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## Almost Kenmotsu Manifold Equipped with $M$ -Projective Curvature Tensor

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### Abstract

The object of the present paper is to study  $M$ -projective curvature tensor on almost Kenmotsu manifolds with characteristic vector field  $\xi$  belonging to some nullity distributions.

**Key Words:** Almost Kenmotsu manifold, nullity distribution,  $M$ -projective curvature tensor, semisymmetric, Ricci-semisymmetric, Einstien manifold.

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### 1. Introduction

In 1972 Kenmotsu [8] introduced and studied a new class of almost contact metric manifolds called Kenmotsu manifolds. Almost contact metric manifold  $M^{2n+1}$  with 1-form  $\eta$  and fundamental 2-form  $\Phi$  defined by  $\Phi(X, Y) = g(X, \phi Y)$ , where  $\phi$  is a  $(1, 1)$  tensor field such that  $d\eta = 0$  and  $d\Phi = 2\eta \wedge \Phi$  is called almost Kenmotsu manifold. The normality of an almost contact metric manifold is given by vanishing the  $(1, 2)$ -type torsion tensor  $N = [\phi, \phi] + 2d\eta \otimes \xi$ , where  $[\phi, \phi]$  is the Nijenhuis torsion of  $\phi$  [1]. According to [8], the normality of an almost Kenmotsu manifold is given by

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \quad (1.1)$$

for every vector fields  $X, Y$  on  $M^{2n+1}$ . On the other hand, Gray [7] and Tanno [13] introduced the notion of  $k$ -nullity distribution, which is defined for any  $p \in M^{2n+1}$  and  $k \in \mathbb{R}$  as follows:

$$N_p(k) = \{Z \in T_p M^{2n+1} : R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y]\} \quad (1.2)$$

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for any  $X, Y \in T_p M$ , where  $T_p M^{2n+1}$  denotes the tangent vector space of  $M^{2n+1}$  at any point  $p \in M^{2n+1}$  and  $R$  denotes the Riemannian curvature tensor of type  $(1, 3)$ . Moreover, if  $k$  is a smooth function then the distribution is called generalized  $k$ -nullity distribution. Later Blair, Koufogiorgos and Papantoniou [2] introduced a generalized notion of the  $(k, \mu)$ -nullity distribution on a contact metric manifold  $M^{2n+1}$  which is defined for any  $p \in M^{2n+1}$  and  $(k, \mu) \in \mathbb{R}^2$  as follows:

$$N_p(k, \mu) = \{Z \in T_p M^{2n+1} : R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y] + \mu[g(Y, Z)hX - g(X, Z)hY]\}, \quad (1.3)$$

where  $h = \frac{1}{2}\mathcal{L}_\xi\phi$  and  $\mathcal{L}$  denotes the Lie differentiation. After this, Dileo and Pastore [5] introduced another generalized notion of the  $k$ -nullity distribution called  $(k, \mu)'$ -nullity distribution on an almost Kenmotsu manifold  $M^{2n+1}$  and is defined for any  $p \in M^{2n+1}$  and  $(k, \mu) \in \mathbb{R}^2$  as follows:

$$N_p(k, \mu)' = \{Z \in T_p M^{2n+1} : R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y] + \mu[g(Y, Z)h'X - g(X, Z)h'Y]\}, \quad (1.4)$$

where  $h' = h \circ \phi$ . Recently, Dileo et al. ([5], [6], [7]), Wang et al. ([15], [16], [17]) and De et al. ([3], [9], [14]) obtained some important results on almost Kenmotsu manifolds with characteristic vector field  $\xi$  belonging to some nullity distributions. In this paper we investigate some results on almost Kenmotsu manifolds with  $M$ -projective curvature tensor  $W^*$  defined by [11]

$$W^*(X, Y)Z = R(X, Y)Z - \frac{1}{4n}[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY], \quad (1.5)$$

where  $X, Y, Z$  are any vector fields and  $r$  is the scalar curvature. The paper is organized as follows: In Section 2, we give some basic formulas and properties of almost Kenmotsu manifolds according to Dileo and Pastore ([5], [7]). In Section 3, we study  $M$ -projectively semisymmetric almost Kenmotsu manifolds with  $\xi$  belonging to  $(k, \mu)'$ -nullity distribution. The Section 4 deals with the study of  $M$ -projectively Ricci-semisymmetric almost Kenmotsu manifolds with  $\xi$  belonging to  $(k, \mu)'$ -nullity distribution. The Section 5 and 6 are concerned with the study of  $M$ -projective curvature tensor on almost Kenmotsu manifolds with  $\xi$  belonging to  $(k, \mu)$ -nullity distribution.

## 2. Almost Kenmotsu manifolds and nullity distributions

Let  $M^{2n+1}$  be an almost Kenmotsu manifold with structure  $(\phi, \xi, \eta, g)$ , where  $\phi$  is a  $(1, 1)$  tensor field,  $\xi$  is a characteristic vector field and  $\eta$  is a 1-form

and  $g$  is a Riemannian metric such that [1]

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta \circ \phi = 0, \quad \phi\xi = 0, \quad g(X, \xi) = \eta(X), \quad (2.1)$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad (2.2)$$

for all vector fields  $X, Y$  on  $M^{2n+1}$ . Let  $D$  be the distribution orthogonal to  $\xi$  and defined by  $D = \text{Ker}(\eta) = \text{Im}(\phi)$ . The two tensor fields  $h = \frac{1}{2}\mathcal{L}_\xi\phi$  and  $l = R(\cdot, \xi)\xi$  on an almost Kenmotsu manifold  $M^{2n+1}$  are symmetric and satisfy the following relations [10]

$$h\xi = 0, \quad l\xi = 0, \quad \text{tr}(h\phi) = 0, \quad h\phi + \phi h = 0, \quad (2.3)$$

$$\nabla_X\xi = -\phi^2 X - \phi h X, \quad (2.4)$$

$$\phi l\phi - l = 2(h^2 - \phi^2), \quad (2.5)$$

$$\text{tr}(l) = S(\xi, \xi) = g(Q\xi, \xi) = -2n - \text{tr}h^2, \quad (2.6)$$

$$R(X, Y)\xi = \eta(X)(Y - \phi h Y) - \eta(Y)(X - \phi h X) + (\nabla_Y\phi h)X - (\nabla_X\phi h)Y, \quad (2.7)$$

for all vector fields  $X, Y$  on  $M^{2n+1}$ . Now we give some basic properties of almost Kenmotsu manifolds with  $\xi$  belonging to the  $(k, \mu)'$ -nullity distribution.

The  $(1, 1)$ -type tensor field  $h'$  satisfies  $h'\phi + \phi h' = 0$  and  $h'\xi = 0$ . Also it is known that

$$h = 0 \Leftrightarrow h' = 0, \quad h'^2 = (k+1)\phi^2 (\Leftrightarrow h^2 = (k+1)\phi^2). \quad (2.8)$$

For an almost Kenmotsu manifold, we have from (1.4)

$$R(X, Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)h'X - \eta(X)h'Y] \quad \text{and} \quad (2.9)$$

$$R(\xi, X)Y = k[g(X, Y)\xi - \eta(Y)X] + \mu[g(h'X, Y)\xi - \eta(Y)h'X], \quad (2.10)$$

where  $k, \mu \in \mathbb{R}$ .

Contracting  $Y$  in (2.10), we get

$$S(X, \xi) = 2nk\eta(X). \quad (2.11)$$

Let  $X \in D$  be the eigen vector of  $h'$  corresponding to the eigen value  $\lambda$  orthogonal to  $\xi$ . It follows from (2.8) that  $\lambda^2 = -(k+1)$ , a constant. Therefore,  $k \leq -1$  and  $\lambda = \pm\sqrt{-k-1}$ . We denote  $[\lambda]'$  and  $[-\lambda]'$  as the corresponding eigenspaces associated with  $h'$  corresponding to the non-zero eigen values  $\lambda$  and  $-\lambda$  respectively. We have the following lemmas.

**Lemma 2.1.** ([5], Proposition 4.1) Let  $(M^{2n+1}, \phi, \xi, \eta, g)$  be an almost Kenmotsu manifold such that  $\xi$  belongs to the  $(k, \mu)'$ -nullity distribution and  $h' \neq 0$ . Then  $k < -1$ ,  $\mu = -2$  and  $Spec(h') = \{0, \lambda, -\lambda\}$  with 0 as simple eigen value and  $\lambda = \sqrt{-k-1}$ . The distributions  $[\xi] \oplus [\lambda]'$  and  $[\xi] \oplus [-\lambda]'$  are integrable with totally geodesic leaves. The distributions  $[\lambda]'$  and  $[-\lambda]'$  are integrable with totally umbilical leaves.

**Lemma 2.2.** ([5], Lemma 4.1) Let  $(M^{2n+1}, \phi, \xi, \eta, g)$  be an almost Kenmotsu manifold with  $h' \neq 0$  and  $\xi$  belongs to the  $(k, -2)'$ -nullity distribution. Then for every  $X, Y \in T_p M$ ,

$$(\nabla_X h')Y = -g(h'X + h'^2 X, Y)\xi - \eta(Y)(h'X + h'^2 X). \quad (2.12)$$

According to Takahashi [12] and De et al. [4], we have the following definitions:

**Definition 2.1.** An almost Kenmotsu manifold is said to be  $\phi$ -symmetric if it satisfies

$$\phi^2((\nabla_W R)(X, Y)Z) = 0, \quad (2.13)$$

for all vector fields  $W, X, Y, Z \in T_p M^{2n+1}$ . In addition, if the vector fields  $W, X, Y, Z$  are orthogonal to  $\xi$ , then the manifold is called locally  $\phi$ -symmetric

**Definition 2.2.** An almost Kenmotsu manifold is said to be  $\phi$ -recurrent if it satisfies

$$\phi^2((\nabla_W R)(X, Y)Z) = A(W)R(X, Y)Z \quad (2.14)$$

for any vector fields  $W, X, Y, Z \in T_p M^{2n+1}$ . In relation (2.14) if the vector fields  $W, X, Y, Z$  are orthogonal to  $\xi$ , then the manifold is called locally  $\phi$ -recurrent.

### 3. $M$ -projectively semisymmetric almost Kenmotsu manifold with $\xi$ belongs to $(k, \mu)'$ -nullity distribution

Let  $M^{2n+1}$  be  $M$ -projectively semisymmetric almost Kenmotsu manifold with  $\xi$  belongs to  $(k, \mu)'$ -nullity distribution. Then we have

$$(R(X, Y) \cdot W^*)(U, V)W = 0, \quad (3.1)$$

for all vector fields  $X, Y, U, V, W$ . This implies

$$\begin{aligned} & R(X, Y)W^*(U, V)W - W^*(R(X, Y)U, V)W \\ & - W^*(U, R(X, Y)V)W - W^*(U, V)R(X, Y)W = 0. \end{aligned} \quad (3.2)$$

Putting  $X = U = \xi$  in (3.2), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W - W^*(R(\xi, Y)\xi, V)W \\ - W^*(\xi, R(\xi, Y)V)W - W^*(\xi, V)R(\xi, Y)W = 0. \end{aligned} \quad (3.3)$$

Using (2.10) and lemma (2.1) in (1.5), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W = k\{(k + 2nk + \frac{1}{2})g(V, W)\eta(Y)\xi - \frac{3}{2}g(h'V, W)\eta(Y)\xi \\ - \frac{(k + 1)}{2}g(Y, V)\eta(W)\xi + \frac{3}{2}g(Y, h'V)\eta(W)\xi \\ - (k + 2nk + \frac{1}{2})g(V, W)Y + \frac{3}{2}g(h'V, W)Y \\ - 2\{-\frac{(k + 1)}{2}g(h'Y, V)\eta(W)\xi + \frac{3}{2}g(h'Y, h'V)\eta(W)\xi \\ + \frac{(k + 1)}{2}\eta(V)\eta(W)Y\} - (k + 2nk + \frac{1}{2})g(V, W)h'Y \\ + \frac{3}{2}g(h'V, W)h'Y + \frac{(k + 1)}{2}\eta(V)\eta(W)h'Y\}. \end{aligned} \quad (3.4)$$

Making use of (2.10) and (1.5), we obtain the following:

$$\begin{aligned} W^*(R(\xi, Y)\xi, V)W = k\{(k + 2nk + \frac{1}{2})g(V, W)\eta(Y)\xi - \frac{3}{2}g(h'V, W)\eta(Y)\xi \\ - \frac{(k + 1)}{2}\eta(W)\eta(Y)V + \frac{3}{2}\eta(Y)\eta(W)h'V \\ - W^*(Y, V)W\} + 2W^*(h'Y, V)W, \end{aligned} \quad (3.5)$$

$$\begin{aligned} W^*(\xi, R(\xi, Y)V)W = k\{(\frac{k}{2} + 2nk)g(Y, V)\eta(W)\xi - (k + 2nk + \frac{1}{2})g(Y, W) \\ \eta(V)\xi + \frac{3}{2}g(h'Y, W)\eta(V)\xi + \frac{(k + 1)}{2}\eta(V)\eta(W)Y \\ - \frac{3}{2}\eta(V)\eta(W)h'Y - (4n + 1)g(h'Y, V)\eta(W)\xi\} \\ + 2(k + 2nk + \frac{1}{2})g(h'Y, W)\eta(V)\xi - (k + 1) \\ [3g(Y, W)\eta(V)\xi - 6\eta(Y)\eta(V)\eta(W)\xi \\ + \eta(V)\eta(W)h'Y + 3\eta(V)\eta(W)Y] \end{aligned} \quad (3.6)$$

and

$$\begin{aligned}
W^*(\xi, V)R(\xi, Y)W &= k\left\{ \left(\frac{k}{2} + 2nk\right)g(Y, W)\eta(V)\xi + \frac{3}{2}g(Y, W)h'V \right. \\
&\quad - \left(k + 2nk + \frac{1}{2}\right)g(Y, V)\eta(W)\xi + \frac{3}{2}g(h'V, Y)\eta(W)\xi \\
&\quad + \frac{(k+1)}{2}\eta(Y)\eta(W)V - \frac{3}{2}\eta(W)\eta(Y)h'V \} \\
&\quad - 2\left\{ \left(\frac{k}{2} + 2nk\right)g(h'Y, W)\eta(V)\xi + \frac{3}{2}g(h'Y, W)h'V \right. \\
&\quad - \left(k + 2nk + \frac{1}{2}\right)g(h'Y, V)\eta(W)\xi + \frac{3}{2}g(h'V, h'Y)\eta(W)\xi \\
&\quad \left. \left. + \frac{(k+1)}{2}\eta(Y)\eta(W)V \right\} \right. \\
&\quad \left. \left. \right. \right. \quad (3.7)
\end{aligned}$$

Using (3.4)-(3.7) in (3.3), we obtain

$$\begin{aligned}
&kW^*(Y, V)W + (k+1)\left(\frac{k}{2} + 3\right)g(Y, W)\eta(V)\xi - k(k+2nk + \frac{1}{2})g(V, W)Y \\
&+ 3(k+1)\eta(V)\eta(W)Y - 6(k+1)\eta(Y)\eta(V)\eta(W)\xi + \frac{3k}{2}g(h'V, W)Y \\
&- \left(\frac{5k}{2} + 1\right)g(h'Y, W)\eta(V)\xi + 2(k+2nk + \frac{k}{2})g(V, W)h'Y \\
&- \frac{3k}{2}g(Y, W)h'V + \frac{3k}{2}\eta(V)\eta(W)h'Y - 3g(h'V, W)h'Y \\
&+ 3g(h'Y, W)h'V - 2W^*(h'Y, V)W = 0. \quad (3.8)
\end{aligned}$$

Letting  $Y, W \in [\lambda]'$  and  $V \in [-\lambda]'$ , we obtain from (3.8) that

$$W^*(Y, V)W = (k+1)g(Y, W)V \text{ and } W^*(h'Y, V)W = \lambda(k+1)g(Y, W)V. \quad (3.9)$$

In view of (3.9), (3.8) becomes

$$\left[ \frac{3k\lambda}{2} - 3\lambda^2 + (k+1)(k-2\lambda) \right]g(Y, W)V = 0. \quad (3.10)$$

Using the relation  $\lambda^2 = -(k+1)$  in (3.10), we get

$$\lambda(\lambda-1)(2\lambda^2 + 3\lambda - 1) = 0. \quad (3.11)$$

If  $\lambda = 0$ , then  $k = -1$  and hence from (2.8) we have  $h' = 0$ , which is a contradiction to our assumption that  $h' \neq 0$ . Then from (3.11) without loss of generality we may take  $\lambda = 1$  and hence  $k = -2$ . Hence from propositions 4.2 and 4.3 of [5], we have the following:

**Theorem 3.1.** The  $M$ -projective semisymmetric almost Kenmotsu manifold  $(M^{2n+1}, \phi, \xi, \eta, g)$  ( $n > 1$ ) with  $\xi$  belonging to the  $(k, \mu)'$ -nullity distribution and

$h' \neq 0$  is locally isometric to the Riemannian product of an  $(n+1)$ -dimensional manifold of constant sectional curvature -4 and a flat  $n$ -dimensional manifold.

#### 4. $M$ -projectively Ricci-semisymmetric almost Kenmotsu manifold with $\xi$ belongs to $(k, \mu)'$ -nullity distribution

**Definition 4.1.** An almost Kenmotsu manifold is  $M$ -projectively Ricci-semisymmetric if

$$(W^*(X, Y) \cdot S)(U, V) = 0, \quad (4.1)$$

for all vector fields  $X, Y, U, V$ .

From (4.1), we have

$$S(W^*(X, Y)U, V) + S(U, W^*(X, Y)V) = 0. \quad (4.2)$$

Setting  $X = U = \xi$  in (4.2), we get

$$S(W^*(\xi, Y)\xi, V) + S(\xi, W^*(\xi, Y)V) = 0. \quad (4.3)$$

Using (1.5) and (2.11) in (4.3), we obtain

$$\begin{aligned} & -\left(\frac{k+1}{2}\right)S(Y, V) + \frac{3}{2}S(h'Y, V) + 2nk(k+2nk+\frac{1}{2})g(Y, V) \\ & - 3nkg(h'Y, V) + nk^2(4n+1)\eta(Y)\eta(V) = 0. \end{aligned} \quad (4.4)$$

Replacing  $Y$  by  $h'Y$  in (4.4), we get

$$(k+1)\{-3S(Y, V) - S(h'Y, V) + 6nkg(Y, V)\} + 4nk(k+2nk+\frac{1}{2})g(h'Y, V) = 0. \quad (4.5)$$

Letting  $Y, V \in [\lambda]'$  in (4.5) gives that

$$S(Y, V) = \frac{nk}{\lambda+3}\left[3 + \frac{2\lambda(4nk+2k+1)}{k+1}\right]g(Y, V). \quad (4.6)$$

Using lemma (2.1), from (4.6) we can state the following:

**Theorem 4.1.** An  $M$ -projectively Ricci-semisymmetric almost Kenmotsu manifold  $M^{2n+1}$  with  $\xi$  belongs to  $(k, \mu)'$ -nullity distribution is an Einstein manifold provided  $\lambda \neq -3$ .

#### 5. $M$ -projectively semisymmetric almost Kenmotsu manifold with $\xi$ belongs to $(k, \mu)$ -nullity distribution

For  $\xi \in (k, \mu)$ -nullity distribution, we have from (1.3) that

$$R(X, Y)Z = k[g(Y, Z)X - g(X, Z)Y] + \mu[g(Y, Z)hX - g(X, Z)hY]. \quad (5.1)$$

Using theorem 4.1 of [5], it follows from (5.1) that

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \quad (5.2)$$

$$R(\xi, X)Y = -g(X, Y)\xi + \eta(Y)X, \quad (5.3)$$

$$S(X, \xi) = -2n\eta(X). \quad (5.4)$$

Applying (5.3) and (5.4) in (1.5), we get

$$W^*(\xi, Y)Z = -g(Y, Z)\xi + \eta(Z)Y - \frac{1}{4n}[S(Y, Z)\xi + 2n\eta(Z)Y + g(Y, Z)Q\xi - \eta(Z)QY].$$

Suppose

$$(R(X, Y) \cdot W^*)(U, V)W = 0 \quad (5.5)$$

for all vector fields  $X, Y, U, V, W$ . Then we have

$$\begin{aligned} & R(X, Y)W^*(U, V)W - W^*(R(X, Y)U, V)W \\ & - W^*(U, R(X, Y)V)W - W^*(U, V)R(X, Y)W = 0. \end{aligned} \quad (5.6)$$

Putting  $X = U = \xi$  in (5.6), we get

$$\begin{aligned} & R(\xi, Y)W^*(\xi, V)W - W^*(R(\xi, Y)\xi, V)W \\ & - W^*(\xi, R(\xi, Y)V)W - W^*(\xi, V)R(\xi, Y)W = 0. \end{aligned} \quad (5.7)$$

Using (5.3), (5.4) and (5.5), we get

$$\begin{aligned} R(\xi, Y)W^*(\xi, V)W &= \frac{1}{2}g(V, W)\eta(Y)\xi - \frac{1}{2}g(Y, V)\eta(W)\xi - \frac{1}{4n}S(V, W)\eta(Y)\xi \\ & - \frac{1}{4n}S(Y, V)\eta(W)\xi - \frac{1}{2}g(V, W)Y - \frac{1}{4n}S(V, W)Y, \end{aligned} \quad (5.8)$$

$$\begin{aligned} W^*(R(\xi, Y)\xi, V)W &= W^*(Y, V)W - \frac{1}{2}g(V, W)\eta(Y)\xi + \frac{1}{2}\eta(W)\eta(Y)V \\ & - \frac{1}{4n}[S(V, W)\eta(Y)\xi - \eta(Y)\eta(W)QV], \end{aligned} \quad (5.9)$$

$$\begin{aligned} W^*(\xi, R(\xi, Y)V)W &= -g(Y, W)\eta(V)\xi + \eta(V)\eta(W)Y - \frac{1}{4n}[S(Y, W)\eta(V)\xi \\ & + 2n\eta(V)\eta(W)Y + g(Y, W)\eta(V)Q\xi - \eta(W)\eta(V)QY] \end{aligned} \quad (5.10)$$

and

$$\begin{aligned} W^*(\xi, V)R(\xi, Y)W &= -\frac{1}{2}g(Y, W)V - \frac{1}{4n}g(Y, W)QV - \frac{1}{2}g(Y, V)\eta(W)\xi \\ & + \frac{1}{2}\eta(Y)\eta(W)V - \frac{1}{4n}S(Y, V)\eta(W)\xi + \frac{1}{4n}\eta(W)\eta(Y)QV. \end{aligned} \quad (5.11)$$

Using (5.8)-(5.11) in (5.7), we obtain

$$\begin{aligned}
 W^*(Y, V)W &= g(V, W)\eta(Y)\xi + \frac{1}{2n}S(V, W)\eta(Y)\xi - \frac{1}{2}g(V, W)Y \\
 &\quad - \eta(Y)\eta(W)V - \frac{1}{2n}\eta(Y)\eta(W)QV + \frac{1}{2}g(Y, W)\eta(V)\xi \\
 &\quad - \frac{1}{2}\eta(V)\eta(W)Y + \frac{1}{2}g(Y, W)V + \frac{1}{4n}[S(Y, W)\eta(V)\xi \\
 &\quad - S(V, W)Y - \eta(V)\eta(W)QY + g(Y, W)QV]. \tag{5.12}
 \end{aligned}$$

By the influence of (1.5), one can get from (5.12) that

$$\begin{aligned}
 R(Y, V)W &= g(V, W)\eta(Y)\xi + \frac{1}{2n}S(V, W)\eta(Y)\xi - \frac{1}{2}g(V, W)Y \\
 &\quad - \eta(Y)\eta(W)V - \frac{1}{2n}\eta(Y)\eta(W)QV + \frac{1}{2}g(Y, W)\eta(V)\xi \\
 &\quad - \frac{1}{2}\eta(V)\eta(W)Y + \frac{1}{2}g(Y, W)V + \frac{1}{4n}[S(Y, W)\eta(V)\xi \\
 &\quad - S(Y, W)V - \eta(V)\eta(W)QY + g(V, W)QY]. \tag{5.13}
 \end{aligned}$$

Contracting (5.13) over  $Y$ , we get

$$S(V, W) = \frac{(r + 4n - 4n^2)}{4n - 1}g(V, W) - \frac{(r + 2n + 4n^2)}{4n - 1}\eta(V)\eta(W). \tag{5.14}$$

Again contracting (5.14) over  $V$  and  $W$ , we get  $r = -2n - 4n^2$  and substituting this in (5.14), we get

$$S(V, W) = -2ng(V, W). \tag{5.15}$$

On account of (5.15), it follows from (5.13) that

$$R(Y, V)W = -[g(V, W)Y - g(Y, W)V]. \tag{5.16}$$

Hence from (5.16), we can state the following:

**Theorem 5.1.** An  $M$ -projectively semisymmetric almost Kenmotsu manifold  $M^{2n+1}$  with  $\xi$  belongs to  $(k, \mu)$ -nullity distribution is an Einstein manifold and moreover the manifold is of constant curvature  $-1$ .

## 6. $M$ -projectively Ricci-semisymmetric almost Kenmotsu manifold with $\xi$ belongs to $(k, \mu)$ -nullity distribution

**Definition 6.1.** An almost Kenmotsu manifold is  $M$ -projectively Ricci-semisymmetric if

$$(W^*(X, Y) \cdot S)(U, V) = 0, \tag{6.1}$$

for all vector fields  $X, Y, U, V$ .

From (4.3), we have

$$S(W^*(\xi, Y)\xi, V) + S(\xi, W^*(\xi, Y)V) = 0. \quad (6.2)$$

Using (1.5) and theorem 4.1 of [5] in (6.2), we obtain

$$S(Y, V) = -ng(Y, V) - \frac{1}{4n}S(QY, V) \quad (6.3)$$

Taking  $V = \xi$  in (6.3), we get

$$QY = -2nY \quad (6.4)$$

Therefore (6.3) becomes

$$S(Y, V) = -2ng(Y, V). \quad (6.5)$$

This implies that the manifold is Einstien manifold.

Conversely let the manifold be Einstien manifold of the form (6.5). Then clearly  $W^* \cdot S = 0$ . Hence we can state the following:

**Theorem 6.1.** An almost Kenmotsu manifold  $M^{2n+1}$  with  $\xi$  belongs to  $(k, \mu)$ -nullity distribution is an Einstein manifold if and only if it is  $M$ -projectively Ricci-semisymmetric.

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