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Submanifolds of Kenmotsu Manifolds and Ricci Solitons

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Abstract

The present paper deals with the study of Ricci solitons on submanifolds, specially invariant and anti-invariant submanifolds of Kenmotsu manifolds with respect to Riemannian connection, quarter symmetric metric connection and quarter symmetric non-metric ϕ-connection, respectively.

Keywords: Invariant submanifold, anti-invariant submanifold, Ricci solitons, quarter symmetric metric connection, quarter symmetric non-metric ϕ- connection.
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1. Introduction

In 1982, Hamilton [11] introduced the notion of Ricci flow to find the canonical metric on a smooth manifold. Then Ricci flow has become a powerful tool for the study of Riemannian manifolds, especially for those manifolds with positive curvature. Perelman ([25], [26]) used Ricci flow and its surgery to prove Poincare conjecture. The Ricci flow is an evolution equation for metrics on a Riemannian manifold defined as follows:

$$\frac{\partial}{\partial t} g_{ij}(t) = -2R_{ij}$$

A Ricci soliton emerges as the limit of the solutions of the Ricci flow. A solution to the Ricci flow is called Ricci soliton if it moves only by one parameter group of diffeomorphism and scaling. A Ricci solitons (g, V, λ) on a Riemannian manifold (M, g) is a generalization of an Einstein metric such that [12]

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{1.1}$$

where S is Ricci tensor, $\pounds_V$ is the Lie derivative operator along the vector field V on M and λ is a real number. The Ricci soliton is said to be shrinking, steady and expanding according as λ is negative, zero and positive respectively.

During the last two decades, the geometry of Ricci solitons has been the focus of attention of many mathematicians. In particular it has become more important after Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904. In [27] Sharma studied the Ricci solitons in contact geometry. Thereafter, Ricci solitons in contact metric manifolds have been studied by various authors such as Bejan and Crasmareanu [2], Hui et al. ([4], [17]-[21]), Chen and Deshmukh [5], Deshmukh et al. [7], He and Zhu [13], Nagaraja and Premalata [23], Tripathi [29] and many others.

In [28] Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He proved that they could be divided into three classes: (i) homogeneous normal contact Riemannian manifolds with $c > 0$, (ii) global Riemannian products of a line or a circle with a Kähler manifold of constant holomorphic sectional curvature if $c = 0$ and (iii) a warped product space $\mathbb{R} \times_f \mathbb{C}^n$ if $c < 0$. It is known that the manifolds of class (i) are characterized by admitting a Sasakian structure. The manifolds of class (ii) are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu [22] characterized the differential geometric properties of the manifolds of class (iii) which are nowadays called Kenmotsu manifolds and later studied by several authors ([14]-[16]) etc.

As a generalization of both Sasakian and Kenmotsu manifolds, Oubiña [24] introduced the notion of trans-Sasakian manifolds, which are closely related to the locally conformal Kähler manifolds. A trans-Sasakian manifold of type (0, 0), $(\alpha, 0)$ and $(0, \beta)$ are called the cosympletic, α-Sasakian and β-Kenmotsu manifolds respectively, α, β being scalar functions. In particular, if $\alpha = 0, \beta = 1$; and $\alpha = 1, \beta = 0$ then a trans-Sasakian manifold will be a Kenmotsu and Sasakian manifold respectively.

In modern analysis, the geometry of submanifolds has become a subject of growing interest for its significant applications in applied mathematics and theoretical physics. Recently De and Majhi [6] studied invariant submanifolds of

Kenmotsu manifolds. The present paper deals with the study of Ricci solitons on submanifolds of Kenmotsu manifolds. The paper is organized as follows. Section 2 is concerned with some preliminaries. Section 3 is devoted to the study of Ricci solitons on submanifolds of Kenmotsu manifolds.

In [8] Friedmann and Schouten introduced the notion of semisymmetric linear connection on a differentiable manifold. Then in 1932 Hayden [10] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semisymmetric metric connection on a Riemannian manifold has been given by Yano in 1970 [30]. In 1975, Golab [9] introduced the idea of a quarter symmetric linear connection in differentiable manifolds.

A linear connection $\overline{\nabla}$ in an n-dimensional differentiable manifold M is said to be a quarter symmetric connection [9] if its torsion tensor τ of the connection $\overline{\nabla}$ is of the form

$$\tau(X,Y) = \overline{\nabla}_X Y - \overline{\nabla}_Y X - [X,Y] = \eta(Y)\phi X - \eta(X)\phi Y, \tag{1.2}$$

where η is a 1-form and ϕ is a tensor of type (1,1). In particular, if $\phi X = X$ then the quarter symmetric connection reduces to the semi-symmetric connection. Thus the notion of quarter symmetric connection generalizes the notion of the semi-symmetric connection. Again if the quarter symmetric connection $\overline{\nabla}$ satisfies the condition $(\overline{\nabla}_X g)(Y,Z) = 0$ for all X, Y, $Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields on the manifold M, then $\overline{\nabla}$ is said to be a quarter symmetric metric connection. And if $(\overline{\nabla}_X g)(Y,Z) \neq 0$, then $\overline{\nabla}$ is said to be a quarter symmetric non-metric connection. Furthermore, a quarter symmetric non-metric connection is said to be a quarter symmetric non-metric ϕ-connection [1] if $(\overline{\nabla}_X \phi)(Y) = 0$ for all $X, Y \in \chi(M)$.

Recently Hui et al. [20] studied Ricci solitons on Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection. The Ricci solitons on invariant and anti-invariants submanifolds of Kenmotsu manifolds with respect to quarter symmetric metric connection are studied in this paper. Also in this section, we have studied Ricci solitons on invariant and anti-invariant submanifold of Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection.

2. Preliminaries

An odd dimensional smooth manifold $(\widetilde{M}^{2n+1}, g)$ is said to be an almost contact metric manifold [3] if it admits a $(1,1)$ tensor field ϕ, a vector field ξ, an 1-form η and a Riemannian metric g which satisfy

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad \phi^2 X = -X + \eta(X)\xi, \tag{2.1}$$

$$g(\phi X, Y) = -g(X, \phi Y), \quad \eta(X) = g(X, \xi), \quad \eta(\xi) = 1, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.3}$$

for all vector fields X, Y on M.

An almost contact metric manifold $\widetilde{M}^{2n+1}(\phi, \xi, \eta, g)$ is said to be Kenmotsu manifold if the following conditions hold [22]:

$$\widetilde{\nabla}_X \xi = X - \eta(X)\xi, \tag{2.4}$$

$$(\widetilde{\nabla}_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.5}$$

where $\widetilde{\nabla}$ denotes the Riemannian connection of g.

In a Kenmotsu manifold, the following relations hold [22]:

$$(\widetilde{\nabla}_X \eta)(Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.6}$$

$$\widetilde{R}(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.7}$$

$$\widetilde{R}(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{2.8}$$

$$\widetilde{S}(X, \xi) = -2n\eta(X) \tag{2.9}$$

for any vector field X, Y on $\widetilde{M}$ and $\widetilde{R}$ is the Riemannian curvature tensor and $\widetilde{S}$ is the Ricci tensor of type $(0, 2)$.

Let M be a $(2m+1)$-dimensional $(m < n)$ submanifold of a Kenmotsu manifold $\widetilde{M}$. Let ∇ and $\nabla^{\perp}$ are the induced connections on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M respectively. Then the Gauss and Weingarten formulae are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{2.10}$$

and

$$\widetilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V \tag{2.11}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, where h and A_V are second fundamental form and the shape operator (corresponding to the normal vector field V) respectively for the immersion of M into $\widetilde{M}$. The second fundamental form h and the shape operator A_V are related by

$$g(h(X, Y), V) = g(A_V X, Y) \tag{2.12}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, where g is the Riemannian metric on $\widetilde{M}$ as well as on M.

A submanifold M of a Kenmotsu manifold $\widetilde{M}$ is said to be totally umbilical if

$$h(X, Y) = g(X, Y)H \tag{2.13}$$

for all $X, Y \in \Gamma(TM)$, where H is the mean curvature of M. Moreover, if $H = 0$ then M is minimal in $\widetilde{M}$.

Analogous to almost Hermitian manifolds, the invariant and anti-invariant submanifolds are depend on the behaviour of almost contact metric structure ϕ.

A submanifold M of an almost contact metric manifold $\widetilde{M}$ is said to be invariant if the structure vector field ξ is tangent to M at every point of M and ϕX is tangent to M for any vector field X tangent to M at every point of M, that is $\phi(TM) \subset TM$ at every point of M.

On the otherhand, M is said to be anti-invariant if for any X tangent to M, then ϕX is normal to M, i.e., $\phi(TM) \subset T^{\perp}M$ at every point of M, where $T^{\perp}M$ is the normal bundle of M.

Let $\overline{\widetilde{\nabla}}$ be a quarter symmetric metric connection on a Kenmotsu manifold $\widetilde{M}$. Then we have [16]

$$\overline{\widetilde{\nabla}}_X Y = \widetilde{\nabla}_X Y - \eta(X)\phi Y. \tag{2.14}$$

Here the induced connection on submanifold M of a Kenmotsu manifold $\widetilde{M}$ from the connection $\overline{\widetilde{\nabla}}$ is denoted by $\overline{\nabla}$. The corresponding Gauss formula with respect to quarter symmetric metric connection is

$$\overline{\widetilde{\nabla}}_X Y = \overline{\nabla}_X Y + \overline{h}(X, Y), \tag{2.15}$$

where $\overline{h}$ is the second fundamental form with respect to quarter symmetric metric connection.

Recently Hui et al. [20] studied Ricci solitons on Kenmotsu manifolds with respect to quarter symmetric non-metric ϕ-connection. The quarter symmetric non-metric ϕ-connection on a Kenmotsu manifold $\widetilde{M}$ is given by ([1], [20])

$$\widetilde{\nabla}'_X Y = \widetilde{\nabla}_X Y + \eta(X)\phi Y + g(X, Y)\xi - \eta(Y)X - \eta(X)Y, \tag{2.16}$$

where the induced connection on submanifold M of a Kenmotsu manifold $\widetilde{M}$ from the connection $\widetilde{\nabla}'$ is denoted by ∇'. The corresponding Gauss formula with respect to quarter symmetric non-metric ϕ-connection is

$$\widetilde{\nabla}'_X Y = \nabla'_X Y + h'(X, Y), \tag{2.17}$$

where h' is the second fundamental form with respect to quarter symmetric non-metric ϕ-connection.

3. Ricci Solitons on Submanifolds of Kenmotsu Manifolds

Let us take (g, ξ, λ) be a Ricci soliton on a submanifold M of a Kenmotsu manifold $\widetilde{M}$. Then we have

$$(\pounds_\xi g)(Y, Z) + 2S(Y, Z) + 2\lambda g(Y, Z) = 0. \tag{3.1}$$

from (2.4) and (2.10), we get

$$\begin{aligned} X - \eta(X)\xi &= \widetilde{\nabla}_X \xi \\ &= \nabla_X \xi + h(X, \xi). \end{aligned} \tag{3.2}$$

Equating tangential and normal components of (3.2), we get

$$\nabla_X \xi = X - \eta(X)\xi \quad \text{and} \quad h(X, \xi) = 0. \tag{3.3}$$

From (2.1), (2.2) and (3.3), we get

$$\begin{aligned} (\pounds_\xi g)(Y, Z) &= g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) \\ &= 2[g(Y, Z) - \eta(Y)\eta(Z)]. \end{aligned} \tag{3.4}$$

In view of (3.4), (3.1) yields

$$S(Y, Z) = -(\lambda + 1)g(Y, Z) + \eta(Y)\eta(Z), \tag{3.5}$$

which implies that M is Einstein. Thus we can state the following:

Theorem 3.1. If (g, ξ, λ) is a Ricci soliton on a submanifold M of a Kenmotsu manifold $\widetilde{M}$ then M is η-Einstein.

From (3.5), we get

$$S(Y, \xi) = -(\lambda - 2)\eta(Y) \quad \text{for all Y.} \tag{3.6}$$

Again using (3.3) we can prove that

$$S(Y, \xi) = -2m\eta(Y). \tag{3.7}$$

From (3.6) and (3.7) we get $\lambda - 2 = 2m$, i.e., $\lambda = 2(m + 1) > 0$ always. This leads to the following:

Theorem 3.2. A Ricci soliton (g, ξ, λ) on a submanifold M of a Kenmotsu manifold $\widetilde{M}$ is always expanding.

From (2.10), (2.14) and (2.15) we get

$$(3.8)\qquad \begin{aligned}\overline{\nabla}_X Y + \overline{h}(X,Y) &= \widetilde{\nabla}_X Y - \eta(X)\phi Y\\ &= \nabla_X Y - \eta(X)\phi Y + h(X,Y).\end{aligned}$$

If M is invariant submanifold of $\widetilde{M}$ then $\phi X \in TM$ for any $X \in TM$ and therefore equating normal and tangential parts from (3.8), we get

$$(3.9)\qquad \overline{h}(X,Y) = h(X,Y)$$

and

$$(3.10)\qquad \overline{\nabla}_X \xi = \nabla_X Y - \eta(X)\phi Y,$$

which means that M admits a quarter symmetric metric connection.
This leads to the following:

Theorem 3.3. If M is an invariant submanifolds of a Kenmotsu manifolds $\widetilde{M}$ with respect to quarter symmetric metric connection, then
(i) M admits quarter symmetric metric connection,
(ii) the second fundamental forms with respect to quarter symmetric metric connection and Riemannian connection are same.

Let us now consider (g, ξ, λ) be a Ricci soliton on a submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric metric connection. Then we have

$$(3.11)\qquad (\overline{\mathcal{L}}_\xi g)(Y,Z) + 2\overline{S}(Y,Z) + 2\lambda g(Y,Z) = 0.$$

Also from (3.10), we get

$$(3.12)\qquad \overline{\nabla}_X \xi = X - \eta(X)\xi$$

and hence

$$(3.13)\qquad (\overline{\mathcal{L}}_\xi g)(Y,Z) = 2[g(Y,Z) - \eta(Y)\eta(Z)]$$

Again it is known that [16]

$$(3.14)\qquad \overline{S}(Y,Z) = S(Y,Z) - 2d\eta(\phi Z, Y) + \omega(Y,Z)) + \psi\eta(Y)\eta(Z),$$

where $\omega(Y,Z) = g(\phi Y, Z)$ and $\psi = \text{tr. } \omega$.

In view of (3.13) and (3.14), (3.11) yields

$$(3.15)\qquad \begin{aligned}S(Y,Z) &= -(\lambda+1)g(Y,Z) - (\psi - 1)\eta(Y)\eta(Z)\\ &+ 2d\eta(\phi Z, Y) - \omega(Y,Z).\end{aligned}$$

This leads to the following:

Theorem 3.4. If (g, ξ, λ) is a Ricci soliton on an invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric metric connection, then its Ricci tensor S with respect to Riemannian connection is of the form (3.15).

By virtue of (2.1) and (2.2) we have from (3.15) that

$$S(Y, \xi) = -(\lambda + \psi)\eta(Y). \tag{3.16}$$

From (3.7) and (3.16), it follows that

$$\lambda = 2m - \psi,$$

which means that the Ricci soliton (g, ξ, λ) is shrinking, steady and expanding according as $\psi > 2m$, $\psi = 2m$ or $\psi < 2m$, respectively.
This leads the following:

Theorem 3.5. A Ricci soliton (g, ξ, λ) on an invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric metric connection is shrinking, steady and expanding according as $\psi > 2m$, $\psi = 2m$ or $\psi < 2m$, respectively.

If M is an anti-invariant submanifold of $\widetilde{M}$, then $\phi X \in T^{\perp}M$ for all $X \in TM$ and therefore equating tangential part from (3.8), we get

$$\overline{\nabla}_X Y = \nabla_X Y; \tag{3.17}$$

which means that quarter symmetric metric connection and Riemannian connection are same. Hence $\overline{S}(X, Y) = S(X, Y)$ for all $X, Y \in M$.
Again $\overline{\nabla}_X \xi = \nabla_X \xi = X - \eta(X)\xi$ and hence (3.13) holds. Consequently, by virtue of Theorem 3.1, we can state the following:

Theorem 3.6. If (g, ξ, λ) is a Ricci soliton on an anti-invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric metric connection, then M is η-Einstein.

In similar as above Theorem 3.2, we can state the following:

Theorem 3.7. A Ricci soliton (g, ξ, λ) is on an anti-invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric metric connection is always expanding.

We can consider (g, ξ, λ) is a Ricci soliton on a submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric non-metric ϕ-connection. Then we have

$$(\pounds'_{\xi} g)(Y,Z) + 2S'(Y,Z) + 2\lambda g(Y,Z) = 0. \tag{3.18}$$

From (2.10), (2.16) and (2.17), we get

$$\begin{aligned} \nabla'_X Y + h'(X,Y) &= \widetilde{\nabla}_X Y + \eta(X)\phi Y + g(X,Y)\xi \\ &- \eta(Y)X - \eta(X)Y \\ &= \nabla_X Y + h(X,Y) + \eta(X)\phi Y \\ &+ g(X,Y)\xi - \eta(Y)X - \eta(X)Y. \end{aligned} \tag{3.19}$$

If M is an invariant submanifold of $\widetilde{M}$ then $\phi X \in TM$ for any $X \in TM$ and $\xi \in TM$. Therefore equating the tangential parts of (3.19), we get

$$\nabla'_X Y = \nabla_X Y + \eta(X)\phi Y + g(X,Y)\xi - \eta(Y)X - \eta(X)Y, \tag{3.20}$$

which means that M admits quarter symmetric non-metric ϕ-connection.
From (3.20), we obtain

$$\nabla'_X \xi = -\eta(X)\xi \tag{3.21}$$

and hence

$$\begin{aligned} (\pounds'_{\xi} g)(Y,Z) &= g(\nabla'_Y \xi, Z) + g(Y, \nabla'_Z \xi) \\ &= -2\eta(Y)\eta(Z). \end{aligned} \tag{3.22}$$

Also using (3.20), we can calculate that

$$S'(Y,Z) = S(Y,Z) + 2m[g(Y,Z) + \eta(Y)\eta(Z)], \tag{3.23}$$

where S' and S are the Ricci tensor of invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric non-metric ϕ-connection and Riemannian connection respectively.
By virtue of (3.22) and (3.23), it follows from (3.18) that

$$S(Y,Z) = -(\lambda + 2m)g(Y,Z) - (2m-1)\eta(Y)\eta(Z), \tag{3.24}$$

which implies that M is η-Einstein. This leads to the following:

Theorem 3.8. If (g, ξ, λ) is a Ricci soliton on an invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric non-metric ϕ-connection, then M is η-Einstein.

By virtue of (2.1) and (2.2), we have from (3.24) that

$$S(Y,\xi) = -(\lambda + 4m - 1)\eta(Y). \tag{3.25}$$

From (3.7) and (3.25), we have $\lambda = -2(m - \frac{1}{2}) < 0$.
This leads the following:

Theorem 3.9. A Ricci soliton (g, ξ, λ) on an invariant submanifold M of a Kenmotsu manifold $\widetilde{M}$ with respect to quarter symmetric non-metric ϕ-connection is always shrinking.

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