept of atomic measure on measure space $(R^n, \tau, \Sigma, \mu_A)$.

Definition 2.5. *:Atom*
Given a measurable space (R^n, τ, Σ) and a measure μ_A on that space, a set A in Σ is called an atom if $\mu_A(A) > 0$ and for any measurable subset B of A has either $\mu_A(B)$=0 or $\mu_A(B)$=$\mu_A(A)$.

Definition 2.6. *: **Atomic Measure Space*** $(R^n, \tau, \Sigma, \mu_A)$
Let $(R^n, \tau, \Sigma, \mu_A)$ be a measure space. A measure μ_A is called atomic if every set A $\in \Sigma$ such that $\mu_A(A) > 0$ contains an atom that is for any measurable subset B of A has either $\mu_A(B)$=0 or $\mu_A(B)$=$\mu_A(A)$.The measure space $(R^n, \tau, \Sigma, \mu_A)$ with atomic measure μ_A is called an atomic measure space.

In [3] S. C. P. Halakatti has introduced the concepts of measure chart, measure atlas, measurable and measure manifolds.

Definition 2.7. ***:Measure Chart***
A measurable chart $((U, \tau_{1/U}, \Sigma_{1/U}), \phi)$ equipped with a measure $\mu_{1/U}$ is called a measure chart, denoted by $((U, \tau_{1/U}, \Sigma_{1/U}, \mu_{1/U}), \phi)$ is called a measure chart.

Definition 2.8. *: **Measure Atlas***
By an R^n measure atlas of class C^k on M we mean a countable collection $(A, \tau_{1/A}, \Sigma_{1/A}, \mu_{1/A})$ of n-dimensional measure charts $((U_n, \tau_{1/U_n}, \Sigma_{1/U_n}, \mu_{1/U_n}),$ $\phi_n)$ for all n $\in$ N on $(M, \tau_1, \Sigma_1, \mu_1)$ satisfying the following conditions:
(a_1) $U_{n=1}^{\infty}$ $((U_n, \tau_{1/U_n}, \Sigma_{1/U_n}, \mu_{1/U_n}), \phi_n)$=M
that is, the countable union of the measure charts in $(A, \tau_{1/A}, \Sigma_{1/A}, \mu_{1/A})$ cover $(M, \tau_1, \Sigma_1, \mu_1)$
(a_2) for any pair of measure charts $((U_n, \tau_{1/U_n}, \Sigma_{1/U_n}, \mu_{1/U_n}), \phi_n)$ and $((U_m, \tau_{1/U_m},$ $\Sigma_{1/U_m}, \mu_{1/U_m}), \phi_m)$ in $(A, \tau_{1/A}, \Sigma_{1/A}, \mu_{1/A})$, the transition maps $\phi_n \circ \phi_m^{-1}$ and $\phi_m \circ \phi_n^{-1}$ are
(1) differentiable maps of class C^k (k$\geq$1)that is,
$\phi_n \circ \phi_m^{-1}$:$\phi_m(U_n \cap U_m) \to \phi_n(U_n \cap U_m) \subseteq (R^n, \tau, \Sigma, \mu)$
$\phi_m \circ \phi_n^{-1}$:$\phi_n(U_n \cap U_m) \to \phi_m(U_n \cap U_m) \subseteq (R^n, \tau, \Sigma, \mu)$
are differential maps of class C^k (k$\geq$1)
(2) Measurable that is, these two transition maps $\phi_n \circ \phi_m^{-1}$ and $\phi_m \circ \phi_n^{-1}$ are measurable functions if,
(c) For any measurable subset K $\subseteq \phi_n(U_n \cap U_m)$,
$(\phi_n \circ \phi_m^{-1})^{-1}$(K) $\in \phi_m(U_n \cap U_m)$ is also measurable.
(d) For any measurable subset S $\subseteq \phi_m(U_n \cap U_m)$,
$(\phi_m \circ \phi_n^{-1})^{-1}$(S) $\in \phi_n(U_n \cap U_m)$ is also measurable.
(a_3) For any two measure atlases $(A_1, \tau_{1/A_1}, \Sigma_{1/A_1}, \mu_{1/A_1})$ and $(A_2, \tau_{1/A_2}, \Sigma_{1/A_2},$ $\mu_{1/A_2})$ we say that a mapping $T : A_1 \to A_2$ is measurable if $T^{-1}(E)$ is measurable for every measurable chart $E = ((U, \tau_{1/U}, \Sigma_{1/U}), \phi) \subset (A_2, \tau_{1/A_2}, \Sigma_{1/A_2}, \mu_{1/A_2})$, and the mapping is measure preserving if $\mu_{1/A_1}(T^{-1}(E))$= $\mu_{1/A_2}(E)$,where A_1 $\sim A_2$ and μ_{1/A_1}=μ_{1/A_2}.
Then we call T a transformation.
(a_4) If a measurable transformation $T : A \to A$ preserves a measure μ_1 then we

say that μ_1 is T-invariant (or invariant under T). If T is invariant and if both T and T^{-1} are measurable and measure preserving then we call T an invertible measure preserving transformation.

Definition 2.9. *: **Measure Manifold***
A non-empty set M equipped with differentiable structure, topological structure and algebraic structure σ-algebra is called measurable manifold. A measure μ_1 defined on (M, τ_1, Σ_1) and the quadruple $(M, \tau_1, \Sigma_1, \mu_1)$ is called measure manifold.

Definition 2.10. *: **Atomic** T_2**-Space/**$AT_2 - Space$*
An atomic measure space $(R^n, \tau, \Sigma, \mu_A)$ is said to be an atomic T_2-Space if for every p, q $\in (R^n, \tau, \Sigma, \mu_A)$ with q $\neq$ p $\exists$ atomic Borel open sets A, $B \in \Sigma$ such that $p \in A$ and $p \notin B$, $q \in B$ and $q \notin$ Asuch that $A \cap B$=$\emptyset$ satisfying the atomic measure conditions:
i) for $p \in A$ and $p \notin B \; \exists \; C \in \Sigma$, $C \subset A$ and has either $\mu_A(C)$=0 or $\mu_A(C)$=$\mu_A(A)$
ii) for $q \in B$ and $q \notin$ A $\exists$ $D \in \Sigma$, $D \subset B$ and has either $\mu_A(D)$=0 or $\mu_A(D)$=$\mu_A(B)$ and $\mu_A(A \cap B)$=$\mu_A(\emptyset)$=0.

Definition 2.11. *: **Atomic Regular Space/AR-Space***
An atomic measure space $(R^n, \tau, \Sigma, \mu_A)$ is said to be atomic regular, if for any point p and closed set F in $(R^n, \tau, \Sigma, \mu_A)$ such that p $\notin F$, there exists atomic Borel open sets A and $B \in \Sigma$ such that $p \in A$, $F \subset B$ and $A \cap B$=$\emptyset$ satisfying the atomic measure conditions:
i) for $p \in A \; \exists \; C \in \Sigma$, $C \subset A$ and has either $\mu_A(C)$=0 or $\mu_A(C)$=$\mu_A(A)$
ii) for $F \subset B \; \exists \; D \in \Sigma$, $F \subset B$ and $D \subset B$ and has either $\mu_A(D)$=0 or $\mu_A(D)$=$\mu_A(B)$ and $\mu_A(A \cap B)$=$\mu_A(\emptyset)$=0.

Proposition 2.1. For every topological homeomorphism with reference to topological structure τ_1 on measure space $(R^n, \tau_1, \Sigma_1, \mu_1)$ that is $T : (R^n, \tau_1, \Sigma_1, \mu_1) \to (R^m, \tau_2, \Sigma_2, \mu_2)$ there exists a measure invariant transformation with reference to σ-algebraic structure Σ_1 on a measure space $(R^n, \tau_1, \Sigma_1, \mu_1)$.

Theorem 2.1. On a measure spae $(R^n, \tau_1, \Sigma_1, \mu_1)$ if Hausdorff(T_2) property is a topological invariant under homeomorphism then it is invariant under measure transformation.

Theorem 2.2. On a measure spae $(R^n, \tau_1, \Sigma_1, \mu_1)$ if regularity is a topological invariant under homeomorphism then it is invariant under measure transformation.

Theorem 2.3. Atomic Hausdorff property is invariant under measure transformation.

Theorem 2.4. Atomic regularity is invariant under measure transformation.

3. Main Results

The interesting feature of this research is that the atomic measure space $(R^n, \tau, \Sigma, \mu_A)$ carries two structure one is topological and another is σ-algebra on which atomic measure μ_A is well dfined. In this paper it will be shown that the extended topological properties on atomic measure space $(R^n, \tau, \Sigma, \mu_A)$ are not only invariant under homeomorphism but they are invariant under measure invariant transformation also. Such results have deeper implications in the study of measure manifold introduced by S. C. P. Halakatti[3]. The measure manifold confirms the measure invariance of the following extended topological properties.

Theorem 3.1. *:* If $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ is atomic Hausdorff space then the C^∞ measurable homeomorphism $\phi : (M, \tau, \Sigma, \mu_A) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ generates a atomic Hausdorff measure manifold (M, τ, Σ, μ_A).

Proof: Let $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ be atomic Hausdorff space.
Let $\phi : (M, \tau, \Sigma, \mu_A) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ be a C^∞ measurable homeomorphism and measure invariant function.
Now let us show that there exists a mapping, $\phi : (M, \tau, \Sigma, \mu_A) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ which preserves the atomic Hausdorff property under measure transformation.
since $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ is atomic Hausdorff space, for every points p, $q \in R^n$ $\exists$ atomic Borel open sets A, $B \in \Sigma_1$ such that $p \in A$, $p \notin B$ and $q \in B$, $q \notin A$ such that $A \cap B = \emptyset$ and satisfy the atomic measure conditions:
i) for $p \in A$ and $p \notin B$ $\exists$ $C \in \Sigma$, $C \subset A$ and has either $\mu_{A_1}(C)$=0 or $\mu_{A_1}(C)$=$\mu_{A_1}(A)$
ii) for $q \in B$ and $q \notin A$ $\exists$ $D \in \Sigma$, $D \subset B$ and has either $\mu_{A_1}(D)$=0 or $\mu_{A_1}(D)$=$\mu_{A_1}(B)$ and $\mu_{A_1}(A \cap B)$=$\mu_{A_1}(\emptyset)$=0.
By theorem 3[1], theorem 3.6[2] and by the existence of the inverse function theorem for measure spaces, for every C^∞ function $\phi : U \subset M \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ $\exists$ a C^∞ inverse function $\phi^{-1} : V \subset (R^n, \tau_1, \Sigma_1, \mu_{A_1}) \longrightarrow U \subset M$
since $\phi : U\ (= \cap_{i=1}^{\infty} U_i) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ is a bijective map, $\forall$ p, $q \in (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ $\exists$ $\phi^{-1}(p)$, $\phi^{-1}(q) \in (M, \tau, \Sigma, \mu_A)$ [by theorem 3.6[2]]
$\Rightarrow$ ϕ and ϕ^{-1} are homeomorphism, measurable and measure invariant.
Then the pair (U, ϕ) where U is a subset of M and ϕ a homeomorphism, measurable and measure invariant is an n-dimensional atomic measure chart in M generated by ϕ such that for every points $\phi^{-1}(p)$, $\phi^{-1}(q) \in (M, \tau, \Sigma, \mu_A)$ with

$\phi^{-1}(p) \neq \phi^{-1}(q)$ and atomic measure charts (U_1, ϕ_1), $(U_2, \phi_2) \in \Sigma$ such that $\phi^{-1}(p) \in (U_1, \phi_1)$ and $\phi^{-1}(p) \notin (U_2, \phi_2)$, $\phi^{-1}(q) \in (U_2, \phi_2)$ and $\phi^{-1}(q) \notin (U_1, \phi_1)$ such that $U_1 \cap U_2 = \phi$ satisfying the atomic measure conditions:
i) for $\phi^{-1}(p) \in (U_1, \phi_1)$ and $\phi^{-1}(p) \notin (U_2, \phi_2)$ $\exists$ $(U_3, \phi_3) \in \Sigma$, $(U_3, \phi_3) \subset (U_1, \phi_1)$ and has either $\mu_A(U_3)$=0 or $\mu_A(U_3)$=$\mu_A(U_1)$
ii) for $\phi^{-1}(q) \in (U_2, \phi_2)$ and $\phi^{-1}(q) \notin (U_1, \phi_1)$ $\exists$ $(U_4, \phi_4) \in \Sigma$, $(U_4, \phi_4) \subset (U_2, \phi_2)$ and has either $\mu_A(U_4)$=0 or $\mu_A(U_4)$=$\mu_A(U_2)$ and $\mu_A(U_1 \cap U_2)$=$\mu_A(\emptyset)$=0.
Hence the proof.

Theorem 3.2. *:* If $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ is atomic regular space then the C^∞ measurable homeomorphism $\phi : (M, \tau, \Sigma, \mu_A) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ generates a atomic regular measure manifold (M, τ, Σ, μ_A).

Proof: Let $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ be atomic regular space.
Let $\phi : (M, \tau, \Sigma, \mu_A) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ be a C^∞ measurable homeomorphism and measure invariant function.
Now let us show that there exists a mapping, $\phi : (M, \tau, \Sigma, \mu_A) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ which preserves the atomic regular property under measure transformation.
since $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ is atomic regular space, for every point p and closed set $F \in R^n$ such that $p \notin F$, there exists atomic Borel open sets A and $B \in \Sigma_1$ such that $p \in A$, $F \subset B$ and $A \cap B = \emptyset$ satisfying the atomic measure conditions:
i) for $p \in A$ $\exists$ $C \in \Sigma_1$, $C \subset A$ and has either $\mu_{A_1}(C)$=0 or $\mu_{A_1}(C)$=$\mu_{A_1}(A)$.
ii) for $F \subset B$ $\exists$ $D \in \Sigma_1$, $F \subset B$ and has either $\mu_{A_1}(D)$=0 or $\mu_{A_1}(D)$=$\mu_{A_1}(B)$ and $\mu_{A_1}(A \cap B)$=$\mu_{A_1}(\emptyset)$=0.
By theorem 4[1], theorem 3.6[2] and by the existence of the inverse function theorem for measure spaces, for every C^∞ function $\phi : U \subset M \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ $\exists$ a C^∞ inverse function $\phi^{-1} : V \subset (R^n, \tau_1, \Sigma_1, \mu_{A_1}) \longrightarrow U \subset M$
since $\phi : U\ (= \cap_{i=1}^{\infty} U_i) \longrightarrow (R^n, \tau_1, \Sigma_1, \mu_{A_1})$ is a bijective map, such that $\forall$ p and Borel closed set F in $(R^n, \tau_1, \Sigma_1, \mu_{A_1})$ $\exists$ $\phi^{-1}(p)$ and $\phi^{-1}(F)$ in (M, τ, Σ, μ_A) [by theorem 3.6[2]]
$\Rightarrow$ ϕ and ϕ^{-1} are homeomorphism, measurable and measure invariant.
Then the pair (U, ϕ) where U is a subset of M and ϕ a homeomorphism, measurable and measure invariant is an n-dimensional atomic measure chart in M generated by ϕ such that for every point $\phi^{-1}(p)$ and Borel closed set $\phi^{-1}(F)$ in (M, τ, Σ, μ_A) with $\phi^{-1}(p) \neq \phi^{-1}(F)$ and atomic measure charts (U_1, ϕ_1), (U_2, ϕ_2) $\in \Sigma$ such that $\phi^{-1}(p) \in (U_1, \phi_1)$ and $\phi^{-1}(F) \subset (U_2, \phi_2)$ with $U_1 \cap U_2 = \phi$ satisfying the atomic measure conditions:
i) for $\phi^{-1}(p) \in (U_1, \phi_1)$ $\exists$ $(U_3, \phi_3) \in \Sigma_1$, $(U_3, \phi_3) \subset (U_1, \phi_1)$ and has either $\mu_A(U_3)$=0 or $\mu_A(U_3)$=$\mu_A(U_1)$

ii) for $\phi^{-1}(F) \subset (U_2, \phi_2) \exists (U_4, \phi_4) \in \Sigma_1$, $\phi^{-1}F \subset (U_2, \phi_2)$ and $(U_4, \phi_4) \subset (U_2, \phi_2)$ and has either $\mu_A(U_4)$=0 or $\mu_A(U_4)$=$\mu_A(U_2)$ and $\mu_A(U_1 \cap U_2)$=$\mu_A(\emptyset)$=0. Hence the proof.

Definition 3.1. *: Atomic Measure Condition on Measurable Manifold*
Given a measurable manifold (M, τ, Σ) admitting a atomic measure μ_A, a measurable chart (U, ϕ) in M is called an atom if $\mu_A(U) > 0$ and satisfying the following condition:
$\mu_A(V)$=0 or $\mu_A(V)$=$\mu_A(U)$, for any measurable subset $V \subseteq (U, \phi)$.

Definition 3.2. *: Atomic Measure Chart*
A measure $\mu_{A/_U}$ on a measurable chart $((U, \tau_{1/U}, \Sigma_{1/U}), \phi)$ is called atomic measure chart, denoted by $((U, \tau_{1/U}, \Sigma_{1/U}, \mu_{A/U}), \phi)$ satisfying the following atomic measure conditions:
(i) for any measurable chart $((V, \tau_{1/V}, \Sigma_{1/V}), \phi) \subseteq ((U, \tau_{1/U}, \Sigma_{1/U}), \phi)$ has either $\mu_A(V)$=0 or $\mu_A(V)$=$\mu_A(U)$
(ii) ϕ is homeomorphism,
(iii) ϕ is measurable,
(iv) ϕ is atomic measure invariant.
then, the structure $((U, \tau_{1/U}, \Sigma_{1/U}, \mu_{A/U}), \phi)$ is called atomic measure chart.

Definition 3.3. *: Atomic Measure Atlas*
By an R^n atomic measure atlas of class C^k on M, we mean a countable collection $(A, \tau_{1/_A}, \Sigma_{1/_A}, \mu_{A/_A})$ of n-dimensional atomic measure charts $((U_i, \tau_{1/U_i}, \Sigma_{1/U_i},$ $\mu_{A/U_i}), \phi_i)$ for all $i \in$ I on $(M, \tau_1, \Sigma, \mu_A)$ satisfying the following atomic measure conditions:
(a_1) for any atomic measure chart
$((V, \tau_{1/V}, \Sigma_{1/V}, \mu_{A/V}), \phi) \subseteq ((U, \tau_{1/U}, \Sigma_{1/U}, \mu_{A/V}), \phi)$ has either $\mu_A(V)$=0 or $\mu_A(V)$=$\mu_A(U)$
(a_2) $\cup_{i=1}^{\infty} ((U_i, \tau_{1/U_i}, \Sigma_{1/U_i}, \mu_{A/U_i}), \phi_i) =$ M
that is, the countable union of the atomic measure charts in $(A, \tau_{1/_A}, \Sigma_{1/_A}, \mu_{A/_A})$ cover $(M, \tau_1, \Sigma_1, \mu_A)$.
(a_3) for any pair of atomic measure charts $((U_i, \tau_{1/U_i}, \Sigma_{1/U_i}, \mu_{A/U_i}), \phi_i)$ and $((U_j, \tau_{1/U_j}, \Sigma_{1/U_j}, \mu_{A/U_j}), \phi_j)$ in $(A, \tau_{1/_A}, \Sigma_{1/_A}, \mu_{A/_A})$, the transition maps $\phi_i \circ \phi_j^{-1}$ and $\phi_j \circ \phi_i^{-1}$ are
(1) differentiable maps of class C^k(k≥1)that is ,
$\phi_i \circ \phi_j^{-1}$: $\phi_j(U_i \cap U_j) \rightarrow \phi_i(U_i \cap U_j) \subseteq (R^n, \tau, \Sigma, \mu_A)$
$\phi_j \circ \phi_i^{-1}$: $\phi_i(U_i \cap U_j) \rightarrow \phi_j(U_i \cap U_j) \subseteq (R^n, \tau, \Sigma, \mu_A)$ are differential maps of class C^k(k≥1),

(2) Measurable and atomic measure invariant. That is,
(i) any Borel subset $K \subseteq \phi_i(U_i \cap U_j)$ is measurable and atomic measure invariant then $(\phi_i \circ \phi_j^{-1})^{-1}(K) \in \phi_j(U_i \cap U_j)$ is also measurable and atomic measure invariant, that is, $\mu_A((\phi_i \circ \phi_j^{-1})^{-1}(K))$=$\mu_A(K)$.
(ii) for any Borel subset $S \subseteq \phi_j(U_i \cap U_j)$, is measurable and atomic measure invariant then $(\phi_j \circ \phi_i^{-1})^{-1}(S) \in \phi_i(U_i \cap U_j)$ is measurable and atomic measure invariant, that is $\mu_A(S)$=$\mu_A(\phi_j \circ \phi_i^{-1})^{-1}(S)$.
(a_4) Any two atlases $(A_1, \tau_{1/_{A_1}}, \Sigma_{1/_{A_1}}, \mu_{A/_{A_1}})$ and
$(A_2, \tau_{1/_{A_2}}, \Sigma_{1/_{A_2}}, \mu_{A/_{A_2}})$ are compatible on (M, τ, Σ, μ_A) satisfying the two equivalence relations:
(i) $A_1 \sim A_2$, iff $A_1 \cup A_2 \in A^k(M)$
(ii) $A_1 \sim A_2$, iff $\mu_A(A_1)$=$\mu_A(A_2)$.

Definition 3.4. ***: Atomic Measure Manifold***
A measure manifold (M, τ, Σ, μ_A) admitting atomic measure μ_A with two equivalence relations $A_1 \sim A_2$, iff $A_1 \cup A_2 \in A^k(M)$ and $\mu_A(A_1) = \mu_A(A_2)$ is called atomic measure manifold (M, τ, Σ, μ_A).

4. Conclusion

In our study, it is observed that the different categories like atomic completely regular measure manifold, atomic normal measure manifold, atomic quotient measure manifold, atomic network measure manifolds etc., can be studied. Also, we can construct different categories of atomic measure manifold (M, τ, Σ, μ_A) depending on the different topological structures by using the tools of inverse function theorem and pull back function on measure manifold.

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