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Some Anisotropic Dark Energy model in Five Dimensional Bianchi Type-V Space-time

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Abstract

Five dimensional Bianchi type-V space-time has been considered with the introduction of four different skewness parameters along spatial directions to quantify the deviation of pressure from isotropy. We have assumed that the four skewness parameters are time dependent to study the anisotropic nature of the dark energy which has the dynamical energy density. For the phantom model it is found that the Universe achieves the flatness. The physical properties of the Universe has also been discussed in this paper.

Keywords : Five dimensional Bianchi-V-space-time, Five dimensional Hubbles Parameter, Deceleration parameter, Anisotropic Dark energy, Dynamical energy density, Dark energy having energy density.
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1. Introduction

Many authors have studied the nature of anisotropic dark energy. Bianchi type-I cosmological models with constant deceleration parameter (DP) in the presence of anisotropic dark energy (DE) and perfect fluid have been studied by Kumar and Singh (2010). They have assumed phenomenological parametrization of minimally interacting DE in terms of it's effective equation of state (EoS) and time dependent skewness parameters $\alpha(t), \beta(t), \gamma(t)$. Bianchi type-V universe is the generalization of the open universe in FRW cosmology. The isotropic DE model with variable effective equation of state (EoS) parameter in Bianchi type-V space-time have been recently studied by Kumar and Yadav (2010), they have found that the Universe is dominated by DE at present epoch and after dominance of DE, Universe achieves flatness. Recently Pradhan et al (2011a, 2011b) have studied anisotropic DE models in different physical contexts in four dimensional space-time. They have found that in the earlier stage EoS parameter was positive and it evolves with negative sign at present epoch. Yadav et al. (2011b) have studied anisotropic DE models with variable EoS parameter. They have suggested that the present acceleration of Universe is described by the dynamic of EoS parameters.

In the paper [1], Anil Kumar Yadav has studied some physically realistic and totally anisotropic four dimensional Bianchi-V models with anisotropic dark energy (DE). To study the anisotropic nature of DE, he has assumed three time dependent skewness parameters, which modify effective equation of state (EoS). It is interesting to note that the time dependent form of the skewness parameters provide exact solutions of Einsteins field equations together with the special law of variation of Hubble's parameter.

Thus in this paper, we propose to solve Einsteins field equations together with the special law of variation of Hubble's parameter in higher five dimensional space-time using the time dependent form of the skewness parameters.

2. Five dimensional Einsteins field equations

We consider the spatially homogeneous and anisotropic Bianchi-V space-time in V_5 with the metric

$$ds^2 = -dt^2 + A^2dx^2 + e^{2\eta x}[B^2dy^2 + C^2(dz^2 + du^2)] \tag{1}$$

where A, B and C are the metric functions of cosmic time t and η is a constant.

We define the average Hubbles parameter as

$$H = \frac{\dot{a}}{a} = \frac{1}{4}\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{2\,\dot{C}}{C}\right] \tag{2}$$

where $a = (ABC^2)^{1/4}$ is the average scale factor of the space-time (1) and an over dot denotes derivative with respect to the cosmic time t.

The directional Hubbles parameters along x, y, z and u coordinate axes are defined as

$$H_x = \frac{\dot{A}}{A}, \qquad H_y = \frac{\dot{B}}{B}, \qquad H_z = \frac{\dot{C}}{C}, \qquad H_u = \frac{\dot{C}}{C}. \tag{3}$$

The five dimensional Einsteins field equations are given by

$$R_{ij} - \frac{1}{2} g_{ij} R = -T_{ij}, \qquad (i, j = 1, 2, 3, 4, 5), \tag{4}$$

where

$$\begin{aligned} T^i_j &= \mathrm{diag}[-\rho, p_x, p_y, p_z, p_u] \\ &= \mathrm{diag}[-1, \omega_x, \omega_y, \omega_z, \omega_u]\rho = \mathrm{diag}[-1, \omega+\alpha, \omega+\beta, \omega+\gamma, \omega+\delta]\rho \end{aligned} \tag{5}$$

in which we have used the equation of state $p = \omega\rho$, where ρ is the energy density of the DE component; $\alpha(t), \beta(t), \gamma(t)$ and $\delta(t)$ are skewness parameters, which modify EoS (hence pressure) of the DE component and are functions of the cosmic time $t; \omega$ is the EoS parameter of DE; $\omega_x, \omega_y,$ ω_z and ω_u are the directional EoS parameters along x, y, z and u coordinate axes, respectively and we assume the five velocity vector $v^i = (1, 0, 0, 0, 0)$ satisfying $v^i v_i = -1$.

In a co moving coordinate system, the five dimensional field equations (4) become

$$\frac{\ddot{B}}{B} + \frac{2\ddot{C}}{C} + \frac{2\dot{B}\dot{C}}{BC} + \frac{\dot{C}^2}{C^2} - \frac{3\eta^2}{A^2} = -(\omega+\alpha)\rho, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{2\ddot{C}}{C} + \frac{2\dot{A}\dot{C}}{AC} + \frac{\dot{C}^2}{C^2} - \frac{3\eta^2}{A^2} = -(\omega+\beta)\rho, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\eta^2}{A^2} = -(\omega+\gamma)\rho, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{3\eta^2}{A^2} = -(\omega+\delta)\rho, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{2\dot{B}\dot{C}}{BC} + \frac{2\dot{A}\dot{C}}{AC} + \frac{\dot{C}^2}{C^2} - \frac{6\eta^2}{A^2} = \rho, \tag{10}$$

$$\frac{3\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{2\dot{C}}{C} = 0. \tag{11}$$

The left hand side of equation (8) and (9) are identical because of the metric function C is common along z and u directions in the metric (1) which are again in the format of Yadav (2011).

The energy conservation equation $T^{ij}_{;j} = 0$ gives

$$\dot{\rho} + 4\rho(\omega+1)H + \rho(\alpha H_x + \beta H_y + \gamma H_z + \delta H_u) = 0. \tag{12}$$

3. Solution of Einstein's field Equations in V_5

Recently Kumar (2010) obtained some FRW cosmological models with constant DP and shown that the constant DP models stand adequately for our present view of different phases of the evolution of Universe. In the paper[1], he has shown that the behaviour of the constant deceleration parameter models with the four dimensional Bianchi-V space-time in presence of anisotropic dark energy. In this paper we extend the work of A. K. Yadav (2011) to five dimensional space-time (1). In this way we show that how the constant deceleration parameter model with the line element (1) play the role in presence of anisotropic dark energy. According to Singh et al (2008), the law of variation of average Hubbles parameter is given by

$$H = Da^{-n} \tag{13}$$

where $D > 0$ and $n \geq 0$ are constants.

In general equation (12) is very complicated to obtain the solution. Therefore, in the present paper, we obtain some particular solutions of the equation (12) with some assumption as follows:

Case (i): Here we consider α, β, γ and δ are different constants. With this assumption the dark energy density scales obtained as

$$\rho\, a^{-4(\omega+1)} A^{-\alpha} B^{-\beta} C^{-\gamma} C^{-\delta}.$$

But this form of ρ is not useful to solve the field equations (6)-(12) combine with (13).

Case (ii): Here we consider α, β, γ and δ are equal constants ($\alpha = \beta = \gamma = \delta = k$). With this assumption the dark energy density scales obtained as

$$\rho \sim a^{-4(\omega+k+1)}.$$

With this form of ρ the Einsteins field equations can be solved. But the resulting model would not possess the anisotropy of DE due to the equality of skewness parameters.

Thus, in order to study the anisotropic nature of dynamical DE, we have to assume that skewness parameters are time-dependent quantities. At the same time, we have to ensure that the time-dependant forms of the skewness parameters provide exact solutions of the field equations (6)-(11) together with (13). Therefore, we consider the following time-dependent forms of the skewness parameters as under :

$$\alpha(t) = \eta(H_y + H_z + H_u), \tag{14}$$

$$\beta(t) = -\eta(H_x), \tag{15}$$

$$\gamma(t) = -\eta(H_x), \tag{16}$$

$$\delta(t) = -\eta(H_x) \tag{17}$$

where η is an arbitrary constant, which parameterizes the anisotropy of the DE.

We consider that equation of state parameter of dark energy (ω = constant). Therefore, we can study different models related to the DE by choosing different values of ω, such that $\omega < -1$: phantom, $\omega = -1$:cosmological constant and $\omega > -1$: quintessence.

In view of the assumptions (14)-(17) and ω = const., after integrating equation (12), we obtain

$$\rho(t) = \rho_0 a^{-4(\omega+1)} \tag{18}$$

where ρ_0 is a positive constant of integration.

Integrating (12) and absorbing the constant of integration in B or C, without loss of generality, we obtain

$$A^3 = BC^2. \tag{19}$$

Subtracting (7) from (8), (7) from (10), (8) from (10) and taking second integral of each, we get the following three relations respectively

$$\frac{A}{B} = d_1 \exp[x_1 \int a^{-4} dt - \frac{\eta\rho_0}{\omega} \int a^{-4(\omega+1)} dt], \tag{20}$$

$$\frac{A}{C} = d_2 \exp[x_2 \int a^{-4} dt - \frac{\eta\rho_0}{\omega} \int a^{-4(\omega+1)} dt], \tag{21}$$

$$\frac{B}{C} = d_3 \exp[x_3 \int a^{-4} dt], \tag{22}$$

where d_1, x_1, d_2, x_2, d_3 and x_3 are constants of integration. From equations (20)-(22) and (19), the metric functions explicitly obtained as

$$A(t) = a \exp[-\frac{3\eta\rho_0}{4\omega} \int a^{-4(\omega+1)} dt], \tag{23}$$

$$B(t) = m^2 a \exp[2l \int a^{-4} dt + \frac{\eta\rho_0}{4\omega} \int a^{-4(\omega+1)} dt], \tag{24}$$

$$C(t) = m^{-1} a \exp[-l \int a^{-4} dt + \frac{\eta\rho_0}{4\omega} \int a^{-4(\omega+1)} dt], \tag{25}$$

where

$$m = (d_2 d_2)^{1/4}, \qquad l = \frac{x_2 + x_3}{4} \tag{26}$$

with

$$d_2^2 = d_1^{-1}, \qquad 2x_2 = -x_1. \tag{27}$$

It has been observed that all the resulting metric functions in higher five dimensional space-time are obtained in the format of Yadav (2011).

Conclusion : We conclude that the equations (23), (24), (25) are the exact solutions of Einstein's field equations (4) for anisotropic five dimensional Bianchi type V space-time filled with dark energy.

Now, we discuss the dark energy cosmologies for $n \neq 0$ and $n = 0$ in higher five dimensional space-time.

3.1. Dark Energy Cosmology for $n \neq 0$ in five dimensional space-time

This model is based on the exact solutions of Einsteins field equations for anisotropic five dimensional Bianchi type V space-time filled with dark energy. After integration of (13), we have

$$a(t) = (nDt)^{1/n} \tag{28}$$

where the constant of integration has been omitted by assuming that $a = 0$ at $t = 0$.

Using (27) into (22)-(24), we obtain the following expressions for scale factors:

$$A(t) = (nDt)^{1/n} \exp[\frac{3\eta\rho_0}{4\omega D(n-4\omega-4)}(rDt)^{n-4\omega-4}], \tag{29}$$

$$B(t) = m^2(nDt)^{1/r} \exp[\frac{2l}{D(r-4)}(rDt)^{\frac{n-4}{n}} + \frac{\eta\rho_0}{4\omega D(n-4\omega-4)}(rDt)^{n-4\omega-4}], \tag{30}$$

$$C(t) = m^{-1}(nDt)^{1/n} \exp[\frac{-l}{D(r-4)}(nDt)^{\frac{n-4}{n}} + \frac{\eta\rho_0}{4\omega D(n-4\omega-4)}(rDt)^{n-4\omega-4}], \tag{31}$$

where $n \neq 4$.

The physical parameters such as directional Hubble's parameters (H_x, H_y, H_z, H_u), average Hubble parameter (H), anisotropy parameter ($\overline{A}$), expansion scalar (θ) and spatial volume(V) are, respectively, given by

$$H_x = (nt)^{-1} - \frac{3\eta\rho_0}{4\omega}(rDt)^{\frac{-4(\omega+1)}{n}}, \tag{32}$$

$$H_y = (nt)^{-1} + 2l(nDt)^{\frac{-4}{n}} + \frac{\eta\rho_0}{4\omega}(nDt)^{\frac{-4(\omega+1)}{n}}, \tag{33}$$

$$H_z = (nt)^{-1} + l(nDt)^{\frac{-4}{n}} + \frac{\eta\rho_0}{4\omega}(nDt)^{\frac{-4(\omega+1)}{n}}, \tag{34}$$

$$H_u = (nt)^{-1} + l(nDt)^{\frac{-4}{n}} + \frac{\eta\rho_0}{4\omega}(nDt)^{\frac{-4(\omega+1)}{n}}, \tag{35}$$

$$H = (nt)^{-1}, \tag{36}$$

$$\overline{A} = \frac{3}{4D^2}[2l^2(nDt)^{\frac{2(n-4)}{n}} + \frac{\eta^2\rho_0^2}{4\omega^2}(nDt)^{\frac{2(n-4\omega-4)}{n}}, \tag{37}$$

$$\theta = 4(nt)^{-1}, \tag{38}$$

$$V = (nDt)^{4/n}\exp(3\eta x). \tag{39}$$

Shear scalar of the model is given by

$$\sigma^2 = [3l^2(nDt)^{\frac{-8}{n}} + \frac{3\eta^2\rho_0^2}{8\omega^2}(nDt)^{\frac{8(\omega+1)}{n}}]. \tag{40}$$

The value of deceleration parameter (q) is found to be

$$q = n - 1 \tag{41}$$

which is a constant. The sign of q indicates whether the model inflates or not. A positive sign of q, i.e., $n > 1$ corresponds to the standard decelerating model whereas the negative sign of q, i.e., $0 < n < 1$ indicates inflation. The expansion of the universe at a constant rate corresponds to $q = 0$, i.e, $n = 1$. Also, recent observations of SN Ia, reveal that the present Universe is accelerating and value of DP lies somewhere in the range $-1 < q < 0$.

It follows that in the derived model, we can choose the values of DP consistent with the observations. Putting the values of $H_x, H_y, H_z \text{and} H_u$ in (14), (15), (16) and (17) the skewness parameters of DE are obtained as under:

$$\alpha(t) = \eta[3(nt)^{-1} + \frac{3\eta\rho_0}{4\omega}(rDt)^{\frac{-4(\omega+1)}{n}}], \tag{42}$$

$$\beta(t) = \gamma(t) = \delta(t) = -\eta[(nt)^{-1} - \frac{3\eta\rho_0}{4\omega}(nDt)^{\frac{-4(\omega+1)}{n}}]. \tag{43}$$

In view of (5), the directional EoS parameters of DE are given by

$$\omega_x = \omega + \eta[3(nt)^{-1} + \frac{3\eta\rho_0}{4\omega}(nDt)^{\frac{-4(\omega+1)}{n}}], \tag{44}$$

$$\omega_y = \omega_z = \omega_u = \omega - \eta[(nt)^{-1} - \frac{3\eta\rho_0}{4\omega}(nDt)^{\frac{-4(\omega+1)}{n}}]. \tag{45}$$

The energy density and pressure of the DE are obtained as

$$\rho = \rho_0(nDt)^{\frac{-4(\omega+1)}{n}}, \tag{46}$$

$$p = \omega\rho_0(nDt)^{\frac{-4(\omega+1)}{n}}. \tag{47}$$

The critical density (ρ_c) and density parameter (Ω) are obtained as

$$\rho_c = 3(nt)^{-2}, \tag{48}$$

$$\Omega = \frac{\rho_0}{3}(D)^{\frac{-4(\omega+1)}{n}}(nt)^{\frac{2n-4(\omega+1)}{n}}. \tag{49}$$

It is observed that at $t = 0$, the spatial volume vanishes while all other parameters diverge. Therefore, the derived model starts expanding with big bang singularity at $t = 0$. This singularity is point type because the directional scale factors $A(t), B(t), C(t)$ vanishes at initial moment.

The solutions for the scale factors have a combination of a power-law term and exponential term in the product form. The DE term appears in exponential form and thus affects their evolution significantly. For $\eta < 0$ and $n > 4\omega + 4$, the DE contributes to the expansion of $A(t)$ while opposing to the expansion of $B(t)$ and $C(t)$. Similarly for $\eta > 0$ and $n > 4\omega + 4$, the anisotropic DE opposes the expansion of $A(t)$ while contributing to the expansion of $B(t)$ and $C(t)$.

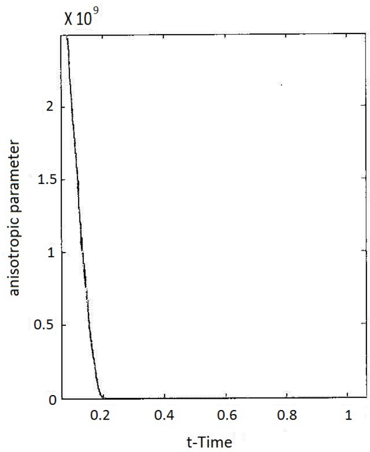


Fig. 1 : Plot of anisotropic parameter$\bar{A}$ versus time (t)

The difference between the directional EoS parameters and hence the pressures of the DE, along $x-$axis and $y-$axis (or$z-$axis, or $u-$axis) is $4\eta(nt)^{-1}$, which decreases as t increases. Therefore, the anisotropy of DE decreases as t increases and finally drops to zero at late time. The variation of mean anisotropic parameter $(\overline{A})$ versus cosmic time t has been graphed in Fig. 1 by choosing $D = 2$, $\omega = -1.1$, $n = 0.6$ and other constant as unity. Since the current observations strongly recommend that the present Universe is accelerating (i.e $q < 0$). Now we assume $n = 0.6$ i.e. $q = -0.4$ in the remaining discussion of the model.

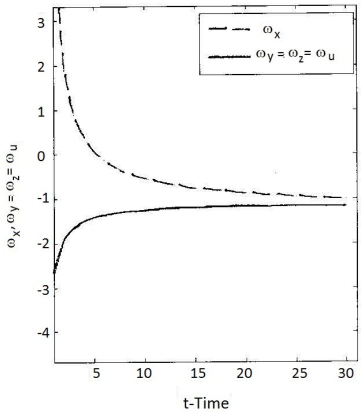


Fig. 2 : Plot of directional EoS parameter versus time

Fig. 2 shows the variation of directional EoS parameters i.e., $(\omega_x, \omega_y, \omega_z, \omega_u)$ versus cosmic time t. We observe that directional EoS parameter along $x-$axis (i.e. ω_x) is decreasing function of time while directional EoS parameters along $y-$axis, $z-$axis and $u-$axis are increasing function of time. At the later stage of evolution, all the directional EoS parameters approaches to -1 in five dimensional space-time also. The same is predicted by current observations.

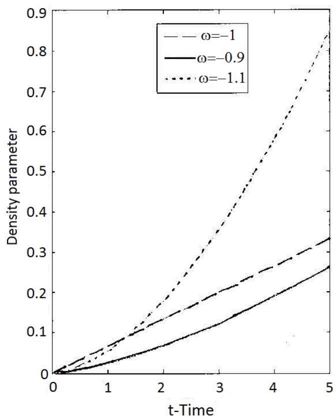


Fig. 3 : Single plot of density parameter (Ω) versus time (t)

Fig. 3 shows the variation of density parameter (Ω) of DE versus cosmic time t during evolution of Universe for $\omega = -0.9$ (dot line), $\omega = -1$ (dash line) and $\omega = -1.1$ (solid line). It is observed that for $\omega = -1.1$, the density parameter (Ω) approaches to 1 at late time. This shows that the derived five dimensional model predicts a flat Universe at the present epoch.

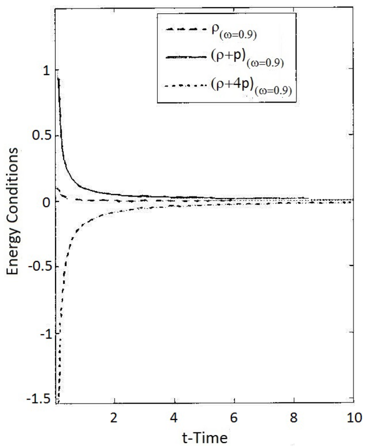


Fig. 4(a) : Single plot energy condition versus time (t) for Quintessence model

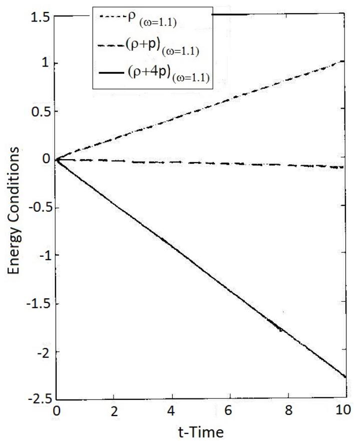


Fig. 4(b) : Single plot energy condition versus time (t) for phantom model

The energy conditions are given by

$$\rho \geq 0, \qquad \rho + p \geq 0 : \qquad \text{(weak energy condition)}$$

$$\rho + p \geq 0, \qquad \rho + 4p \geq 0 : \text{(strong energy condition)}$$

The left hand side of energy conditions have graphed in Fig. 4. From Fig. 4 for ($\omega = -0.9$) i.e. quintessence model, we observe that

(i) $\rho \geq 0$ (ii) $\rho + p \geq 0$ (iii) $\rho + 4p < 0$.

Thus the derived five dimensional quintessence model violates the strong energy conditions.

Further, for $\omega = -1.1$ (i.e. phantom model) it is observed that

(i) $\rho > 0$ (ii) $\rho + p < 0$ (iii) $\rho + 4p < 0$.

Thus the derived five dimensional phantom model violates the weak as well as strong energy conditions, The same is predicted by current astronomical observations.

Conclusion : We conclude that the derived model is based on the exact solution of Einsteins field equations for anisotropic five dimensional Bianchi-type V space-time filled with dark energy which is singular and describes dynamics of the Universe from big-bang to present epoch. The anisotropic dark energy contributes to the expansion of one (or two) of the scale factors while it opposes the expansion of the other two (or one) leading the geometry of Universe and further we observed that the anisotropy of dark energy vanishes at late time for this model.

3.2. Dark Energy Cosmology for $n = 0$ in five dimensional space-time

This model is based on the exact solutions of Einsteins field equations for anisotropic five dimensional Bianchi type V space-time filled with dark energy.

In this case, integration of (13) yields

$$a(t) = c_1 e^{Dt} \tag{50}$$

where c_1 is a positive constant of integration.

The metric functions, therefore, become

$$A(t) = c_1 \exp[Dt + \frac{3\eta\rho_0}{16\omega D(\omega+1)c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}], \tag{51}$$

$$B(t) = m^2 c_1 \exp[Dt - \frac{l}{2Dc_1^4} e^{-4Dt} - \frac{\eta\rho_0}{16\omega D(\omega+1)c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}], \tag{52}$$

$$C(t) = m^{-1} c_1 \exp[Dt + \frac{l}{4Dc_1^4} e^{-4Dt} - \frac{\eta\rho_0}{16\omega D(\omega+1)c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}], \tag{53}$$

provided $\omega \neq -1$.

For $\omega = -1$, we have

$$A(t) = c_1 \exp[Dt + \frac{3\eta\rho_0}{4} t], \tag{54}$$

$$B(t) = m^2 \exp[Dt - \frac{l}{2Dc_1^4} e^{-4Dt} - \frac{\eta\rho_0}{4} t], \tag{55}$$

$$C(t) = m^{-1} c_1 \exp[Dt + \frac{l}{4Dc_1^4} e^{-4Dt} - \frac{\eta\rho_0}{4} t]. \tag{56}$$

The other cosmological parameters of the model are obtained as under:

$$H_x = D - \frac{3\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}, \tag{57}$$

$$H_y = D + \frac{2l}{c_1^4} e^{-4Dt} + \frac{\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}, \tag{58}$$

$$H_z = D - \frac{l}{c_1^4} e^{-4Dt} + \frac{\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}, \tag{59}$$

$$H_u = D - \frac{l}{c_1^4} e^{-4Dt} + \frac{\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}, \tag{60}$$

$$H = D, \tag{61}$$

$$\overline{A} = \frac{3}{4D^2}[\frac{2l^2}{c_1^8} e^{-8Dt} + \frac{\eta^2\rho_0^2}{4\omega^2 c_1^{8(\omega+1)}} e^{-8D(\omega+1)t}], \tag{62}$$

$$\theta = 4D, \tag{63}$$

$$V = c_1^4 e^{4Dt}, \tag{64}$$

$$\sigma^2 = \frac{3l^2}{c_1^8} e^{-8Dt} + \frac{3\eta^2\rho_0^2}{8\omega^2 c_1^{8(\omega+1)}} e^{-8D(\omega+1)t}. \tag{65}$$

The skewness parameters and the directional EoS parameters of DE, respectively, are given by

$$\alpha(t) = \eta[3D + \frac{3\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}], \tag{66}$$

$$\beta(t) = \gamma(t) = \delta(t) = -\eta[D - \frac{3\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}], \tag{67}$$

$$\omega_x = \omega + \eta[3D + \frac{3\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}], \tag{68}$$

$$\omega_y = \omega_z = \omega_u = \omega - \eta[D - \frac{3\eta\rho_0}{4\omega c_1^{4(\omega+1)}} e^{-4D(\omega+1)t}]. \tag{69}$$

The energy density and pressure of the DE are given by

$$\rho = \rho_0 c_1^{-4(\omega+1)} e^{-4D(\omega+1)t}, \tag{70}$$

$$p = \omega\rho_0 c_1^{-4(\omega+1)} e^{-4D(\omega+1)t}. \tag{71}$$

The critical density (ρ_c) and density parameter(Ω) of the DE are obtained as

$$\rho_c = 3D^2, \tag{72}$$

$$\Omega = \frac{\rho_0 c_1^{-4(\omega+1)}}{3D^2} e^{-4D(\omega+1)t}. \tag{73}$$

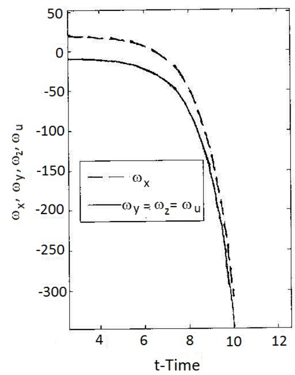
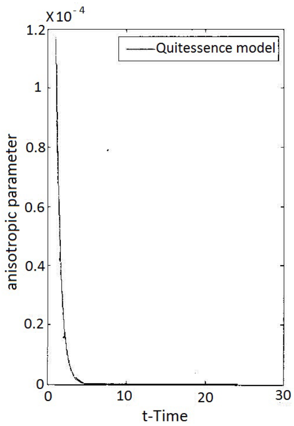


Fig. 5 : Plot of directional EoS parameters versus time (t)

Fig. 5 shows the variation of directional EoS parameter versus time for $n = 0$ in five dimensional space-time. We observed that $\omega_x, \omega_y, \omega_z, \omega_u$ are evolving with negative sign as expected



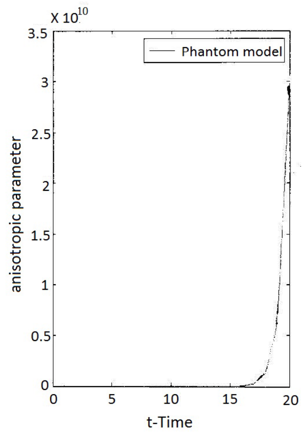


Fig. 6(a) : Plot of anisotropic parameter $\overline{A}$ versus time (t) for $n = 0$

Fig. 6(b) : Plot of anisotropic parameter $\overline{A}$ versus time (t) for $n = 0$

Fig. 6(a, b) shows the plot of anisotropy parameter $(\overline{A})$ verses time t, in both quintessence model and phantom model. We see that in quintessence model, the anisotropy parameter decreases as time increases and finally drop to zero at late time. But in phantom model the anisotropy parameter does not vanish during the evolution of universe in five dimensional space-time.

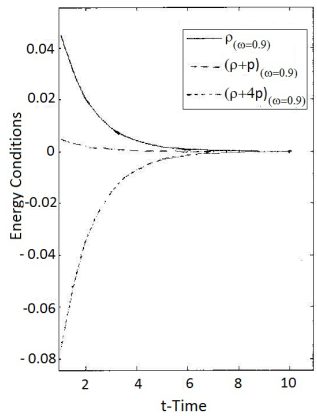


Fig. 7 : Single plot energy condition in quintessence model for $n = 0$

The left hand side of energy conditions have graphed if Fig. 7. for quintessence model and It is observed that the quintessence model violates strong energy condition in five dimensional space-time.

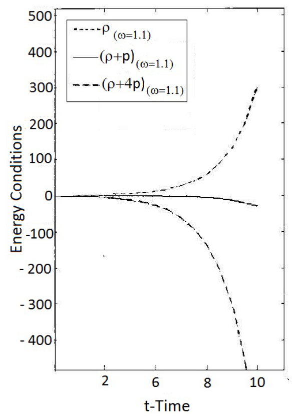


Fig. 8 : Single plot energy condition in phantom model for $n = 0$

The left hand side of energy conditions have graphed in Fig. 8 for phantom model. It is observed that the phantom model violate weak energy condition as well as strong energy condition in five dimensional space-time.

We observed that the five dimensional model has no initial singularity. The directional scale factors and all other physical quantities are constant at $t = 0$. The directional scale factors and volume of the universe increase exponentially with the cosmic time whereas the average Hubbles parameter and expansion scalar are constants through out the evolution. Therefore, uniform exponential expansion happen.

Further we observe that the DE term appears in exponential form in the scale factors and thus effects their evolution singnificantly. Thus, the spatial geometry of the universe is affected by the anisotropic DE. The difference between the directional EoS parameters of the DE and hence the pressures of the DE, along x-axis and y-axis(orz-axis, or u-axis) is $4\eta D$, which is constant through out the evolution of the universe. Therefore, the anisotropy of the DE does not vanish during the evolution of universe.

As $t \to \infty$, the directional scale factors and volume of the universe tends to infinity whereas the skewness and directional EoS parameters, directional Hubbles parameters become constants. The pressure and energy density of the DE drops to zero in quintessence model (i.e. $\omega > -1$).

For $n = 0$, we get $q = -1$; incidentally this value of deceleration parameter leads to $\frac{dH}{dt} = 0$, which implies the greatest value of Hubbles parameter and the fastest rate of expansion of the universe. Therefore, the derived model can be utilized to describe the dynamics of the late time evolution of the actual universe. Therefore, we emphasize upon the late time behavior of the derived model. From(68) and (69). It is observed that if we take equation $\rho_c = \rho_0$, $c_1 = 1$ and $\omega = -1$ then $\Omega \approx 1$. Thus the model predicts a flat Universe after dominance of vacuum energy.

Conclusion : We conclude that the derived model is based on the exact solution of Einsteins field equations for anisotropic five dimensional Bianchi-type V space-time filled with dark energy which is non-singular and describes dynamics of the future Universe. The anisotropic dark energy contributes to the expansion of one (or two) of the scale factors while it opposes the expansion of the other two (or one) leading the geometry of Universe and further we observed that the anisotropy of dark energy occurs at early stage (i.e., quintessence model) or at later time of Universe (i.e., phantom model)

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