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Two-Layered Two-Phase Magnetohydro-Dynamic Fluid Model of Blood Flow Through Narrow Circular Vessels

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Abstract

A two-layered two-phase magnetohydrodynamic model for blood flow through a narrow circular vessel under the influence of uniform transverse magnetic field has been developed. It is assumed that the core region is the suspension of all the erythrocytes in plasma fluid and a peripheral layer of cell-free plasma. Analytical expressions for velocity profiles of plasma and cells in both regions along with flow rate, effective viscosity and resistance to flow have been obtained. Results have been evaluated numerically for various values of the rheological parameters available from published works and discussed graphically. It may be noticed that the effective viscosity decreases/increases with increasing pheripheral layer thickness/Hartman number. This is good agreement to the published works related to dialysis.

Key Words : Two-phase model, Hartman number, Hematocrit, Blood flow, Magnetic field.

1. Introduction

The study of blood flow through the vessels of the circulatory system has been the subject of scientific research. When blood flows through tubes, the two-phase nature of blood as a suspension becomes important as the diameter of the red blood cell (RBC) becomes comparable to the tube diameter. It is well known that blood is a multiphase suspension of red blood cells (RBC), white blood cells (WBC), and platelets, suspended in plasma. The red blood cells, one of the main constituents of blood constitute about 40-45% of the total blood volume play an important role for blood flowing through small vessels of diameter 2400-8μm (Srivastava and Srivastava [1]). The experimental and theoretical studies of blood flow phenomena are very useful for the diagnosis of cardiovascular diseases and development of pathological patterns in human or animal physiology and for other clinical purposes.

Several researchers Bayliss [2], McDonald [3], Copley and Stainsby [4], Whitmore [5] have proposed a single phase homogeneous Newtonian model of blood. This classical approach does not account for the presence of red cells in blood. Some experimental studies (Cokelet [6], Haynes [7]) on blood flow indicate that blood can no longer be treated as a single phase homogeneous viscous fluid when the diameter of blood vessel is smaller than 1000m. Thus, in dealing with the problem of microcirculation, the red blood cells cannot be ignored. It seems to be important and necessary to consider the whole blood as a particle-fluid system flowing through small vessels. Many investigations have been conducted in the literature using particulate suspension theory to describe the flow of blood in small vessels (Maithili Sharan and Aleksander S. Popel[8], Nair et al. [9], Seshadri and Jaffrin [10] Gupta et al. [11] and Srivastava [12]). Srivastava and Srivastava [1] proposed a two-phase theoretical model to address pulsatile blood flow in the entrance region of an artery.

It has been shown by Woodcock [13] that red cells which contain hemoglobin (an iron compound) have negative electric charge. Hence, blood can be considered as an electrically conducting fluid (Rosensweig [14]). From MHD studies, it is well known that magnetic field could be used to control the movement of charged particle (Boyd and Sanderson [15]). Many workers have studied the effect of magnetic field on blood flow through narrow vessels (Chaturani and Bhartiya [16] and Tiwari [17]) Chaturani and Bhartiya [16] studied two-layered magnetohydrodynamic flow through parallel plates with applications to blood flow and noticed that magnetic field help in reducing the cell injury and the dialysis time.

With the above discussions in mind we propose to study the flow of blood in small vessels involving a two-fluid two-phase model with transverse effect of magnetic field. The mathematical model considered as a two-layered model of blood, consisting of a core region of suspension of all the erythrocytes (RBC), assumed to be a particle fluid suspension (i.e. a suspension of red cells in plasma) and a peripheral layer of plasma. The present study involves the effect of hematocrit, Hartman number and peripheral layer thickness on the flow characteristics of blood through small vessels.

2. Mathematical Formulation and Analysis

Consider axially symmetric, laminar steady flow of blood in an uniform rigid circular tube of radius R (Fig.1). Blood is represented by a two-fluid model consisting of a core region (central layer) of suspension of all the erythrocytes assumed to be a particle-fluid mixture (i.e. suspension of red cells in plasma) of radius $(R-\delta)$ and a peripheral cell-depleted layer near the wall of plasma of thickness δ. It is assumed that blood and plasma both are Newtonian fluid. Since red cells are magnetic in nature, blood can be regarded as two phase magnetic fluid in the core region. The homogeneous transverse magnetic field is considered.

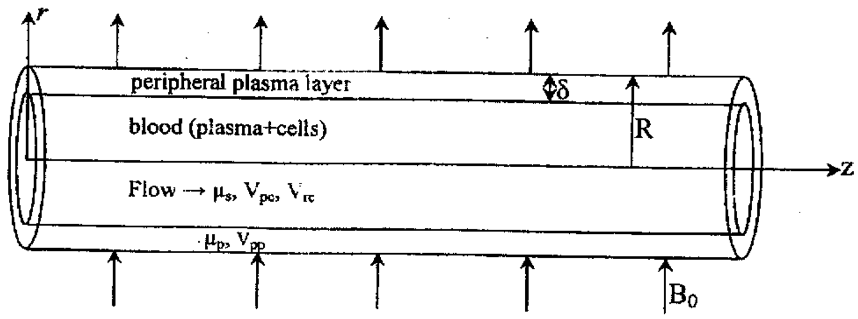

Fig.1. Flow geometry of two-layered magnetic fluid model.

The governing equations of motion under the above assumptions under no body forces in the presence of transverse magnetic field (Nayteh [18]) are

$$-\frac{dp}{dz}+\mu_p\left(\frac{d^2V_{pp}}{dr^2}+\frac{1}{r}\frac{dV_{pp}}{dr}\right)=0 \qquad (1)$$

in cell free peripheral layer $(R-\delta \leq r \leq R)$

$$(1-\phi)\left[-\frac{dp}{dz}+\mu_s\left(\frac{d^2V_{pc}}{dr^2}+\frac{1}{r}\frac{dV_{pc}}{dr}\right)\right]+\phi S(V_{rc}-V_{pc})=0 \qquad (2)$$

in core-region $(0 \leq r \leq R-\delta)$, for fluid phase (plasma) and

$$\phi\left[-\frac{dp}{dz}+\mu_s\left(\frac{d^2V_{pc}}{dr^2}+\frac{1}{r}\frac{dV_{pc}}{dr}\right)\right]+\phi S(V_{pc}-V_{rc})-\phi\sigma e B_0^2 V_{pc}=0 \qquad (3)$$

in particulate phase (red cells).

where r and z are radial and axial co-ordinates; V_{pp}, V_{pc} are the velocities of plasma in the peripheral layer and core region respectively; V_{rc} the velocity of red cells in the core; μ_p the fluid (plasma) viscosity; μ_s the suspension (blood) viscosity; ϕ the volume occupied by the red cells per unit volume of the blood called Hematocrit; p the pressure; $B_o = (\mu_e/H_o)$ the electro-magnetic induction; μ_e the magnetic permeability; H_o the intensity of the magnetic field; σ_e the conductivity of the fluid and S the drag coefficient of the interaction between the two phase (fluid and particle).

The expression for drag coefficient of interaction S is selected (Charm and Kurland [19] as

$$S = \frac{9}{2}\frac{\mu_p}{a^2}\frac{\left[4 + 3(8\phi - 3\phi^2)^{1/2} + 3\phi\right]}{(2 - 3\phi)^2} \tag{4}$$

where a is the radius of a particle.

The boundary conditions are

$$V_{pp} = 0 \qquad \text{at } r = R \tag{5}$$

$$V_{pc} \text{ is finite at } r = 0 \tag{6}$$

$$V_{pp} = V_{pc} \qquad \text{at } r = R - \delta \tag{7}$$

The expressions for velocities V_{pp}, V_{pc} and V_{rc} obtained as the solutions of equations (1), (2) and (3) under the boundary conditions (5), (6) and (7) are given as

$$V_{pp} = -\frac{R^2}{4\mu_p}\frac{dp}{dz}\left\{1 - \left(\frac{r}{R}\right)^2\right\} \tag{8}$$

$$V_{pc} = -\frac{R^2}{4\mu_s}\frac{dp}{dz}\left[\frac{4}{K^2M^2} + \left\{\left(1 - \left(1 - \frac{\delta}{R}\right)^2\right)\overline{\mu} - \frac{4}{K^2M^2}\right\}\frac{J_0(iKM\frac{r}{M})}{J_0(iKM(1 - \frac{\delta}{r}))}\right] \tag{9}$$

$$V_{rc} = -\frac{R^2}{4\mu_s}\frac{dp}{dz}\left\{1 - (1 - K^2)\frac{M^2\mu_s}{R^2S}\right\} \times$$
$$\left[\frac{4}{K^2M^2} + \left\{\left(1 - \left(1 - \frac{\delta}{R}\right)^2\right)\overline{\mu} - \frac{4}{K^2M^2}\right\}\frac{J_0(iKM\frac{r}{M})}{J_0(iKM(1 - \frac{\delta}{r}))}\right] \tag{10}$$

where $\overline{\mu} = \mu_s/\mu_p$, $K^2 = \phi$, $M = B_oR\sqrt{\frac{\sigma e}{\mu_s}}$ is Hartman number, and J_0 is the Bessel function of zero order.

Volume flow rate Q is given by

$$Q = 2\pi \int_{R-\delta}^{R} rV_{pp}dr + 2\pi(1-\phi)\int_{\partial}^{R-\delta} rV_{pc}dr + 2\pi\phi\int_{\partial}^{R-\delta} rV_{rc}dr \tag{11}$$

Using equations (8), (9) and (10) into equation (11), the expression for flow rate is obtained as

$$Q = \frac{\pi R^4}{8\mu_s}\big(-\frac{dp}{dz}\big)\Bigg[\Bigg\{1-2\big(1-\frac{\delta}{R}\big)^2+\big(1-\frac{\delta}{R}\big)^4\Bigg\}\overline{\mu}+\Bigg\{1-K^2(1-K^2)\frac{M^2\mu_s}{R^2S}\Bigg\}$$
$$\Bigg[\frac{8}{K^2M^2}\big(1-\frac{\delta}{R}\big)^2-\frac{4i}{KM}\big(1-\frac{\delta}{R}\big)\Bigg\{\big(1-\big(1-\frac{\delta}{R}\big)^2\big)\overline{\mu}-\frac{4}{K^2M^2}\Bigg\}\times$$
$$\frac{J_0(iKM(1-\frac{\delta}{R}))}{J_0(iKM(1-\frac{\delta}{r}))}\Bigg]\Bigg] \tag{12}$$

Effective viscosity eff can be derived by using the formula

$$\mu_{eff} = \frac{\pi R^4}{8Q}\left(-\frac{dp}{dz}\right). \tag{13}$$

Using (12) in (13), we obtain

$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_s}\Bigg[\Bigg\{1-2\big(1-\frac{\delta}{R}\big)^2+\big(1-\frac{\delta}{R}\big)^4\Bigg\}\overline{\mu}+\Bigg\{1-K^2(1-K^2)\frac{M^2\mu_s}{R^2S}\Bigg\}\times$$
$$\Bigg[\frac{8}{K^2M^2}\big(1-\frac{\delta}{R}\big)^2-\frac{4i}{KM}\big(1-\frac{\delta}{R}\big)\Bigg\{\big(1-\big(1-\frac{\delta}{R}\big)^2\big)\overline{\mu}-\frac{4}{K^2M^2}\Bigg\}\times$$
$$\frac{J_0(iKM(1-\frac{\delta}{R}))}{J_0(iKM(1-\frac{\delta}{r}))}\Bigg]\Bigg] \tag{14}$$

The resistance is obtained by using the formula

$$\lambda_R = \left(\frac{-dp/dz}{Q}\right) \tag{15}$$

Now use of (12) in (15), gives the expression for resistance to flow as

$$\frac{1}{\lambda_R} = \frac{\pi R^4}{8\mu_s}\Bigg[\Bigg\{1-2\big(1-\frac{\delta}{R}\big)^2+\big(1-\frac{\delta}{R}\big)^4\Bigg\}\overline{\mu}+\Bigg\{1-K^2(1-K^2)\frac{M^2\mu_s}{R^2S}\Bigg\}\times$$
$$\Bigg[\frac{8}{K^2M^2}\big(1-\frac{\delta}{R}\big)^2-\frac{4i}{KM}\big(1-\frac{\delta}{R}\big)\Bigg\{\big(1-\big(1-\frac{\delta}{R}\big)^2\big)\overline{\mu}-\frac{4}{K^2M^2}\Bigg\}\times$$
$$\frac{J_0(iKM(1-\frac{\delta}{R}))}{J_0(iKM(1-\frac{\delta}{r}))}\Bigg]\Bigg]. \tag{16}$$

For non-magnetic case ($M = 0$) or when volume fraction ϕ is zero, the expressions for total flow rate, effective viscosity and resistance to flow are reduces to :

$$Q = \frac{\pi R^4}{8\mu_s}\left(-\frac{dp}{dz}\right)\left[\left\{1-\left(1-\frac{\delta}{R}\right)^4\right\}\overline{\mu}+2\left(1-\frac{\delta}{R}\right)^3\right] \tag{17}$$

$$\frac{1}{\mu_{eff}} = \frac{1}{\mu_s}\left[\left\{1-\left(1-\frac{\delta}{R}\right)^4\right\}\overline{\mu}+2\left(1-\frac{\delta}{R}\right)^3\right] \tag{18}$$

$$\frac{1}{\lambda_R} = \frac{\pi R^4}{8\mu_s}\left[\left\{1-\left(1-\frac{\delta}{R}\right)^4\right\}\overline{\mu}+2\left(1-\frac{\delta}{R}\right)^3\right] \tag{19}$$

3. Results and Discussion

In order to discuss the results of the theoretical model proposed in the study, the analytical expressions for velocity profiles, flow rate, effective viscosity and resistance to flow have been obtained. It may be noticed that non-magnetic two-layered flow results can be obtained as a special case of the present model by putting $M = 0$.

To get a physical insight of the problem, the flow rate, effective viscosity and resistance to flow obtained analytically in the equations (12), (14) and (16) respectively have been plotted in Figures (2) to (10). The results are displayed graphically for 20% and 40% hematocrit for 40 μm and 70 μm diameter tubes. For numerical calculation we take $\mu_s = 2.18$ cp for 20% and 3.10 cp for 40% hematocrit and peripheral layer thickness $\delta = 4.67$ μm for 20% and 3.12 μm for 40% hematocrit (Haynes [7]; Bugliarello and Sevilla [20]; Sud and Sekhon [21]). The viscosity of plasma $\mu_p = 1.2$ cp and diameter of RBC $d = 6.78\mu m$ also have been considered in the numerical calculations.

The volumetric flow rate Q with pressure gradient $(-dp/dz)$ for 20% hematocrit, 40 μm diameter; 40% hematocrit, 40 μm diameter; 20% hematocrit, 70 μm diameter; 40% hematocrit, 70 μm diameter have been plotted in Figs. 2, 3, 4, 5 respectively. From figures and numerical results it is concluded that the flow rate increases with pressure gradient and decreases with Hartman number. For large values of $M > 7, 5, 8, 5$ for above four cases the flow rate is almost zero. It is also noticed that the flow rate decreases as hematocrit increases and reverse effect as diameter of the tube increases.

Figures 6 and 7 show the variation of effective viscosity with Hartman number for 20% and 40% hematocrit for 40 μm and 70 μm tubes diameter. It is observed that the effective viscosity increases with Hartman number upto M

= 7, 5, 8, 5 for the four cases. Figure 8 shows the variation of effective viscosity with peripheral layer thickness for different values of Hartman number. It is observed that for one-layered MHD flow, the effective viscosity is maximum. As the layer thickness increases, effective viscosity decreases. Thus the effective viscosity of blood can be controlled by varying the strength of magnetic field (Hartman number) and peripheral layer thickness. This is the good agreement to the results obtained by Chaturani and Bhartiya [16] for a dialyser.

The effect of Hartman number (M) on resistance to flow (λ_R) has been plotted in figures 9 and 10. From figures it is concluded that the resistance to flow increases with Hartman number for different cases upto M = 7, 5, 8, 5. Numerical values of λ_R decreases as diameter increases but reverse effect are obtained as hematocrit increases.

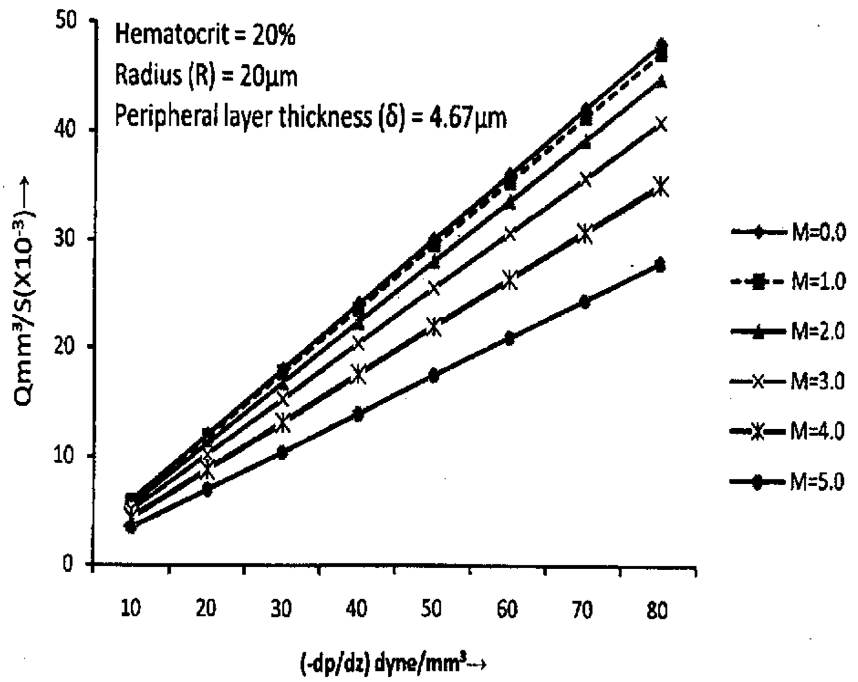

Fig.2:Variation of flow rate with pressure gradient.

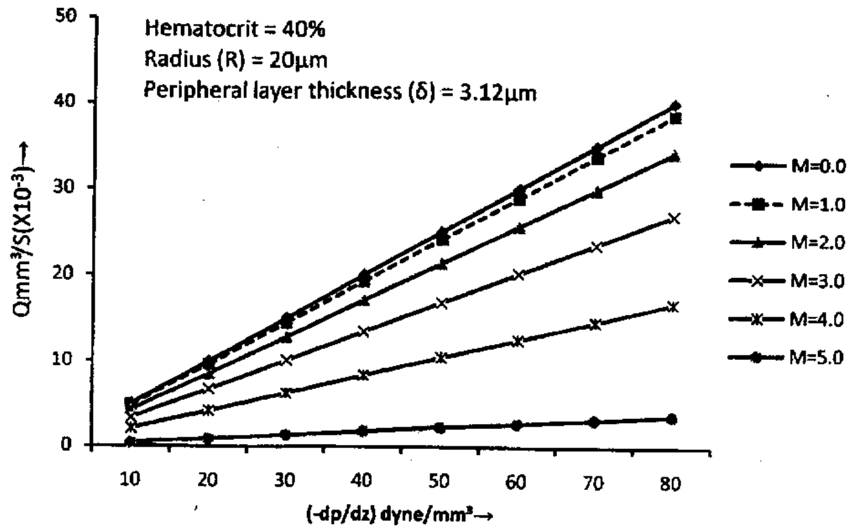

Fig.3:Variation of flow rate with pressure gradient.

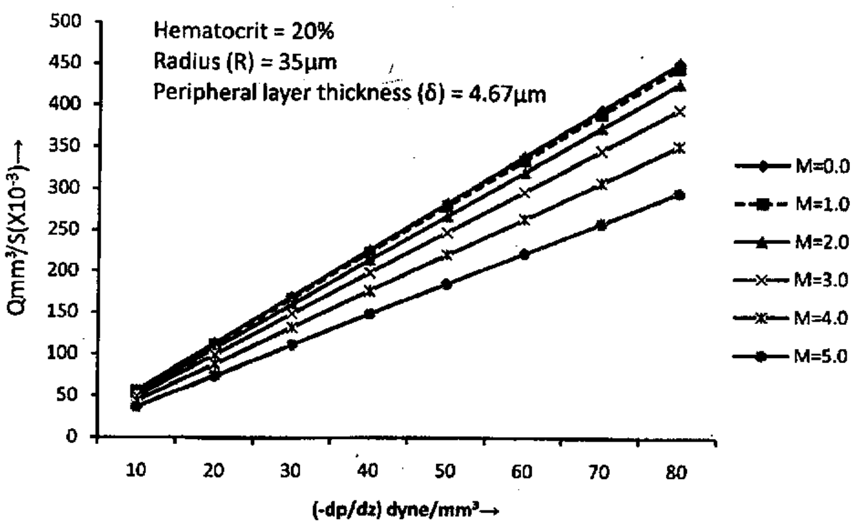

Fig.4:Variation of flow rate with pressure gradient.

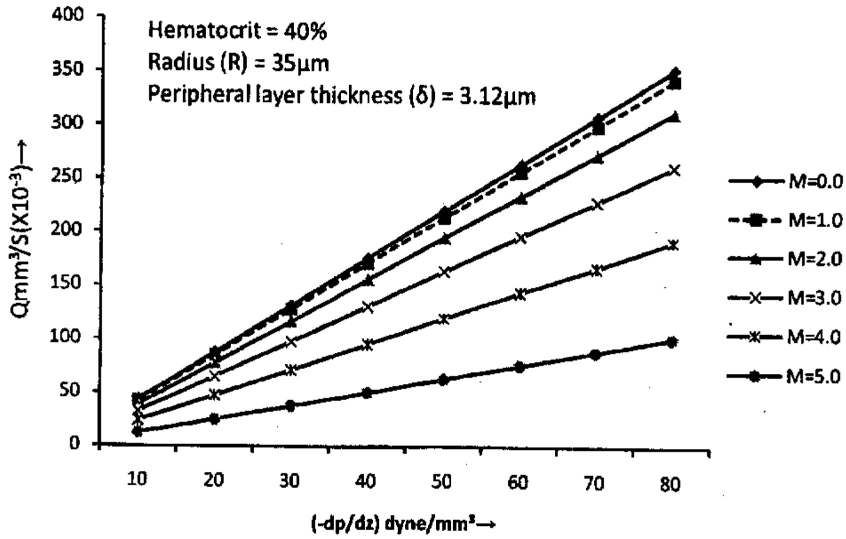

Fig.5: Variation of flow rate with pressure gradient.

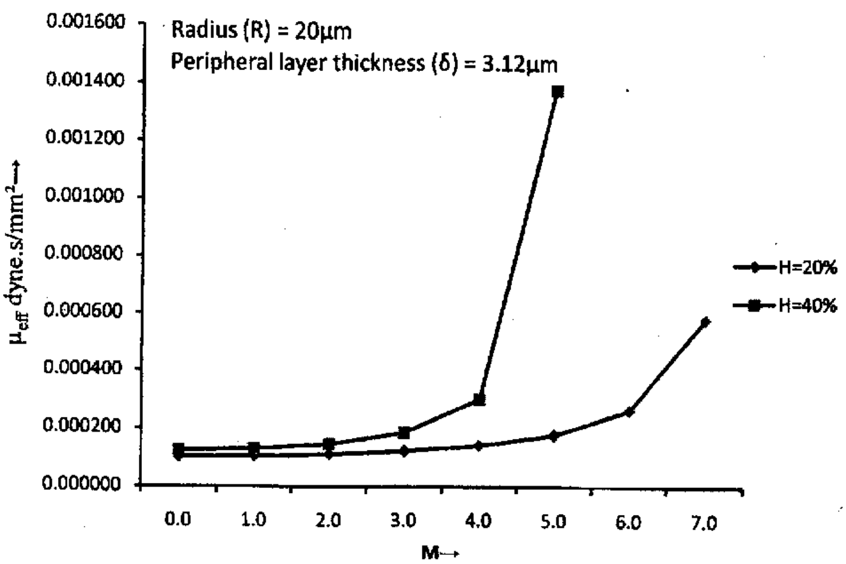

Fig.6: Variation of effective viscosity with Hartman number.

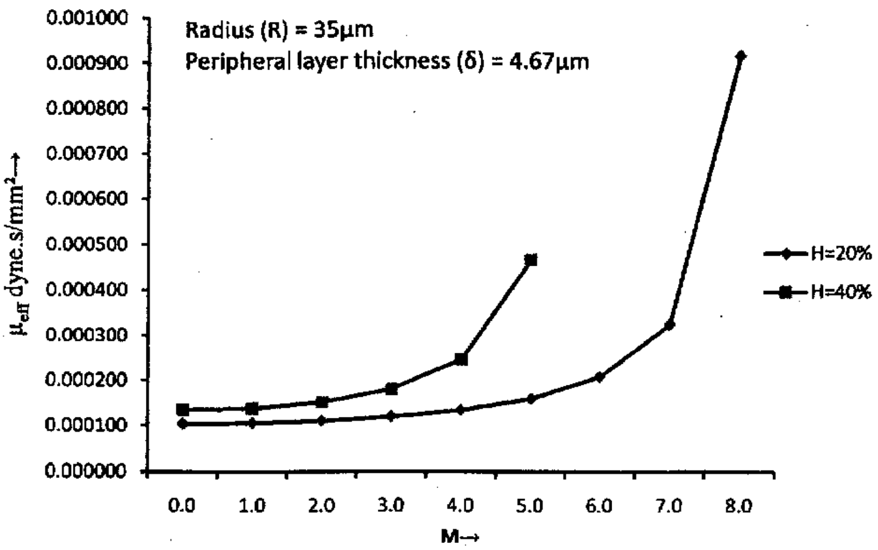

Fig.7:Variation of effective viscosity with Hartman number.

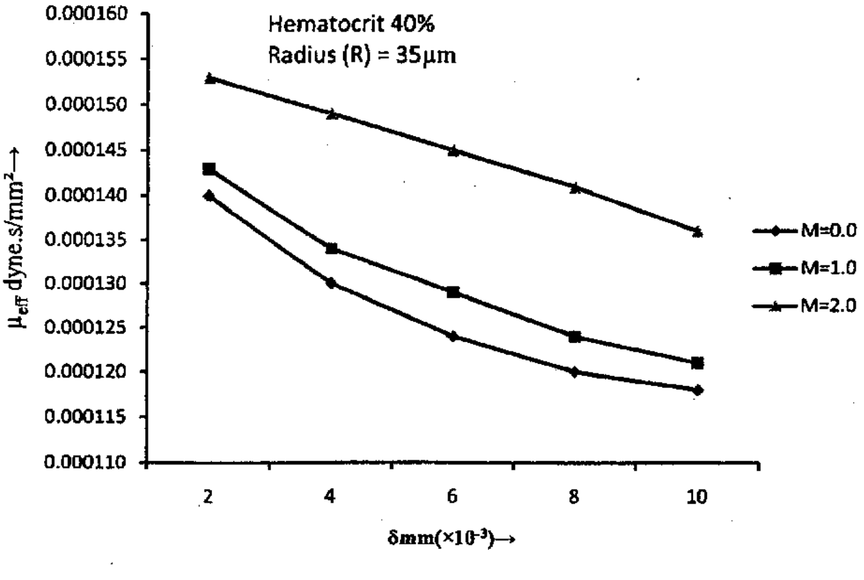

Fig.8:Variation of effective viscosity with peripheral layer thickness.

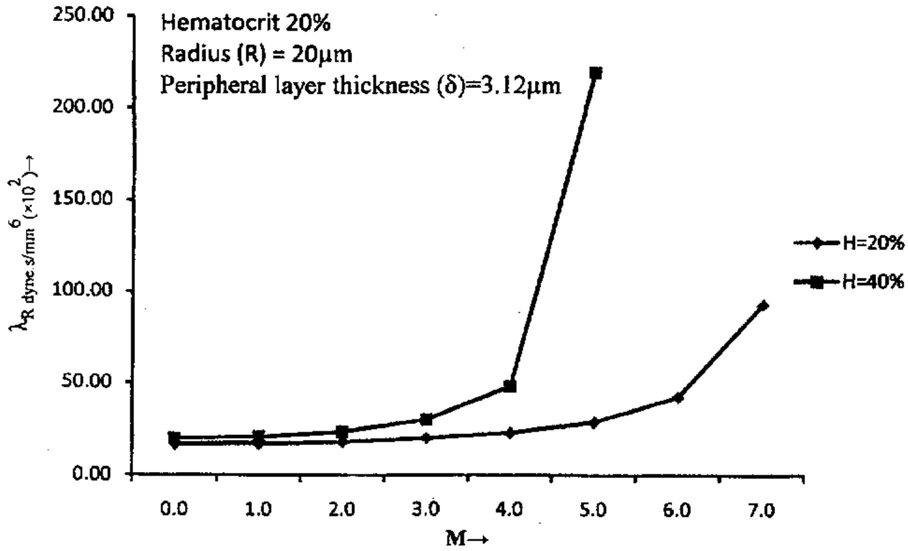

Fig.9:Variation of resistance to flow with Hartman Number.

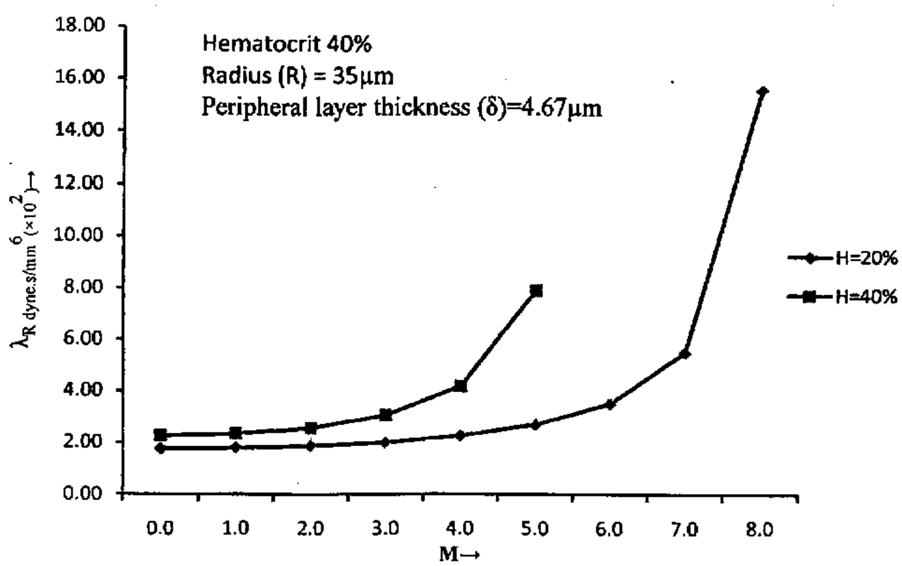

Fig.10:Variation of resistance to flow with Hartman Number.

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