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On LP-Sasakian Manifolds satisfying Certain Curvature Tensors

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Abstract

The object of the present paper is to study some curvature condition on LP-Sasakian manifolds which satisfy $P.\widetilde{W}_2 = 0$, $\widetilde{W}_2.\widetilde{W}_2 = 0$, $L.\widetilde{W}_2 = 0$, $\widetilde{W}_2.L = 0$, $W_2.\widetilde{W}_2 = 0$ and $\widetilde{W}_2.R = 0$.

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1. Introduction

In 1989, K. Matsumoto introduced the notion of LP-Sasakian manifold. Then I. Mihai and R. Rosca [3] introduced the same notion independently and they obtained several results on this manifold. LP-Sasakian manifold have also been studied by K. Matsumoto and I. Mihai [6], B. Prasad [1], U. C. De, K. Matsumoto and A. A. Shaikh [9], M. Trafdar and A. Bhattacharyya [7], Venkatesha and C. S. Bagewadi [10], Shaikh, Prakasha and Ahmad [11], Prasad and Haseeb [12], Berman [13] and others.

2. Preliminaries

A differentiable manifold M^n of dimension n is called LP-Sasakian manifold Matsumoto [4, 6], if it admits a (1,1) tensor field ϕ , a contravariant vector field ξ , a covariant vector field η and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \quad (2.1)$$

$$\phi^2(X) = X + \eta(X)\xi, \quad (2.2)$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad (2.3)$$

$$g(X, \xi) = \eta(X), \quad D_X \xi = \phi X, \quad (2.4)$$

$$(D_X \eta)(Y) = g(\phi X, Y), \quad (2.5)$$

Let us put

$$F(X, Y) = g(\phi X, Y). \quad (2.6)$$

Then the tensor field F of (0,2) type is symmetric i.e.

$$F(X, Y) = F(Y, X), \quad (2.7)$$

$$(D_X \phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi. \quad (2.8)$$

where D denotes the operator of covariant differentiation with respect to the Lorentzian metric g .

It can easily be seen that in an LP-Sasakian manifold the following relation hold:

$$\phi\xi = 0, \quad \eta(\phi X) = 0. \quad (2.9)$$

Also, an LP-Sasakian manifold M^n are said to be Einstein and η -Einstein if its Ricci tensor S is of the form

$$\begin{aligned} S(X, Y) &= a' g(X, Y) \\ \text{and } S(X, Y) &= a' g(X, Y) + b' \eta(X)\eta(Y), \end{aligned} \quad (2.10)$$

for any vector field X, Y where a', b' are function on M^n . Further on such a LP-Sasakian manifold with (ϕ, η, ξ, g) structure the following relation holds (Matsumoto and Mihai [6] and De, Matsumoto and Shaikh [9])

$$g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y), \quad (2.11)$$

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \quad (2.12)$$

$$R(\xi, X)\xi = X + \eta(X), \quad (2.13)$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \quad (2.14)$$

$$S(X, \xi) = (n-1)\eta(X), \quad (2.15)$$

$$Q\xi = (n-1)\xi \quad (2.16)$$

$$S(\phi X, \phi Y) = S(X, Y) + (n-1)\eta(X)\eta(Y). \quad (2.17)$$

The notion of pseudo- W_2 curvature tensor \widetilde{W}_2 was given by (Prasad and Maurya [2])

$$\begin{aligned} \widetilde{W}_2(X, Y)Z &= aR(X, Y)Z + b[g(Y, Z)QX - g(X, Z)QY] - \\ &\quad \frac{r}{n} \left[\frac{a}{n-1} + b \right] [g(Y, Z)X - g(X, Z)Y]. \end{aligned} \quad (2.18)$$

If $a = 1$, $b = -\frac{1}{n-1}$, then (2.18) takes the form

$$\begin{aligned}\widetilde{W}_2(X, Y)Z &= R(X, Y)Z - \frac{1}{n-1}[g(Y, Z)QX - g(X, Z)QY] \\ &= W_2(X, Y)Z.\end{aligned}\quad (2.19)$$

Thus W_2 -curvature is particular case of \widetilde{W}_2 -curvature, where a and b are real constant and R, Q and r are the Riemannian curvature tensor of type (1,3), the Ricci operator defined by $g(QX, Y) = S(X, Y)$ and scalar curvature respectively. Prasad et. al [15] and Kumar [14] extended this notation on LP-Sasakian manifold and LP-Sasakian manifold with coefficient α .

On the other hand, the projective curvature tensor P and the concircular tensor L in a Riemannian manifold (M^n, g) defined by (Mishra [8])

$$P(X, Y)Z = R(X, Y)Z - \frac{1}{n-1}[S(Y, Z)X - S(X, Z)Y], \quad (2.20)$$

and

$$L(X, Y)Z = R(X, Y)Z - \frac{r}{n(n-1)}[g(Y, Z)X - g(X, Z)Y]. \quad (2.21)$$

Putting ξ for X in (2.18), (2.19) (2.20) and (2.21) and using (2.14),(2.15) and (2.16), we have

$$\widetilde{W}_2(\xi, Y)Z = k[g(Y, Z)\xi - \eta(Z)Y] + b[(n-1)g(Y, Z)\xi - \eta(Z)QY], \quad (2.22)$$

where $k = a - \frac{r}{n} \left[\frac{a}{n-1} + b \right]$.

$$W_2(\xi, Y)Z = [g(Y, Z)\xi - \eta(Z)Y] - \frac{1}{n-1}[(n-1)g(Y, Z)\xi - \eta(Z)QY], \quad (2.23)$$

$$P(\xi, Y)Z = g(Y, Z)\xi - \frac{1}{n-1}S(Y, Z)\xi, \quad (2.24)$$

$$L(\xi, Y)Z = \left[1 - \frac{r}{n(n-1)} \right] [g(Y, Z)\xi - \eta(Z)Y]. \quad (2.25)$$

Now, we define $P(X, Y) \cdot \widetilde{W}_2$, $L(X, Y) \cdot \widetilde{W}_2$, $\widetilde{W}_2(X, Y) \cdot \widetilde{W}_2$, $\widetilde{W}_2(X, Y) \cdot L$, $W_2(X, Y) \cdot \widetilde{W}_2$ and $\widetilde{W}_2(X, Y) \cdot R$ as follows:

$$\begin{aligned}(P(X, Y) \cdot \widetilde{W}_2)(U, V)W &= P(X, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(P(X, Y)U, V)W - \\ &\quad \widetilde{W}_2(U, P(X, Y)V)W - \widetilde{W}_2(U, V)P(X, Y)W,\end{aligned}\quad (2.26)$$

$$\begin{aligned}(L(X, Y) \cdot \widetilde{W}_2)(U, V)W &= L(X, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(L(X, Y)U, V)W - \\ &\quad \widetilde{W}_2(U, L(X, Y)V)W - \widetilde{W}_2(U, V)L(X, Y)W,\end{aligned}\quad (2.27)$$

$$\begin{aligned}
(\widetilde{W}_2(X, Y). \widetilde{W}_2)(U, V)W &= \widetilde{W}_2(X, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(\widetilde{W}_2(X, Y)U, V)W - \\
&\quad \widetilde{W}_2(U, \widetilde{W}_2(X, Y)V)W - \widetilde{W}_2(U, V)\widetilde{W}_2(X, Y)W,
\end{aligned} \tag{2.28}$$

$$\begin{aligned}
(W_2(X, Y). \widetilde{W}_2)(U, V)W &= W_2(X, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(W_2(X, Y)U, V)W - \\
&\quad \widetilde{W}_2(U, W_2(X, Y)V)W - \widetilde{W}_2(U, V)W_2(X, Y)W,
\end{aligned} \tag{2.29}$$

$$\begin{aligned}
(\widetilde{W}_2(X, Y). L)(U, V)W &= \widetilde{W}_2(X, Y)L(U, V)W - L(\widetilde{W}_2(X, Y)U, V)W - \\
&\quad L(U, \widetilde{W}_2(X, Y)V)W - L(U, V)\widetilde{W}_2(X, Y)W,
\end{aligned} \tag{2.30}$$

$$\begin{aligned}
(\widetilde{W}_2(X, Y). R)(U, V)W &= \widetilde{W}_2(X, Y)R(U, V)W - R(\widetilde{W}_2(X, Y)U, V)W - \\
&\quad R(U, \widetilde{W}_2(X, Y)V)W - R(U, V)\widetilde{W}_2(X, Y)W.
\end{aligned} \tag{2.31}$$

3. LP-Sasakian manifold satisfying $P(\xi, Y). \widetilde{W}_2$

In this section we consider a LP-Sasakian manifold M^n satisfying the condition

$$P(\xi, Y). \widetilde{W}_2 = 0. \tag{3.1}$$

From (2.26), we have

$$\begin{aligned}
(P(\xi, Y). \widetilde{W}_2)(U, V)W &= P(\xi, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(P(\xi, Y)U, V)W - \\
&\quad \widetilde{W}_2(U, P(\xi, Y)V)W - \widetilde{W}_2(U, V)P(\xi, Y)W,
\end{aligned}$$

Using (3.1), we have

$$\begin{aligned}
P(\xi, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(P(\xi, Y)U, V)W - \\
\widetilde{W}_2(U, P(\xi, Y)V)W - \widetilde{W}_2(U, V)P(\xi, Y)W &= 0.
\end{aligned} \tag{3.2}$$

Taking the inner product with X and using (2.24) and (3.2), we get

$$\begin{aligned}
&[g(Y, \widetilde{W}_2(U, V)W)\eta(X) - g(Y, U)g(\widetilde{W}_2(\xi, V)W, X) - \\
&g(Y, V)g(\widetilde{W}_2(U, \xi)W, X) - g(Y, W)g(\widetilde{W}_2(U, V)\xi, X)] - \\
&\frac{1}{n-1}[S(Y, \widetilde{W}_2(U, V)W)\eta(X) - S(Y, U)g(\widetilde{W}_2(\xi, V)W, X) \\
&- S(Y, V)g(\widetilde{W}_2(U, \xi)W, X) - S(Y, W)g(\widetilde{W}_2(U, V)\xi, X)] = 0.
\end{aligned} \tag{3.3}$$

Taking ξ for U in (3.3) and using (2.4) and (2.15), we have

$$\begin{aligned} & [g(Y, \widetilde{W}_2(\xi, V)W)\eta(X) - g(Y, W)g(\widetilde{W}_2(\xi, V)\xi, X)] - \\ & \frac{1}{n-1} [S(Y, \widetilde{W}_2(\xi, V)W)\eta(X) - S(Y, W)g(\widetilde{W}_2(\xi, V)\xi, X)] = 0, \end{aligned} \quad (3.4)$$

Using (2.23) in (3.4), we have

$$\begin{aligned} & -k[\{g(Y, V)\eta(W)\eta(X) + g(Y, W)\eta(V)\eta(X) + g(Y, W)g(V, X)\} \\ & - \frac{1}{n-1} \{S(Y, V)\eta(W)\eta(X) + S(Y, W)\eta(V)\eta(X) + S(Y, W)g(V, X)\}] \\ & - b(n-1)[\{S(Y, V)\eta(W)\eta(X) + g(Y, W)\eta(V)\eta(X) + g(Y, W)S(V, X)\} \\ & - \frac{1}{n-1} \{S(QY, V)\eta(W)\eta(X) + S(Y, W)\eta(V)\eta(X) + S(Y, W)g(V, X)\}] = 0, \end{aligned} \quad (3.5)$$

where $S(QY, Z) = S^2(Y, Z)$.

Putting e_i for X and W in (3.5), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over e_i , $1 \leq i \leq n$, we get

$$[r - n(n-1)][b(n-1)S(V, X) + kg(V, X) + \{k + b(n-1)\}\eta(V)\eta(X)] = 0. \quad (3.6)$$

From (3.6), we have

$$\begin{aligned} & [r - n(n-1)] = 0, \quad \text{and} \quad b(n-1)S(V, X) + kg(V, X) \\ & + \{k + b(n-1)\}\eta(V)\eta(X) = 0, \end{aligned} \quad (3.7)$$

respectively. In the case of $r - n(n-1) \neq 0$, if $b = 0$, then from (3.7), we get

$$a[r - n(n-1)] = 0.$$

But $r - n(n-1) \neq 0$, we get $a = 0$.

This is the contradiction. Thus we find $b \neq 0$.

In view of (3.7), we have

$$\begin{aligned} S(V, X) = & \left[\frac{a}{b(1-n)} + \frac{r}{n} \left(\frac{a}{b(n-1)^2} \right) \right] g(V, X) + \\ & \left[\frac{a}{b(1-n)} + \frac{r}{n} \left(\frac{a}{b(n-1)^2} \right) + 1 \right] \eta(V)\eta(X), \end{aligned}$$

which is an η -Einstein manifold.

Hence we have the following theorem:

Theorem 3.1. Let M^n be an n -dimensional ($n > 2$) LP-Sasakian manifold satisfying the condition $P(\xi, Y) \cdot \widetilde{W}_2 = 0$. Then M^n is a part of

- (i) $r = n(n-1)$, that is, scalar curvature is constant or
- (ii) an η -Einstein manifold, provided, $a \neq 0$, and $b \neq 0$.

4. LP-Sasakian manifold satisfying $\widetilde{W}_2(\xi, Y) \cdot \widetilde{W}_2$

In this section we consider a LP-Sasakian manifold M^n satisfying the condition

$$\widetilde{W}_2(\xi, Y) \cdot \widetilde{W}_2 = 0. \quad (4.1)$$

From (2.28), we have

$$\begin{aligned} (\widetilde{W}_2(\xi, Y) \cdot \widetilde{W}_2)(U, V)W &= \widetilde{W}_2(\xi, Y) \widetilde{W}_2(U, V)W - \widetilde{W}_2(\widetilde{W}_2(\xi, Y)U, V)W - \\ &\quad \widetilde{W}_2(U, \widetilde{W}_2(\xi, Y)V)W - \widetilde{W}_2(U, V) \widetilde{W}_2(\xi, Y)W, \end{aligned}$$

Using (4.1), we have

$$\begin{aligned} &\widetilde{W}_2(\xi, Y) \widetilde{W}_2(U, V)W - \widetilde{W}_2(\widetilde{W}_2(\xi, Y)U, V)W - \\ &\quad \widetilde{W}_2(U, \widetilde{W}_2(\xi, Y)V)W - \widetilde{W}_2(U, V) \widetilde{W}_2(\xi, Y)W = 0. \end{aligned} \quad (4.2)$$

Taking the inner product with X and putting ξ for U in (4.2), we have

$$\begin{aligned} &g(\widetilde{W}_2(\xi, Y) \widetilde{W}_2(\xi, V)W, X) - g(\widetilde{W}_2(\widetilde{W}_2(\xi, Y)\xi, V)W, X) - \\ &\quad g(\widetilde{W}_2(\xi, \widetilde{W}_2(\xi, Y)V)W, X) - g(\widetilde{W}_2(\xi, V) \widetilde{W}_2(\xi, Y)W, X) = 0. \end{aligned} \quad (4.3)$$

Putting e_i for X in (4.3), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over e_i , $1 \leq i \leq n$, we get

$$\begin{aligned} &g(\widetilde{W}_2(\xi, Y) \widetilde{W}_2(\xi, e_i)W, e_i) - g(\widetilde{W}_2(\widetilde{W}_2(\xi, Y)\xi, e_i)W, e_i) - \\ &\quad g(\widetilde{W}_2(\xi, \widetilde{W}_2(\xi, Y)e_i)W, e_i) - g(\widetilde{W}_2(\xi, e_i) \widetilde{W}_2(\xi, Y)W, e_i) = 0. \end{aligned} \quad (4.4)$$

In view of (2.18), (2.22) and (4.4), we have

$$\begin{aligned} &(a-b) \left[bS(QY, W) - \left(k - \frac{rb}{n} \right) S(Y, W) \right] \\ &\quad + (n-1) [k\{a+b(n-2)\} - b^2\{r-(n-1)\}] g(Y, W) = 0. \end{aligned} \quad (4.5)$$

If $b = 0$ and $k = a \left[1 - \frac{r}{n(n-1)} \right]$, then view of (4.5), we have

$$a \left[\frac{r}{n(n-1)} - 1 \right] [S(Y, W) - (n-1)g(Y, W)] = 0.$$

If $a \neq 0$, then

$$S(Y, W) = (n-1)g(Y, W), \quad \text{or} \quad r = n(n-1).$$

Hence we have the following theorem:

Theorem 4.1. Let M^n be an n -dimensional ($n > 2$) LP-Sasakian manifold satisfying the condition $\widetilde{W}_2(\xi, Y) \cdot \widetilde{W}_2 = 0$. Then we get

- (i) if $b = 0$, then M^n is an Einstein manifold,
- (ii) if $b \neq 0$, then we get

$$\left[bS(QY, W) - \left(\frac{k}{b} - \frac{r}{n} \right) S(Y, W) \right] - \frac{n-1}{(a-b)b} \\ [k\{a + b(n-2)\} - b^2\{r - (n-1)\}] g(Y, W) = 0.$$

5. LP-Sasakian manifold satisfying $L(\xi, Y) \cdot \widetilde{W}_2$

In this section we consider a LP-Sasakian manifold M^n satisfying the condition

$$L(\xi, Y) \cdot \widetilde{W}_2 = 0. \quad (5.1)$$

From (2.27), we have

$$(L(\xi, Y) \cdot \widetilde{W}_2)(U, V)W = L(\xi, Y) \cdot \widetilde{W}_2(U, V)W - \widetilde{W}_2(L(\xi, Y)U, V)W - \\ \widetilde{W}_2(U, L(\xi, Y)V)W - \widetilde{W}_2(U, V)L(\xi, Y)W.$$

Using (5.1), we have

$$L(\xi, Y) \cdot \widetilde{W}_2(U, V)W - \widetilde{W}_2(L(\xi, Y)U, V)W - \\ \widetilde{W}_2(U, L(\xi, Y)V)W - \widetilde{W}_2(U, V)L(\xi, Y)W = 0. \quad (5.2)$$

Taking the inner product with X in (5.2) and using (2.25), we have

$$\left[1 - \frac{r}{n(n-1)} \right] [g(Y, \widetilde{W}_2(U, V)W) \eta(X) - \eta(\widetilde{W}_2(U, V)W) g(X, Y) \\ - g(Y, U) g(\widetilde{W}_2(\xi, V)W, X) + \eta(U) g(\widetilde{W}_2(Y, V)W, X) - \\ g(Y, V) g(\widetilde{W}_2(u, \xi)W, X) + \eta(V) g(\widetilde{W}_2(U, Y)W, X) - \\ g(\widetilde{W}_2(U, V)\xi, X) g(Y, W) + g(\widetilde{W}_2(U, V)Y, X) \eta(W)] = 0. \quad (5.3)$$

Again from (2.25), we have $r \neq n(n-1)$. Thus from (5.3), we have

$$\begin{aligned} & g(Y, \widetilde{W}_2(U, V)W)\eta(X) - \eta(\widetilde{W}_2(U, V)W)g(X, Y) \\ & - g(Y, U)g(\widetilde{W}_2(\xi, V)W, X) + \eta(U)g(\widetilde{W}_2(Y, V)W, X) - \\ & g(Y, V)g(\widetilde{W}_2(U, \xi)W, X) + \eta(V)g(\widetilde{W}_2(U, Y)W, X) - \\ & g(\widetilde{W}_2(U, V)\xi, X)g(Y, W) + g(\widetilde{W}_2(U, V)Y, X)\eta(W) = 0. \end{aligned} \quad (5.4)$$

Taking ξ for U in (5.4) and using (2.15), we have

$$\begin{aligned} & [g(V, W)g(X, Y) - g(Y, W)g(V, X)] + b[(n-1)g(V, W)g(X, Y) - \\ & S(Y, V)\eta(W)\eta(X) + (n-1)g(X, Y)\eta(V)\eta(W) + S(V, X)\eta(Y)\eta(W) \\ & - S(X, Y)\eta(V)\eta(W) - g(Y, W)S(V, X) + (n-1)g(V, Y)\eta(W)\eta(X) \\ & - S(V, X)\eta(Y)\eta(W)] - a'R(Y, V, W, X) - b[S(X, Y)g(V, W) \\ & - S(V, X)g(Y, W)] + \frac{r}{n} \left(\frac{a}{n-1} + b \right) [g(X, Y)g(V, W) \\ & - g(V, X)g(Y, W)] = 0. \end{aligned} \quad (5.5)$$

Putting e_i for Y and W in (5.5), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over e_i , $1 \leq i \leq n$, we get

$$\begin{aligned} (a-b)S(V, W) &= [a(n-1) + bn(n-1) - br]g(V, W) + \\ & b[n(n-1) - r]\eta(V)\eta(W). \end{aligned} \quad (5.6)$$

Hence we have the following theorem :

Theorem 5.1. An n -dimensional ($n > 2$) LP-Sasakian manifold M^n satisfying the condition $L(\xi, Y).\widetilde{W}_2=0$ is an η -Einstein manifold, provided $a-b \neq 0$ and $r \neq n(n-1)$.

6. LP-Sasakian manifold satisfying $\widetilde{W}_2(\xi, Y).L$

In this section we consider a LP-Sasakian manifold M^n satisfying the condition

$$\widetilde{W}_2(\xi, Y).L = 0. \quad (6.1)$$

From (2.30), we have

$$\begin{aligned} (\widetilde{W}_2(\xi, Y).L)(U, V)W &= \widetilde{W}_2(\xi, Y)L(U, V)W - L(\widetilde{W}_2(\xi, Y)U, V)W - \\ & L(U, \widetilde{W}_2(\xi, Y)V)W - L(U, V)\widetilde{W}_2(\xi, Y)W. \end{aligned}$$

Using (6.1), we have

$$\begin{aligned} & \widetilde{W}_2(\xi, Y)L(U, V)W - L(\widetilde{W}_2(\xi, Y)U, V)W - \\ & L(U, \widetilde{W}_2(\xi, Y)V)W - L(U, V)\widetilde{W}_2(\xi, Y)W = 0. \end{aligned} \quad (6.2)$$

Taking the inner product with X in (6.2) and using (2.22), we have

$$\begin{aligned} & (\widetilde{W}_2(\xi, Y)L(U, V)W, X) - g(L(\widetilde{W}_2(\xi, Y)U, V)W, X) - \\ & g(L(U, \widetilde{W}_2(\xi, Y)V)W, X) - g(L(U, V)\widetilde{W}_2(\xi, Y)W, X) = 0. \end{aligned} \quad (6.3)$$

Putting ξ for U in (6.3) and using (2.4), (2.1) and (2.25), we have

$$\begin{aligned} & \left[1 - \frac{r}{n(n-1)}\right] [k + b][g(V, W)g(X, Y) - g(Y, W)g(V, X) + \\ & 2g(V, W)S(X, Y) - (n-1)g(Y, V)\eta(W)\eta(X) + \eta(V)\eta(X)S(Y, W) - \\ & (n-1)g(Y, W)\eta(V)\eta(X) - (n-1)g(Y, W)g(V, X) + \eta(W)\eta(X)S(V, Y)] - \\ & k \left[g(R(Y, V)W, X) - \frac{r}{(n(n-1))} \{g(V, W)g(X, Y) - g(Y, W)g(V, X)\} \right] - \\ & b \left[g(R(QY, V)W, X) - \frac{r}{(n(n-1))} \{g(V, W)S(X, Y) - S(Y, W)g(V, X)\} \right] = 0. \end{aligned} \quad (6.4)$$

Putting e_i for X and V in (6.4), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over e_i , $1 \leq i \leq n$, we get

$$\begin{aligned} & bS(QY, W) - \left[\frac{br}{n-1} - k - b \right] S(Y, W) \\ & - (n-1) \left[k + (n-1)b - \frac{br}{n} \right] g(Y, W) = 0, \end{aligned}$$

when $b = 0$, then above equation can be written as

$$k[S(Y, W) - (n-1)g(Y, W)] = 0,$$

which means that $k[r - n(n-1)] = 0$. From (2.25), we find $r \neq n(n-1)$. Thus we get $k = 0$; namely $a = 0$. This is the contradiction. Therefore we get $b \neq 0$ and

$$\begin{aligned} S(QY, W) &= \left[\frac{r}{n-1} - \frac{k}{b} - 1 \right] S(Y, W) - \\ & (n-1) \left[\frac{r}{n} - \frac{k}{b} - (n-1) \right] g(Y, W). \end{aligned} \quad (6.5)$$

Hence we have the following theorem:

Theorem 6.1. In an n -dimensional ($n > 2$) LP-Sasakian manifold M^n satisfying the condition $\widetilde{W}_2(\xi, Y).L = 0$ holds on M^n , then the equation (6.5) is satisfied on M^n , provided $a \neq 0$ and $b \neq 0$.

7. LP-Sasakian manifold satisfying $W_2(\xi, Y).\widetilde{W}_2$

In this section we consider a LP-Sasakian manifold M^n satisfying the condition

$$W_2(\xi, Y).\widetilde{W}_2 = 0 \quad (7.1)$$

From (2.29), we have

$$\begin{aligned} (W_2(\xi, Y).\widetilde{W}_2)(U, V)W &= W_2(\xi, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(W_2(\xi, Y)U, V)W - \\ &\quad \widetilde{W}_2(U, W_2(\xi, Y)V)W - \widetilde{W}_2(U, V)W_2(\xi, Y)W, \end{aligned}$$

Using (7.1), we have

$$\begin{aligned} W_2(\xi, Y)\widetilde{W}_2(U, V)W - \widetilde{W}_2(W_2(\xi, Y)U, V)W - \\ \widetilde{W}_2(U, W_2(\xi, Y)V)W - \widetilde{W}_2(U, V)W_2(\xi, Y)W = 0. \end{aligned} \quad (7.2)$$

Taking the inner product with X and putting ξ for U , we have

$$\begin{aligned} g(W_2(\xi, Y)\widetilde{W}_2(\xi, V)W, X) - g(\widetilde{W}_2(W_2(\xi, Y)\xi, V)W, X) - \\ g(\widetilde{W}_2(\xi, W_2(\xi, Y)V)W, X) - W(\widetilde{W}_2(\xi, V)W_2(\xi, Y)W, X) = 0. \end{aligned} \quad (7.3)$$

Putting e_i for X and V in (7.3), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over e_i , $1 \leq i \leq n$, we get

$$(a - b) \left[S(QY, W) - \left\{ \frac{r}{n} + (n - 1) \right\} S(Y, W) + \frac{r}{n(n - 1)} g(Y, W) \right] = 0.$$

Hence we have the following theorem:

Theorem 7.1. In an n -dimensional ($n > 2$) LP-Sasakian manifold satisfying the condition $W_2(\xi, Y).\widetilde{W}_2 = 0$, then we get

- (i) if $a - b = 0$, or
- (ii) if $a - b \neq 0$, then

$$S(QY, W) = \left\{ \frac{r}{n} + (n - 1) \right\} S(Y, W) - \frac{r}{n(1 - n)} g(Y, W).$$

8. LP-Sasakian manifold satisfying $\widetilde{W}_2(\xi, Y).R$

In this section we consider a LP-Sasakian manifold M^n satisfying the condition

$$\widetilde{W}_2(\xi, Y).R = 0. \quad (8.1)$$

From (2.31) and (8.1), we have

$$\begin{aligned} (\widetilde{W}_2(\xi, Y).R)(U, V)W = & \widetilde{W}_2(\xi, Y)R(U, V)W - L(\widetilde{W}_2(\xi, Y)U, V)W - \\ & R(U, \widetilde{W}_2(\xi, Y)V)W - R(U, V)\widetilde{W}_2(\xi, Y)W, \end{aligned} \quad (8.2)$$

Taking the inner product with X and putting ξ for U in (8.2), we have

$$\begin{aligned} & k[g(V, W)g(X, Y) - g(R(Y, V)W, X) - g(Y, W)g(V, X) - g(V, X)(W)\eta(Y)] \\ & + b[g(V, W)S(X, Y) - (n-1)\eta(W)\eta(X)g(Y, V) - g(R(QY, V)W, X) \\ & + \eta(V)(X)S(Y, W) - (n-1)\eta(V)(X)g(Y, W) \\ & - (n-1)g(Y, W)g(V, X) + \eta(W)\eta(X)S(Y, W)] = 0. \end{aligned} \quad (8.3)$$

Putting e_i for X and V in (8.3), $\{e_i\}$ is an orthonormal basis of the tangent space at a point of the manifold and taking summation over e_i , $1 \leq i \leq n$, we get

$$bS(QY, W) + kS(Y, W) - (n-1)[k - (n-1)]g(Y, W) = 0. \quad (8.4)$$

If $b = 0$, then equation (8.4) will be

$$a[r - n(n-1)][S(Y, W) - (n-1)g(Y, W)] = 0,$$

Thus, we have

$$S(Y, W) = (n-1)g(Y, W).$$

Hence we have the following theorem:

Theorem 8.1. In an n -dimensional ($n > 2$) LP-Sasakian manifold M^n satisfying the condition $\widetilde{W}_2(\xi, Y).R = 0$, then we get

- (i) if $b = 0$, then the manifold is Einstein manifold, provided $a \neq 0$.
- (ii) if $b \neq 0$, then

$$S(QY, W) = -\frac{k}{b}S(Y, W) + (n-1)[k - (n-1)]g(Y, W).$$

REFERENCES

- [1] Prasad, B.: *A semi-symmetric non-metric p -connection in a LP-sasakian manifold*, Bull. Cal. Math. Soc., 90 (1998), 357-360.
- [2] Prasad, B. and Maurya, A.: *Pseudo W_2 -curvature tensor \widetilde{W}_2 on a Riemannian manifold*, Journal of Pure Math., 21 (2004) 81-85.
- [3] Mihai, I. and Rosca, R.: *On Lorentzian p -Sasakian manifolds*, Classical Analysis, World Scientific Publ. 1992.
- [4] Matsumoto, K.: *On Lorentzian para contact manifolds*, Bull. of Yamagata Univ., Nat. Sci., 12 (1989), 151-156.
- [5] Matsumoto, K. and Mihai, I.: *On Lorentzian p -Sasakian manifolds*, Classical Analysis, world Scientific Publ. 1992.
- [6] Matsumoto, K. and Mihai, I.: *On a certain transformation in Lorentzian para Sasakian manifold*, Tensor, N. S., 47 (1988), 189-197.
- [7] Trafdar, M. and Bhattacharyya, A.: *On Lorentzian para Sasakian manifolds, step in Differential Geometry*, Proceeding of the colloquium on Differential Geometry, 25-30 (2000), Dbrecen, Hungary, 343-348.
- [8] Mishra, R. S.: *Structures on a differentiable manifold and their applications*, Chardrama Prakashan, 50A Bolrampur house, Allahabad, India, (1984).
- [9] De, U. C., Matsumoto, K. and Shaikh, A. A.: *On Lorentzian para Sasakian manifolds*, Rendicontidel Seminario Matematico di Messina, Serie II, Supplementoal n. 3 (1999), 119-158.
- [10] Venkatesha and Bagewadi, C. S.: *On concircular ϕ -recurrent LP-Sasakian manifolds*, Differential Geometry-Dynamical Systems, 10 (2008), 312-319.
- [11] Shaikh, A. A., Prakasha, D. G. and Ahmad, H.: *On generalized ϕ -recurrent LP-Sasakian manifolds*, Journal of E. Math. Soc., 23 (2015), 161-166.
- [12] Prasad, R. and Haseeb, A.: *On a Lorentzian para Sasakian manifold with respect to quarter symmetric metric connection*, Nov. J. Math., 46(2) (2016), 103-116.
- [13] Berman, A.: *On LP-Sasakian manifold a semi symmetric non-metric connection*, KYUNGPOOK Math. J., 8 (2018), 105-116.
- [14] Kumar, R.: *On pseudo \widetilde{W}_2 -flat LP-Sasakian manifold with coefficient α* , Science Vision, 12(1) (2012), 17-21.
- [15] Prasad, B., Gupta, N. and Pandey, S. N.: *Pseudo W_2 -curvature tensor \widetilde{W}_2 on a LP-Sasakian manifold*, JPS, 1(2) (2010).