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3-dimensional Locally ϕ –Concircularly symmetric Lorentzian β –Kenmotsu Manifold

Kripa Sindhu Prasad

Department of Mathematics

Thakur Ram Multiple Campus Birgunj

Tribhuvan University, Nepal

e-mail: kripasindhuchaudhary@gmail.com

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Abstract

The present work deals with the study of 3-dimensional Locally ϕ – concircularly symmetric Lorentzian β –Kenmotsu manifold which generalizes the notion of locally concircularly-symmetric Lorentzian β –Kenmotsu manifold and obtain some interesting results. Also it is proved that a concircularly ϕ –recurrent Lorentzian β –Kenmotsu manifolds is an Einstein manifold and Proved that if a concircularly ϕ –recurrent Lorentzian β –Kenmotsu manifolds $(m^{2n+1}, g), n > 1$, has non zero constant sectional curvature, then it reduces to a concircularly locally ϕ –symmetric manifold.

Keywords and Phrases: Concircularly ϕ –symmetric manifold, concircularly ϕ –recurrent manifold, Einstein manifold, Lorentzian β –Kenmotsu manifolds.

AMS Subject Classification: 53C25, 53C44, 53D10.

1. Introduction

A transformation of an n -dimensional Riemannian manifold M , which transforms every geodesic circle of M into a geodesic circle, is called a concircular transformation. A concircular transformation is always a conformal transformation. Here a geodesic circle means a curve in M whose first curvature is constant and second curvature is identically zero. Thus the geometry of concircular transformations, that is the concircular geometry, is a generalization of inverse geometry in the sense that the change of metric is more general than that introduced by a circle preserving diffeomorphism. An interesting invariant of a concircular transformation is the concircular curvature tensor.

In this paper, we study a concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifold which generalizes the notion of locally concircularly ϕ -symmetric Lorentzian β -Kenmotsu manifold and obtain some interesting results. Again it is proved that a concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifold is an Einstein manifold and proved that if a concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifold (M^{2n+1}, g) , $n > 1$ has non-zero constant sectional curvature, then it reduces to a locally concircularly ϕ -symmetric manifold. Further we study 3-dimensional locally ϕ -concircularly symmetric Lorentzian β -Kenmotsu manifold.

2. Preliminaries

An $(2n + 1)$ dimensional differentiable manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is called a Lorentzian β -Kenmotsu manifold with the structure (ϕ, ξ, η, g) where β is a smooth function on M if it admits a tensor field ϕ of type $(1,1)$, a covariant vector field ξ , one 1 from η and a Lorentzian metric g which satisfy:

$$\phi^2 x = -x + \eta(x)\xi \quad (2.1)$$

$$(a) \quad \eta(\xi) = -1, \quad (b) \quad g(x, \xi) = \eta(x), \quad (c) \quad \eta(\phi x) = 0 \quad (2.2)$$

$$g(\phi x, \phi y) = g(x, y) + \eta(x)\eta(y) \quad (2.3)$$

$$(D_x \phi)(y) = g(x, y)\xi - \eta(y)x \quad (2.4)$$

$$D_x \xi = \beta(x - \eta(x)\xi) \quad (2.5)$$

$$(D_x \eta)(Y) = \beta[(g(x, Y)) - \eta(x)\eta(Y)] \quad (2.6)$$

where D denotes the operator of covariant differentiation with respect to g . Also in Lorentzian β -Kenmotsu manifold, the following holds:

$$\eta(R(X, Y)Z) = g(R(X, Y)Z, \xi) = \beta^2[g(X, Z)\eta(Y) - g(Y, Z)\eta(X)] \quad (2.7)$$

$$R(X, Y)\xi = \beta^2[\eta(X)Y - \eta(Y)X] \quad (2.8)$$

$$S(\phi X, \phi Y) = S(X, Y)2n\beta^2\eta(X)\eta(Y) \quad (2.9)$$

$$S(x, \xi) = 2n\beta^2\eta(X) \quad (2.10)$$

For all vector fields X, Y, Z where S is the Ricci tensor of type $(0,2)$ and R is the Riemannian Curvature tensor of the manifold.

Definition 2.1. A Lorentzian β -Kenmotsu manifold is said to be a locally ϕ -symmetric manifold if.

$$\phi^2((D_w R)(X, Y)Z) = 0. \quad (2.11)$$

For all vector field X, Y, Z, W Orthogonal to ξ .

Definition 2.2. A Lorentzian β -Kenmotsu manifolds is said to be a locally concircularly ϕ -symmetric manifold if.

$$\phi^2((D_w C)(X, Y)Z) = 0. \quad (2.12)$$

For all vector field X, Y, Z, W Orthogonal to ξ .

Definition 2.3. A Lorentzian β -Kenmotsu manifold is said to be concircularly ϕ -recurrent Lorentzian β -Kenmostu manifold if there exists a non -zero 1 form A such that.

$$\phi^2((D_w C)(X, Y)Z) = A(W)C(X, Y)Z, \quad (2.13)$$

for arbitrary vector fields X, Y, Z, W where C is a concircular curvature tensor given by

$$C(X, Y)Z = R(X, Y)Z - \frac{r}{2n(2n+1)}[g(Y, Z)Xg(X, Z)Y], \quad (2.14)$$

where R is the curvature tensor and r is the Scalar curvature. If the 1-form A vanishes, then the manifold reduces to a locally Concircularly ϕ - symmetric manifold.

3. Concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifolds

Let us consider a concircularly ϕ -recurrent Lorentzian β -Kenmostu manifolds. Then by virtue of (2.1) and (2.13) we get

$$(D_w C)(X, Y)Z + \eta((D_w C)(X, Y)Z)\xi = A(W)C(X, Y)Z, \quad (3.1)$$

from which it follows that

$$g((D_w C)(X, Y)Z, U) + \eta((D_w C)(X, Y)Z)\eta(U) = A(W)g(C(X, Y)Z, U) \quad (3.2)$$

Let $\{e_i\}, i = 1, 2, \dots, 2n+1$ be an orthonormal basis of the tangent space at any point of the manifold, Then putting $X = U = e_i$ in (3.2) and taking summation over $i, 1 \leq i \leq 2n+1$, we get.

$$\begin{aligned} (D_w S)(Y, Z) = & -\frac{dr(W)}{2n+1}g(Y, Z) - \frac{dr(W)}{2n+1}g(Y, Z) - \eta(Y)\eta(Z) \\ & + A(W)[S(Y, Z) - \frac{r}{2n+1}g(Y, Z)]. \end{aligned} \quad (3.3)$$

Replacing Z by ξ in (3.3) and using (2.5) and (2.10), we get

$$(D_w S)(Y, \xi) = \frac{dr(W)}{2n+1} \eta(y) - \frac{dr(W)}{2n(2n+1)} 2\eta(y) + A(W) \left[-2n\beta - \frac{r}{2n+1} \right] \eta(Y). \quad (3.4)$$

Now, we have

$$(D_w S)(Y, \xi) = D_w S(Y, \xi) - S(D_w Y, \xi) - S(Y, D_w \xi).$$

Using (2.5) and (2.10) in the above relation, it follows that

$$(D_w S)(Y, \xi) = -2n\beta^2 g(Y, W) \beta S(Y, W). \quad (3.5)$$

In view of (3.4) and (3.5), we get

$$\begin{aligned} S(Y, W) = & -2n\beta^2 g(Y, W) - \frac{dr(W)}{2n+1} + \eta(Y) + \frac{dr(W)}{2n(2n+1)} 2\eta(Y) \\ & - A(W) \left[-2n\beta^2 - \frac{r}{2n+1} \right] \eta(Y). \end{aligned} \quad (3.6)$$

Replacing Y by ϕY and W by ϕW in (3.6) and using (2.3) and (2.9), we get

$$S(Y, W) = -2\beta^2 g(Y, W).$$

Hence we can state the following theorem

Theorem 3.1. A Concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifolds M^{2n+1}, g is an Einstein manifold. Now from (2.14) it follows that

$$(D_w C(X, Y)\xi = (D_w R)(X, Y)\xi - \frac{dr(W)}{2n(2n+1)} [\eta(Y)X - \eta(X)Y]. \quad (3.7)$$

In view of (2.5) and (2.6) it can be easily seen that in a Lorentzian β -Kenmotsu manifold the following relation holds:

$$(D_w R(X, Y)\xi = \beta^3 [g(W, Y)X - g(W, X)Y + R(X, Y)W]. \quad (3.8)$$

Using (3.8) in (3.7), we get

$$\begin{aligned} (D_w C(X, Y)\xi = & \beta^3 [g(W, Y)X - g(W, X)Y + R(X, Y)W] \\ & - \frac{dr(W)}{2n(2n+1)} [\eta(Y)X - \eta(X)Y]. \end{aligned} \quad (3.9)$$

Using (2.7) in (3.9) we have

$$\begin{aligned} (D_w C(X, Y)\xi = & \beta^2(\beta - 1)g(W, Y)X - \beta^2(\beta - 1)g(W, X)Y \\ & - \frac{dr(W)}{2n(2n+1)} [\eta(Y)X - \eta(X)Y]. \end{aligned} \quad (3.10)$$

Now, we suppose that Lorentzian β -Kenmotsu (M^{2n+1}, g) , $n > 1$, is concircularly ϕ -recurrent. Then from (3.1) and (3.10) it follows that

$$(D_w C)(X, Y)Z = -\beta^2(\beta - 1)g(W, Y)g(X, Z) + \beta^2(\beta - 1)g(W, X)g(Y, Z) \\ - \frac{dr(W)}{2n(2n+1)}[\eta(Y)g(X, Z) - \eta(X)g(Y, Z)] + A(W)C(X, Y)Z.$$

Next we suppose that in a concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifold, the sectional curvature of a plane $\pi C Tp(M)$ defined by

$$K_p(\pi) = g(R(X, Y)Y, X), \quad (3.11)$$

is a non zero constant K , where (X, Y) is any orthonormal basis of π . Then we have

$$g((D_z R)(X, Y)Y, X) = 0. \quad (3.12)$$

Again from (2.14) we get

$$(D_z C)(X, Y)Y = (D_z R)(X, Y)Y - \frac{dr(Z)}{2n(2n+1)}[g(Y, Y)X - g(X, Y)Y]. \quad (3.13)$$

In view of (3.12) it follow from (3.13) that

$$g((D_z C)(X, Y)Y, X) = 0. \quad (3.14)$$

By Virtue of (3.14) and (3.1) we have

$$g(D_z C)(X, Y)Y, \xi) \eta(X) = A(Z)g(C(X, Y)Y, X). \quad (3.15)$$

Since in a concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifold, the relation (3.12) holds good, using (3.10) in (3.15) we get

$$\eta(X)[\beta^2(\beta - 1)g(Z, Y)g(X, Y) + \beta^2(\beta - 1)g(Z, X)g(Y, Y) \\ - \frac{dr(Z)}{2n(2n+1)}\{\eta(Y)g(X, Y) - \eta(X)g(Y, Y)\} \\ - A(Z)\{\beta^2 - \frac{r}{2n(2n+1)}\}\{g(Y, Y)\eta(X) - g(X, Y)\eta(Y)\}] \\ = A(z)[K\beta^2 - \frac{r}{2n(2n+1)}\{g(Y, Y)g(X, X) - g(X, Y)g(X, Y)\}]. \quad (3.16)$$

Putting $Y = Z = \xi$ in (3.16) and using (2.2) and simplifying we get

$$n(\rho) = 0. \quad (3.17)$$

Hence from the relation $A(W) = -\eta(w)\eta(\rho)$, we get

$$A(W) = 0. \quad (3.18)$$

Using (3.18) in (3.1), we get

$$(\phi^2(D_w C)(X, Y)Z) = 0.$$

Hence we can state the following theorem:

Theorem 3.2. If a concircularly ϕ -recurrent Lorentzian β -Kenmotsu manifolds (M^{2n+1}, g) , $(n > 1)$, has a non-zero constant sectional curvature then it reduces to a locally concircularly ϕ -symmetric manifold.

4. 3-dimensional Locally ϕ -Concircularly symmetric Lorentzian β -Kenmotsu manifold

In a 3-dimensional Lorentzian β -Kenmotsu manifold, concircular curvature tensor is given by

$$\begin{aligned} C(X, Y)Z = & \frac{(r+4)}{2}[g(Y, Z)X - g(X, Z)Y] \frac{(r+6)}{2}[g(Y, Z)\eta(X)\xi \\ & - g(X, Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\ & - \frac{r}{6}[g(Y, Z)X - g(X, Z)Y]. \end{aligned} \quad (4.1)$$

Taking Covariant differentiation of (4.1), we get

$$\begin{aligned} (D_w C)(X, Y)Z = & \frac{dr(W)}{2}[g(Y, Z)Xg(X, Z)Y] - \frac{dr(W)}{2}[g(Y, Z)\eta(X)\xi \\ & g(X, Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] - \frac{(r+6)}{2} \times \\ & [g(Y, Z)(D_w \eta)(X)\xi + g(Y, Z)\eta(X)(D_w \xi)g(X, Z)(D_w \eta)(Y)\xi] \\ & - g(X, Z)\eta(Y)(D_w \xi) + (D_w \eta)(Y)\eta(Z)X + (D_w \eta)(Z)\eta(Y)X \\ & - (D_w \eta)(X)\eta(Z)X - (D_w \eta)(Z)\eta(X)Y \\ & - \frac{dr(W)}{6}[g(Y, Z)X - g(X, Z)Y]. \end{aligned} \quad (4.2)$$

Taking X, Y, Z horizontal vector field and using (2.5) and (2.6), we get

$$\begin{aligned} (D_w C)(X, Y)Z = & \frac{dr(W)}{2}[g(Y, Z)Xg(X, Z)Y] - \frac{(r+6)}{2}\beta[g(Y, Z) \\ & g(X, W) - g(X, Z)g(Y, W)] + g(Y, Z)\eta(W)\eta(X) \\ & + g(X, Z)\eta(W)\eta(Y)]\xi. \end{aligned} \quad (4.3)$$

From (4.3) it follows that

$$\phi^2(D_w C)(X, Y)Z = \frac{dr(W)}{3}[g(Y, Z)\phi^2 X - g(X, Z)\phi^2 Y].$$

Now taking X, Y, Z horizontal vector field and using (2.1), (2.5) and (2.6), we get

$$\phi^2(D_w C)(X, Y)Z = \frac{dr(W)}{3}[g(Y, Z)X - g(X, Z)Y].$$

Hence we state the following.

Theorem 4.1. A 3-dimensional Lorentzian β -Kenmotsu manifold is locally ϕ -concircularly symmetric if and only if scalar curvature r is content.

In 1977 [3] has proved that

Corollary 4.1. A 3-dimensional Lorentzian β -Kenmotsu manifold is locally ϕ -Symetric if and only if scalar curvature r is constant.

Using corollary (4.1) we state the following:

Theorem 4.2. A 3-dimensional Lorentzian β -Kenmotsu manifold is locally ϕ -concircularly symmetric if and only if it is Locally ϕ -Symmetric.

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