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Almost Kenmotsu Manifold Equipped with M -Projective Curvature Tensor

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Abstract

The object of the present paper is to study M -projective curvature tensor on almost Kenmotsu manifolds with characteristic vector field ξ belonging to some nullity distributions.

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1. Introduction

In 1972 Kenmotsu [8] introduced and studied a new class of almost contact metric manifolds called Kenmotsu manifolds. Almost contact metric manifold M^{2n+1} with 1-form η and fundamental 2-form Φ defined by $\Phi(X, Y) = g(X, \phi Y)$, where ϕ is a $(1, 1)$ tensor field such that $d\eta = 0$ and $d\Phi = 2\eta \wedge \Phi$ is called almost Kenmotsu manifold. The normality of an almost contact metric manifold is given by vanishing the $(1, 2)$ -type torsion tensor $N = [\phi, \phi] + 2d\eta \otimes \xi$, where $[\phi, \phi]$ is the Nijenhuis torsion of ϕ [1]. According to [8], the normality of an almost Kenmotsu manifold is given by

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \quad (1.1)$$

for every vector fields X, Y on M^{2n+1} . On the other hand, Gray [7] and Tanno [13] introduced the notion of k -nullity distribution, which is defined for any $p \in M^{2n+1}$ and $k \in \mathbb{R}$ as follows:

$$N_p(k) = \{Z \in T_p M^{2n+1} : R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y]\} \quad (1.2)$$

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for any $X, Y \in T_p M$, where $T_p M^{2n+1}$ denotes the tangent vector space of M^{2n+1} at any point $p \in M^{2n+1}$ and R denotes the Riemannian curvature tensor of type $(1, 3)$. Moreover, if k is a smooth function then the distribution is called generalized k -nullity distribution. Later Blair, Koufogiorgos and Papantoniou [2] introduced a generalized notion of the (k, μ) -nullity distribution on a contact metric manifold M^{2n+1} which is defined for any $p \in M^{2n+1}$ and $(k, \mu) \in \mathbb{R}^2$ as follows:

$$N_p(k, \mu) = \{Z \in T_p M^{2n+1} : R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y] + \mu[g(Y, Z)hX - g(X, Z)hY]\}, \quad (1.3)$$

where $h = \frac{1}{2}\mathcal{L}_\xi\phi$ and \mathcal{L} denotes the Lie differentiation. After this, Dileo and Pastore [5] introduced another generalized notion of the k -nullity distribution called $(k, \mu)'$ -nullity distribution on an almost Kenmotsu manifold M^{2n+1} and is defined for any $p \in M^{2n+1}$ and $(k, \mu) \in \mathbb{R}^2$ as follows:

$$N_p(k, \mu)' = \{Z \in T_p M^{2n+1} : R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y] + \mu[g(Y, Z)h'X - g(X, Z)h'Y]\}, \quad (1.4)$$

where $h' = h \circ \phi$. Recently, Dileo et al. ([5], [6], [7]), Wang et al. ([15], [16], [17]) and De et al. ([3], [9], [14]) obtained some important results on almost Kenmotsu manifolds with characteristic vector field ξ belonging to some nullity distributions. In this paper we investigate some results on almost Kenmotsu manifolds with M -projective curvature tensor W^* defined by [11]

$$W^*(X, Y)Z = R(X, Y)Z - \frac{1}{4n}[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY], \quad (1.5)$$

where X, Y, Z are any vector fields and r is the scalar curvature. The paper is organized as follows: In Section 2, we give some basic formulas and properties of almost Kenmotsu manifolds according to Dileo and Pastore ([5], [7]). In Section 3, we study M -projectively semisymmetric almost Kenmotsu manifolds with ξ belonging to $(k, \mu)'$ -nullity distribution. The Section 4 deals with the study of M -projectively Ricci-semisymmetric almost Kenmotsu manifolds with ξ belonging to $(k, \mu)'$ -nullity distribution. The Section 5 and 6 are concerned with the study of M -projective curvature tensor on almost Kenmotsu manifolds with ξ belonging to (k, μ) -nullity distribution.

2. Almost Kenmotsu manifolds and nullity distributions

Let M^{2n+1} be an almost Kenmotsu manifold with structure (ϕ, ξ, η, g) , where ϕ is a $(1, 1)$ tensor field, ξ is a characteristic vector field and η is a 1-form

and g is a Riemannian metric such that [1]

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0, \quad g(X, \xi) = \eta(X), \quad (2.1)$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (2.2)$$

for all vector fields X, Y on M^{2n+1} . Let D be the distribution orthogonal to ξ and defined by $D = \text{Ker}(\eta) = \text{Im}(\phi)$. The two tensor fields $h = \frac{1}{2}\mathcal{L}_\xi\phi$ and $l = R(\cdot, \xi)\xi$ on an almost Kenmotsu manifold M^{2n+1} are symmetric and satisfy the following relations [10]

$$h\xi = 0, \quad l\xi = 0, \quad \text{tr}(h\phi) = 0, \quad h\phi + \phi h = 0, \quad (2.3)$$

$$\nabla_X \xi = -\phi^2 X - \phi h X, \quad (2.4)$$

$$\phi l \phi - l = 2(h^2 - \phi^2), \quad (2.5)$$

$$\text{tr}(l) = S(\xi, \xi) = g(Q\xi, \xi) = -2n - \text{tr} h^2, \quad (2.6)$$

$$R(X, Y)\xi = \eta(X)(Y - \phi h Y) - \eta(Y)(X - \phi h X) + (\nabla_Y \phi h)X - (\nabla_X \phi h)Y, \quad (2.7)$$

for all vector fields X, Y on M^{2n+1} . Now we give some basic properties of almost Kenmotsu manifolds with ξ belonging to the $(k, \mu)'$ -nullity distribution.

The $(1, 1)$ -type tensor field h' satisfies $h'\phi + \phi h' = 0$ and $h'\xi = 0$. Also it is known that

$$h = 0 \Leftrightarrow h' = 0, \quad h'^2 = (k+1)\phi^2 (\Leftrightarrow h^2 = (k+1)\phi^2). \quad (2.8)$$

For an almost Kenmotsu manifold, we have from (1.4)

$$R(X, Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)h'X - \eta(X)h'Y] \quad \text{and} \quad (2.9)$$

$$R(\xi, X)Y = k[g(X, Y)\xi - \eta(Y)X] + \mu[g(h'X, Y)\xi - \eta(Y)h'X], \quad (2.10)$$

where $k, \mu \in \mathbb{R}$.

Contracting Y in (2.10), we get

$$S(X, \xi) = 2nk\eta(X). \quad (2.11)$$

Let $X \in D$ be the eigen vector of h' corresponding to the eigen value λ orthogonal to ξ . It follows from (2.8) that $\lambda^2 = -(k+1)$, a constant. Therefore, $k \leq -1$ and $\lambda = \pm\sqrt{-k-1}$. We denote $[\lambda]'$ and $[-\lambda]'$ as the corresponding eigenspaces associated with h' corresponding to the non-zero eigen values λ and $-\lambda$ respectively. We have the following lemmas.

Lemma 2.1. ([5], Proposition 4.1) Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold such that ξ belongs to the $(k, \mu)'$ -nullity distribution and $h' \neq 0$. Then $k < -1, \mu = -2$ and $\text{Spec}(h') = \{0, \lambda, -\lambda\}$ with 0 as simple eigen value and $\lambda = \sqrt{-k-1}$. The distributions $[\xi] \oplus [\lambda]'$ and $[\xi] \oplus [-\lambda]'$ are integrable with totally geodesic leaves. The distributions $[\lambda]'$ and $[-\lambda]'$ are integrable with totally umbilical leaves.

Lemma 2.2. ([5], Lemma 4.1) Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold with $h' \neq 0$ and ξ belongs to the $(k, -2)'$ -nullity distribution. Then for every $X, Y \in T_p M$,

$$(\nabla_X h')Y = -g(h'X + h'^2 X, Y)\xi - \eta(Y)(h'X + h'^2 X). \quad (2.12)$$

According to Takahashi [12] and De et al. [4], we have the following definitions:

Definition 2.1. An almost Kenmotsu manifold is said to be ϕ -symmetric if it satisfies

$$\phi^2((\nabla_W R)(X, Y)Z) = 0, \quad (2.13)$$

for all vector fields $W, X, Y, Z \in T_p M^{2n+1}$. In addition, if the vector fields W, X, Y, Z are orthogonal to ξ , then the manifold is called locally ϕ -symmetric

Definition 2.2. An almost Kenmotsu manifold is said to be ϕ -recurrent if it satisfies

$$\phi^2((\nabla_W R)(X, Y)Z) = A(W)R(X, Y)Z \quad (2.14)$$

for any vector fields $W, X, Y, Z \in T_p M^{2n+1}$. In relation (2.14) if the vector fields W, X, Y, Z are orthogonal to ξ , then the manifold is called locally ϕ -recurrent.

3. M -projectively semisymmetric almost Kenmotsu manifold with ξ belongs to $(k, \mu)'$ -nullity distribution

Let M^{2n+1} be M -projectively semisymmetric almost Kenmotsu manifold with ξ belongs to $(k, \mu)'$ -nullity distribution. Then we have

$$(R(X, Y) \cdot W^*)(U, V)W = 0, \quad (3.1)$$

for all vector fields X, Y, U, V, W . This implies

$$\begin{aligned} R(X, Y)W^*(U, V)W - W^*(R(X, Y)U, V)W \\ - W^*(U, R(X, Y)V)W - W^*(U, V)R(X, Y)W = 0. \end{aligned} \quad (3.2)$$

Putting $X = U = \xi$ in (3.2), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W - W^*(R(\xi, Y)\xi, V)W \\ - W^*(\xi, R(\xi, Y)V)W - W^*(\xi, V)R(\xi, Y)W = 0. \end{aligned} \quad (3.3)$$

Using (2.10) and lemma (2.1) in (1.5), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W = & k\{(k + 2nk + \frac{1}{2})g(V, W)\eta(Y)\xi - \frac{3}{2}g(h'V, W)\eta(Y)\xi \\ & - \frac{(k+1)}{2}g(Y, V)\eta(W)\xi + \frac{3}{2}g(Y, h'V)\eta(W)\xi \\ & - (k + 2nk + \frac{1}{2})g(V, W)Y + \frac{3}{2}g(h'V, W)Y \\ & - 2\{-\frac{(k+1)}{2}g(h'Y, V)\eta(W)\xi + \frac{3}{2}g(h'Y, h'V)\eta(W)\xi \\ & + \frac{(k+1)}{2}\eta(V)\eta(W)Y\} - (k + 2nk + \frac{1}{2})g(V, W)h'Y \\ & + \frac{3}{2}g(h'V, W)h'Y + \frac{(k+1)}{2}\eta(V)\eta(W)h'Y\}. \end{aligned} \quad (3.4)$$

Making use of (2.10) and (1.5), we obtain the following:

$$\begin{aligned} W^*(R(\xi, Y)\xi, V)W = & k\{(k + 2nk + \frac{1}{2})g(V, W)\eta(Y)\xi - \frac{3}{2}g(h'V, W)\eta(Y)\xi \\ & - \frac{(k+1)}{2}\eta(W)\eta(Y)V + \frac{3}{2}\eta(Y)\eta(W)h'V \\ & - W^*(Y, V)W\} + 2W^*(h'Y, V)W, \end{aligned} \quad (3.5)$$

$$\begin{aligned} W^*(\xi, R(\xi, Y)V)W = & k\{(\frac{k}{2} + 2nk)g(Y, V)\eta(W)\xi - (k + 2nk + \frac{1}{2})g(Y, W) \\ & \eta(V)\xi + \frac{3}{2}g(h'Y, W)\eta(V)\xi + \frac{(k+1)}{2}\eta(V)\eta(W)Y \\ & - \frac{3}{2}\eta(V)\eta(W)h'Y - (4n + 1)g(h'Y, V)\eta(W)\xi\} \\ & + 2(k + 2nk + \frac{1}{2})g(h'Y, W)\eta(V)\xi - (k + 1) \\ & [3g(Y, W)\eta(V)\xi - 6\eta(Y)\eta(V)\eta(W)\xi \\ & + \eta(V)\eta(W)h'Y + 3\eta(V)\eta(W)Y] \end{aligned} \quad (3.6)$$

and

$$\begin{aligned}
W^*(\xi, V)R(\xi, Y)W &= k\left\{\left(\frac{k}{2} + 2nk\right)g(Y, W)\eta(V)\xi + \frac{3}{2}g(Y, W)h'V \right. \\
&\quad - \left(k + 2nk + \frac{1}{2}\right)g(Y, V)\eta(W)\xi + \frac{3}{2}g(h'V, Y)\eta(W)\xi \\
&\quad + \frac{(k+1)}{2}\eta(Y)\eta(W)V - \frac{3}{2}\eta(W)\eta(Y)h'V\} \\
&\quad - 2\left\{\left(\frac{k}{2} + 2nk\right)g(h'Y, W)\eta(V)\xi + \frac{3}{2}g(h'Y, W)h'V \right. \\
&\quad - \left(k + 2nk + \frac{1}{2}\right)g(h'Y, V)\eta(W)\xi + \frac{3}{2}g(h'V, h'Y)\eta(W)\xi \\
&\quad \left. + \frac{(k+1)}{2}\eta(Y)\eta(W)V\right\}. \tag{3.7}
\end{aligned}$$

Using (3.4)-(3.7) in (3.3), we obtain

$$\begin{aligned}
kW^*(Y, V)W + (k+1)\left(\frac{k}{2} + 3\right)g(Y, W)\eta(V)\xi - k\left(k + 2nk + \frac{1}{2}\right)g(V, W)Y \\
+ 3(k+1)\eta(V)\eta(W)Y - 6(k+1)\eta(Y)\eta(V)\eta(W)\xi + \frac{3k}{2}g(h'V, W)Y \\
- \left(\frac{5k}{2} + 1\right)g(h'Y, W)\eta(V)\xi + 2\left(k + 2nk + \frac{k}{2}\right)g(V, W)h'Y \\
- \frac{3k}{2}g(Y, W)h'V + \frac{3k}{2}\eta(V)\eta(W)h'Y - 3g(h'V, W)h'Y \\
+ 3g(h'Y, W)h'V - 2W^*(h'Y, V)W = 0. \tag{3.8}
\end{aligned}$$

Letting $Y, W \in [\lambda]'$ and $V \in [-\lambda]'$, we obtain from (3.8) that

$$W^*(Y, V)W = (k+1)g(Y, W)V \text{ and } W^*(h'Y, V)W = \lambda(k+1)g(Y, W)V. \tag{3.9}$$

In view of (3.9), (3.8) becomes

$$\left[\frac{3k\lambda}{2} - 3\lambda^2 + (k+1)(k-2\lambda)\right]g(Y, W)V = 0. \tag{3.10}$$

Using the relation $\lambda^2 = -(k+1)$ in (3.10), we get

$$\lambda(\lambda-1)(2\lambda^2 + 3\lambda - 1) = 0. \tag{3.11}$$

If $\lambda = 0$, then $k = -1$ and hence from (2.8) we have $h' = 0$, which is a contradiction to our assumption that $h' \neq 0$. Then from (3.11) without loss of generality we may take $\lambda = 1$ and hence $k = -2$. Hence from propositions 4.2 and 4.3 of [5], we have the following:

Theorem 3.1. The M -projective semisymmetric almost Kenmotsu manifold $(M^{2n+1}, \phi, \xi, \eta, g)(n > 1)$ with ξ belonging to the $(k, \mu)'$ -nullity distribution and

$h' \neq 0$ is locally isometric to the Riemannian product of an $(n+1)$ - dimensional manifold of constant sectional curvature -4 and a flat n -dimensional manifold.

4. M —projectively Ricci-semisymmetric almost Kenmotsu manifold with ξ belongs to $(k, \mu)'$ —nullity distribution

Definition 4.1. An almost Kenmotsu manifold is M -projectively Ricci- semisymmetric if

$$(W^*(X, Y) \cdot S)(U, V) = 0, \quad (4.1)$$

for all vector fields X, Y, U, V .

From (4.1), we have

$$S(W^*(X, Y)U, V) + S(U, W^*(X, Y)V) = 0. \quad (4.2)$$

Setting $X = U = \xi$ in (4.2), we get

$$S(W^*(\xi, Y)\xi, V) + S(\xi, W^*(\xi, Y)V) = 0. \quad (4.3)$$

Using (1.5) and (2.11) in (4.3), we obtain

$$\begin{aligned} & -\left(\frac{k+1}{2}\right)S(Y, V) + \frac{3}{2}S(h'Y, V) + 2nk(k + 2nk + \frac{1}{2})g(Y, V) \\ & - 3nkg(h'Y, V) + nk^2(4n+1)\eta(Y)\eta(V) = 0. \end{aligned} \quad (4.4)$$

Replacing Y by $h'Y$ in (4.4), we get

$$(k+1)\{-3S(Y, V) - S(h'Y, V) + 6nkg(Y, V)\} + 4nk(k + 2nk + \frac{1}{2})g(h'Y, V) = 0. \quad (4.5)$$

Letting $Y, V \in [\lambda]'$ in (4.5) gives that

$$S(Y, V) = \frac{nk}{\lambda+3}\left[3 + \frac{2\lambda(4nk+2k+1)}{k+1}\right]g(Y, V). \quad (4.6)$$

Using lemma (2.1), from (4.6) we can state the following:

Theorem 4.1. An M -projectively Ricci-semisymmetric almost Kenmotsu manifold M^{2n+1} with ξ belongs to $(k, \mu)'$ -nullity distribution is an Einstein manifold provided $\lambda \neq -3$.

5. M —projectively semisymmetric almost Kenmotsu manifold with ξ belongs to (k, μ) —nullity distribution

For $\xi \in (k, \mu)$ —nullity distribution, we have from (1.3) that

$$R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y] + \mu[g(Y, Z)hX - g(X, Z)hY]. \quad (5.1)$$

Using theorem 4.1 of [5], it follows from (5.1) that

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \quad (5.2)$$

$$R(\xi, X)Y = -g(X, Y)\xi + \eta(Y)X, \quad (5.3)$$

$$S(X, \xi) = -2n\eta(X). \quad (5.4)$$

Applying (5.3) and (5.4) in (1.5), we get

$$W^*(\xi, Y)Z = -g(Y, Z)\xi + \eta(Z)Y - \frac{1}{4n}[S(Y, Z)\xi + 2n\eta(Z)Y + g(Y, Z)Q\xi - \eta(Z)QY].$$

Suppose

$$(R(X, Y) \cdot W^*)(U, V)W = 0 \quad (5.5)$$

for all vector fields X, Y, U, V, W . Then we have

$$\begin{aligned} R(X, Y)W^*(U, V)W - W^*(R(X, Y)U, V)W \\ - W^*(U, R(X, Y)V)W - W^*(U, V)R(X, Y)W = 0. \end{aligned} \quad (5.6)$$

Putting $X = U = \xi$ in (5.6), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W - W^*(R(\xi, Y)\xi, V)W \\ - W^*(\xi, R(\xi, Y)V)W - W^*(\xi, V)R(\xi, Y)W = 0. \end{aligned} \quad (5.7)$$

Using (5.3), (5.4) and (5.5), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W = \frac{1}{2}g(V, W)\eta(Y)\xi - \frac{1}{2}g(Y, V)\eta(W)\xi - \frac{1}{4n}S(V, W)\eta(Y)\xi \\ - \frac{1}{4n}S(Y, V)\eta(W)\xi - \frac{1}{2}g(V, W)Y - \frac{1}{4n}S(V, W)Y, \end{aligned} \quad (5.8)$$

$$\begin{aligned} W^*(R(\xi, Y)\xi, V)W = W^*(Y, V)W - \frac{1}{2}g(V, W)\eta(Y)\xi + \frac{1}{2}\eta(W)\eta(Y)V \\ - \frac{1}{4n}[S(V, W)\eta(Y)\xi - \eta(Y)\eta(W)QV], \end{aligned} \quad (5.9)$$

$$\begin{aligned} W^*(\xi, R(\xi, Y)V)W = -g(Y, W)\eta(V)\xi + \eta(V)\eta(W)Y - \frac{1}{4n}[S(Y, W)\eta(V)\xi \\ + 2n\eta(V)\eta(W)Y + g(Y, W)\eta(V)Q\xi - \eta(W)\eta(V)QY] \end{aligned} \quad (5.10)$$

and

$$\begin{aligned} W^*(\xi, V)R(\xi, Y)W = -\frac{1}{2}g(Y, W)V - \frac{1}{4n}g(Y, W)QV - \frac{1}{2}g(Y, V)\eta(W)\xi \\ + \frac{1}{2}\eta(Y)\eta(W)V - \frac{1}{4n}S(Y, V)\eta(W)\xi + \frac{1}{4n}\eta(W)\eta(Y)QV. \end{aligned} \quad (5.11)$$

Using (5.8)-(5.11) in (5.7), we obtain

$$\begin{aligned} W^*(Y, V)W &= g(V, W)\eta(Y)\xi + \frac{1}{2n}S(V, W)\eta(Y)\xi - \frac{1}{2}g(V, W)Y \\ &\quad - \eta(Y)\eta(W)V - \frac{1}{2n}\eta(Y)\eta(W)QV + \frac{1}{2}g(Y, W)\eta(V)\xi \\ &\quad - \frac{1}{2}\eta(V)\eta(W)Y + \frac{1}{2}g(Y, W)V + \frac{1}{4n}[S(Y, W)\eta(V)\xi \\ &\quad - S(V, W)Y - \eta(V)\eta(W)QY + g(Y, W)QV]. \end{aligned} \quad (5.12)$$

By the influence of (1.5), one can get from (5.12) that

$$\begin{aligned} R(Y, V)W &= g(V, W)\eta(Y)\xi + \frac{1}{2n}S(V, W)\eta(Y)\xi - \frac{1}{2}g(V, W)Y \\ &\quad - \eta(Y)\eta(W)V - \frac{1}{2n}\eta(Y)\eta(W)QV + \frac{1}{2}g(Y, W)\eta(V)\xi \\ &\quad - \frac{1}{2}\eta(V)\eta(W)Y + \frac{1}{2}g(Y, W)V + \frac{1}{4n}[S(Y, W)\eta(V)\xi \\ &\quad - S(Y, W)V - \eta(V)\eta(W)QY + g(V, W)QY]. \end{aligned} \quad (5.13)$$

Contracting (5.13) over Y , we get

$$S(V, W) = \frac{(r + 4n - 4n^2)}{4n - 1}g(V, W) - \frac{(r + 2n + 4n^2)}{4n - 1}\eta(V)\eta(W). \quad (5.14)$$

Again contracting (5.14) over V and W , we get $r = -2n - 4n^2$ and substituting this in (5.14), we get

$$S(V, W) = -2ng(V, W). \quad (5.15)$$

On account of (5.15), it follows from (5.13) that

$$R(Y, V)W = -[g(V, W)Y - g(Y, W)V]. \quad (5.16)$$

Hence from (5.16), we can state the following:

Theorem 5.1. An M -projectively semisymmetric almost Kenmotsu manifold M^{2n+1} with ξ belongs to (k, μ) -nullity distribution is an Einstein manifold and moreover the manifold is of constant curvature -1 .

6. M -projectively Ricci-semisymmetric almost Kenmotsu manifold with ξ belongs to (k, μ) -nullity distribution

Definition 6.1. An almost Kenmotsu manifold is M -projectively Ricci- semisymmetric if

$$(W^*(X, Y) \cdot S)(U, V) = 0, \quad (6.1)$$

for all vector fields X, Y, U, V .

From (4.3), we have

$$S(W^*(\xi, Y)\xi, V) + S(\xi, W^*(\xi, Y)V) = 0. \quad (6.2)$$

Using (1.5) and theorem 4.1 of [5] in (6.2), we obtain

$$S(Y, V) = -ng(Y, V) - \frac{1}{4n}S(QY, V) \quad (6.3)$$

Taking $V = \xi$ in (6.3), we get

$$QY = -2nY \quad (6.4)$$

Therefore (6.3) becomes

$$S(Y, V) = -2ng(Y, V). \quad (6.5)$$

This implies that the manifold is Einstien manifold.

Conversely let the manifold be Einstien manifold of the form (6.5). Then clearly $W^* \cdot S = 0$. Hence we can state the following:

Theorem 6.1. An almost Kenmotsu manifold M^{2n+1} with ξ belongs to (k, μ) -nullity distribution is an Einstein manifold if and only if it is M -projectively Ricci-semisymmetric.

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