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Kantowski-Sachs Universe in the Varying Speed of Light Theory

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Abstract

In this work we have considered the Kantowski-Sachs (KS) universe in the framework of varying speed of light theory. We have presented the general solution of the gravitational field equations with variable speed of light $c(t)$, gravitational coupling parameter $G(t)$ and decaying vacuum energy $\Lambda(t)$ for the KS model. In the limiting case for the equation of state (EOS) parameter $\gamma = 2$ (stiff fluid with $p = \rho c^2$) and $\gamma = 1$ (dust with $p = 0$), exact solutions of the field equations are obtained. The numerical solutions are also presented for both the cases. Moreover, it is shown that in the limiting case of large time, the mean anisotropy parameter tends to zero for $\gamma = 2$ and $\gamma = 1$. Thus the time variation of the fundamental constants provides an effective mechanism for making the KS universe isotropic.

Keywords: general relativity, Kantowski-Sachs universe, varying speed of light.

1. Introduction

[1] proposed the large number hypothesis (LNH), motivated by the occurrence of large numbers in the universe. However, in this connection an exclusive review on the LNH can be obtained in the work of [2] for further reading. Inspired by this theory several scientists [3]−[21] have been intensively investigated problems on the variable gravitational constant and cosmological constant with variable cosmological term Λ in the cosmological as well as astrophysical realm.

On the other hand, work has been done on the cosmological models with variable cosmological term within a framework of dissipative thermodynamics as well as in the case of perfect fluid [22]-[24]. As G couples geometry to matter, it is reasonable to consider $G = G(t)$ in an evolving universe when one considers $\Lambda = \Lambda(t)$. Many extensions of general relativity with $G = G(t)$ has been made ever since Dirac first considered the possibility of variable G, though none of these theories has gained wide acceptance [25]-[32].

The varying speed of light (VSL) cosmology was proposed by [33] as an alternative to cosmological inflation to provide different basis for resolving the problems of the standard models. He conjectured that a spontaneous breaking of the local Lorentz invariance and dimorphism invariance associated with a first order phase-transition can lead to the variation of the speed of light in the early universe. This idea was later on considerably revived by [34]-[36]. Barrow showed that the conception of VSL can lead to the solution of flatness, horizon and monopole problems if the speed of light falls at an appropriate rate. [37] widely studied the dynamics of VSL in theoretical as well as empirical context. It has, in particular, been speculated that VSL offers new paths of solving the problems of the standard Big Bang cosmology which are distinct from their resolutions in context of the inflationary paradigm [38] or the pre-Big Bang scenario of low energy string theory [39]. Moreover, in contrast to the case of inflationary Universe, VSL may provide an explanation for the relativistic smallness of the various physical constants today.

In connection to VSL theory [35] introduced a Machian scenario in which $c = c_0 a^n$, where a is the scale factor. This has significant advantages to the phase-transition scenario in which the speed of light changes suddenly from c^- to c. On the other hand, [34] have investigated possible consequences of a time variation in the velocity of light in vacuum. The Einstein field equations for FRW space time in the VSL theory have been solved by [35]-[36] for anisotropic model, who also obtained the rate of variation of the speed of light required to solve flatness and cosmological constant problems. Some other work to be mentioned in this field is as follows: (i) By assuming energy conservation of observed matter, [40] have solved the flatness cosmological constant problem with varying speed of light c, gravitational coupling strength G and cosmological parameter Λ, (ii) [41] have found exact constant solutions for cosmological density parameter using generalization of general relativity that incorporates a cosmic time variation of velocity of light in vacuum and Newtonian gravitation constant, and (iii) [23] and [42]-[43], studied perfect fluid Bianchi type I model with variable G, c and Λ.

Besides all the above mentioned major work on VSL theory, it is observed that some authors [44]-[45] have proposed a new generalization of general relativity which also allows arbitrary changes in the speed of light c and the gravitational constant G. However, this has been done in such a way that variation in the speed of light introduces corrections to the curvature tensor in the Einstein equations in the cosmological frame. [44] considered the evolution and dynamics of Bianchi type I and V Universe and obtained exact solutions of the gravitational field equations in a small time scale limit.

The purpose of the present paper is to extend the results previously obtained in the framework of the homogeneous and isotropic FRW cosmological models to the case of anisotropic KS universe. In essence we carried out here an investigation to highlight specific features of KS Universe which basically represent dust solutions to the Einstein field equation and are a widely used family of inhomogeneous cosmological models. Basically, we have generalized the work of [44] by considering that the constants are functions of the volume scale factor and obtained the solutions of the field equations under the framework of VSL theory in the small time limit. The paper is organized as follows: The field equations for KS model are written down in Sec. 2. In Sec. 3 we have obtained exact solutions of the field equations for VSL model corresponding to specific time variation law of constants. In Sec. 4, we conclude our results.

2. The Einstein Field Equations

We consider the KS Universe in the framework of Einstein's general relativity. As an additional condition we impose on the physical constants, are some restrictions as advocated by the LNH and VSL theory. The line element for KS Universe is given by

$$ds^2 = -c^2(t)dt^2 + a_1^2(t)dr^2 + a_2^2(d\theta^2 + sin^2\theta d\phi^2), \tag{1}$$

where a_1, a_2 are the scale factors.

The Einstein field equations take the usual form

$$R_{ij} - \frac{1}{2}g_{ij}R = -\frac{8\pi G(t)}{c^4(t)}T_{ij} + \Lambda(t)g_{ij}. \tag{2}$$

The energy momentum tensor can be written as

$$T_{ij} = (p + \rho\, c^2) u_i u_j + p g_{ij}, \tag{3}$$

where u^i $(i = 0, 1, 2, 3)$ be the four velocity, p and ρ are respectively the fluid pressure and energy density.

For the KS metric (1), the Einstein field Eq. (2) can be written as

$$\frac{\dot{a_2}^2}{a_2^2} + \frac{2\dot{a_1}\dot{a_2}}{a_1 a_2} + \frac{c^2}{a_2^2} = 8\pi G\rho + \Lambda c^2, \tag{4}$$

$$\frac{2\ddot{a_2}}{a_2} + \frac{\dot{a_2}^2}{a_2^2} - \frac{2\dot{a_2}\dot{c}}{a_2 c} + \frac{c^2}{a_2^2} = -\frac{8\pi G p}{c^2} + \Lambda c^2, \tag{5}$$

$$\frac{\ddot{a_1}}{a_1} + \frac{\ddot{a_2}}{a_2} + \frac{\dot{a_1}\dot{a_2}}{a_1 a_2} - \frac{\dot{a_2}\dot{c}}{a_2 c} - \frac{\dot{a_1}\dot{c}}{a_1 c} = -\frac{8\pi G p}{c^2} + \Lambda c^2. \tag{6}$$

We assume that the thermodynamic pressure p of the cosmological fluid obeys a linear equation of state

$$p = (\gamma - 1)\rho c^2(t), \tag{7}$$

where equation of state parameter $\gamma =$ constant and $1 \leq \gamma \leq 2$.

The conservation equation gives

$$\dot{\rho} + \left(\frac{\dot{a_1}}{a_1} + \frac{2\dot{a_2}}{a_2}\right)\gamma\rho = -\frac{c^2\dot{\Lambda}}{8\pi G} - \frac{\dot{G}}{G}\rho + \frac{2\dot{c}}{c}\rho. \tag{8}$$

We assume that $\left(T^{j}_{i;j}\right) = 0$, which leads to the following two equations:

$$\dot{\rho} + \left(\frac{\dot{a_1}}{a_1} + \frac{2\dot{a_2}}{a_2}\right)\gamma\rho = 0, \tag{9}$$

$$-\frac{c^2\dot{\Lambda}}{8\pi G} - \frac{\dot{G}}{G}\rho + \frac{2\dot{c}}{c}\rho = 0. \tag{10}$$

For later convenience we introduce the following variables:

$$V = a_1 a_2^2, \tag{11}$$

$$H_i = \frac{\dot{a_i}}{a_i}, \tag{12}$$

$$H = \frac{1}{3}(H_1 + 2H_2). \tag{13}$$

We further define

$$\Delta H_i = H_i - H, \tag{14}$$

where $i = 1, 2, 3$ and $H_2 = H_3 = \frac{\dot{a_2}}{a_2}$, the factors V, H_i and H being the volume scale factor, directional Hubble parameters and mean Hubble parameter respectively.

We obtain

$$H = \frac{\dot{V}}{3V}. \tag{15}$$

In addition to this we introduce the physical quantities related to cosmology as follows:

$$\theta = 3H, \tag{16}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{1}{3}\sum_{i=1}^{3}\frac{(H_i - H)^2}{H^2}, \tag{17}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ik}\sigma^{ik} = \frac{1}{2}\left(\sum_{i=1}^{3}H_i^2 - 3H^2\right) = \frac{3AH^2}{2}, \tag{18}$$

$$q = -H^{-2}(\dot{H} + H^2) = \frac{d}{dt}\left(\frac{1}{H}\right) - 1, \tag{19}$$

where θ, A, σ^2 and q are the scalar expansion, mean anisotropy parameter, shear scalar and deceleration parameter respectively.

Note that $A = 0$ for isotropic expansion. Moreover, the signature of the deceleration parameter indicates how the Universe expands. In fact the positive sign corresponds to 'standard' deceleration model whereas a negative sign indicates an accelerating Universe.

3. An Exact Solution for the KS Universe

In this section we find the exact solutions of the field equations.
From Eq. (5) and (6), we get

$$\frac{\dot{a_2}^2}{a_2^2} + \frac{\ddot{a_2}}{a_2} + \frac{c^2}{a_2^2} - \frac{\dot{a_2}\dot{c}}{a_2 c} + \frac{\dot{a_1}\dot{c}}{a_1 c} = \frac{\dot{a_1}\dot{a_2}}{a_1 a_2} + \frac{\ddot{a_1}}{a_1}. \tag{20}$$

Adding Eq. (4) and (5) and using Eq. (20) in it, we obtain

$$\frac{1}{V}\frac{d}{dt}(VH_1) - \frac{\dot{c}}{c}H_1 = 4\pi\ G\left(\rho - \frac{p}{c^2}\right) + \Lambda c^2. \tag{21}$$

Also adding Eq. (4) and (6) and using Eq. (20), we get

$$\frac{1}{V}\frac{d}{dt}(VH_2) - \frac{\dot{c}}{c}H_2 = 4\pi\ G\left(\rho - \frac{p}{c^2}\right) + \Lambda c^2 - \frac{c^2 a_1}{V}. \tag{22}$$

From Eqs. (21) and (22) with the help of Eq. (12) one can get

$$3\dot{H} + H_1^2 + 2H_2^2 - 3\frac{\dot{c}}{c}H = -4\pi\ G\left(\rho + 3\frac{p}{c^2}\right) + \Lambda c^2. \tag{23}$$

Again adding Eq. (21) and (22), we get

$$\begin{aligned}&\frac{1}{V}\frac{d}{dt}(V[H_1 + 2H_2]) - \frac{\dot{c}}{c}(H_1 + 2H_2)\\ &= 12\pi\ G\left(\rho - \frac{p}{c^2}\right) + 3\Lambda c^2 - \frac{2c^2 a_1}{V}.\end{aligned} \tag{24}$$

Now using Eq. (13) and (15) in Eq. (24), with (7) we get

$$\ddot{V} - \dot{V}\frac{\dot{c}}{c} = 3\Lambda V c^2 + 12\pi\ G(2-\gamma)\rho V - 2a_1^2 c^2. \tag{25}$$

Now we assume that the 'constants' G, c and Λ are decreasing functions of time and to describe their variations we use the following simple phenomenological law given by [44]

$$G = G_0 + \frac{G_1}{V^\alpha}, \quad \Lambda = \Lambda_0 + \frac{\Lambda_1}{V^\beta}, \quad c = c_0 + \frac{c_1}{V^\eta}. \tag{26}$$

where $G_0 > 0, G_1 \geq 0, \Lambda_0 > 0, \Lambda_1 \geq 0, c_0 > 0, c_1 > 0, \alpha > 0, \beta > 0$ and $\eta > 0$ are all constants.

For $t \to 0$, V is extremely small, then

$$G \approx \frac{G_1}{V^\alpha},\ \Lambda \approx \frac{\Lambda_1}{V^\beta},\ c \approx \frac{c_1}{V^\eta}. \tag{27}$$

From the conservation Eq. (9), ρ can be expressed as

$$\rho = \rho_0 V^{-\gamma}, \tag{28}$$

where ρ_0 is constant of integration.

After using Eq. (27) and (28) in Eq. (10), we get the consistency conditions relating the constants $\alpha, \beta, \gamma, \eta, G_1, \Lambda_1$ and c_1 as

$$\alpha - \beta + \gamma - 2\eta = 0, \tag{29}$$

$$\alpha - 2\eta = \frac{-c_1^2 \Lambda_1 \beta}{8\pi G_1 \rho_0}. \tag{30}$$

Hence Eq. (29) and (30), yield

$$\beta = \left(1 + \frac{c_1^2 \Lambda_1}{8\pi G_1 \rho_0}\right)^{-1}. \tag{31}$$

Now by using Eq. (27) and (28) in Eq. (25), we get

$$V\ddot{V} + \eta \dot{V}^2 = \frac{3c_1^2 \Lambda_1}{V^{2\eta+\beta-2}} + \frac{12\pi G_1 \rho_0 (2-\gamma)}{V^{\alpha+\gamma-2}} - \frac{2a_1 c_1^2}{V^{2\eta} - 1} \tag{32}$$

From Eq. (32), after first integration we get

$$\dot{V}^2 = V^{-2\eta} F(V), \tag{33}$$

where

$$F(V) = \left(\frac{6c_1^2 \Lambda_1}{2-\beta} V^{2-\beta} + \frac{24\pi G_1 \rho_0 (2-\gamma)}{2\eta - \alpha - \gamma + 2} V^{(2\eta-\alpha-\gamma+2)} + C\right). \tag{34}$$

where $C = -4c_1^2 \int a_1 dV + c_2$ and $c_2 > 0$ is a constant of integration.

From Eqs. (21) and (22) after integration, we get

$$log[V(H_1 - H_2)] = logc + logK, \tag{35}$$

where $K = exp\left[\int \frac{c^2 a_1}{V(H_1 - H_2)} dt\right]$.

We note that [46] considered $K = \int a_1 dt$ in the conventional Einstein's theory. In our case the value of K is similar to this value for $\alpha = \beta = \eta = 0$ and $c_1 = 1$.

After simplification Eq. (35) provides

$$H_1 = H + \frac{2}{3}\frac{Kc}{V}, \tag{36}$$

and

$$H_2 = H - \frac{Kc}{3V}. \tag{37}$$

If we substitute Eqs. (36) and (37) in Eq. (23), then with the help of Eqs. (29) and (30) we get

$$\frac{c_1^2\Lambda_1\beta[\gamma(\eta+\beta)-2\eta+2]}{(\beta-1)(\beta-2)V^{(2\eta+\beta-2)}} - \frac{2}{V^{2\eta}}\left(a_1c_1^2V+\frac{c_2}{3}\right) + \frac{8c_1^2}{3V^{2\eta}}\int a_1 dV + \frac{2}{3}K^2 = 0. \tag{38}$$

For $\alpha = \beta = \eta = c_2 = 0$ and $c_1 = 1$, the above equation reduces to the equation of the form

$$-2a_1V + \frac{8}{3}\int a_1 dV + \frac{2}{3}\left(\int a_1 dt\right)^2 = 0,$$

as obtained earlier by [46] in the general theory of relativity.

By taking $V \geq 0$, as a parameter, we can obtain the general solution for KS model with physical quantities as follows:

$$t - t_0 = \int \frac{V^\eta}{F(V)^{1/2}} dV, \tag{39}$$

$$\theta = 3H = \frac{F(V)^{1/2}}{V^{\eta+1}}, \tag{40}$$

$$a_1 = V^{1/3} a_{01}\ exp\left[\frac{2}{3}Kc_1\int\frac{dV}{VF(V)^{1/2}}\right], \tag{41}$$

$$a_2 = V^{1/3} a_{02}\ exp\left[\frac{-1}{3}Kc_1\int\frac{dV}{VF(V)^{1/2}}\right], \tag{42}$$

$$A = \frac{2K^2c_1^2}{F(V)}, \tag{43}$$

$$\sigma^2 = \frac{K_1^2c_1^2}{3V^{2(\eta+1)}}. \tag{44}$$

$$q = 3\eta + 2 + \tfrac{3V}{2F(V)} \times \left[-6\Lambda_1c_1^2V^{1-\beta} + 24\pi G_1\rho_0(\gamma-2)V^{(2\eta-\alpha-\gamma+1)} + 4c_1^2a_1\right], \tag{45}$$

where t_0, a_{01} and a_{02} are constants of integration.

3.1. **Case I : $\gamma = 2$.** In this case for extremely small V, Eq. (34) can be written as

$$F(V) = \frac{6c_1^2\Lambda_1}{2-\beta}V^{2-\beta} + B_1, \tag{46}$$

where B_1 is constant.

Using Eq. (46) in Eq. (33) we get

$$\dot{V}^2 = V^{-2\eta}\left(\alpha_0 V^{2-\beta} + B_1\right), \tag{47}$$

where $\alpha_0 = \left(\frac{6c_1^2\Lambda_1}{2-\beta}\right), \quad \beta \neq 2.$

After integrating Eq. (47) with the condition $\alpha_0 V^{2-\beta} >> B_1$ and $t_0 = 0$, it gives

$$V \approx t^{\frac{2}{2\eta+\beta}} \Longrightarrow H \approx \frac{1}{t}. \tag{48}$$

Hence the scale factors a_1 and a_2 can be obtained as

$$a_i \approx t^{\frac{2}{3(2\eta+\beta)}}, \tag{49}$$

for $i = 1, 2$.

Using Eqs. (43) and (48) we get

$$A \approx t^{\frac{2(\beta-2)}{(2\eta+\beta)}}, \quad \beta < 2. \tag{50}$$

$$\sigma^2 \approx \left(\frac{1}{t^{\frac{4(\eta+1)}{2\eta+\beta}}}\right). \tag{51}$$

$$q = 3\left(\eta + \frac{\beta}{2}\right) - 1. \tag{52}$$

Thus q tends to a constant value. The analogous solutions for extremely small value of V are discussed by [44] for Bianchi type I model.

3.2. **Case II : $\gamma = 1$.** Using Eq. (34) with $\Lambda_1 = 0$ for extremely small value of V, we get

$$F(V) = b_0 V^{2\eta-\alpha+1} + B_3, \tag{53}$$

where $b_0 = \left(\frac{24\pi G_1\rho_0}{(2\eta-\alpha+1)}\right)$ and B_3= constant.

Hence using Eq. (53) in Eq. (33), we get

$$\dot{V}^2 = V^{-2\eta}\left(b_0 V^{(2\eta-\alpha+1)} + B_3\right). \tag{54}$$

After integrating Eq. (54) with the condition $b_0V^{2\eta-\alpha+1} >> B_3$ and $t_0 = 0$, it gives

$$V \approx t^{\frac{2}{3-\alpha}} \Longrightarrow H \approx \frac{1}{t}. \tag{55}$$

The other variables can be calculated straightforwardly as follows:

$$a_i \approx t^{\frac{2}{3(3-\alpha)}},\ \alpha \neq 3, \tag{56}$$

for $i = 1, 2$.

$$A \approx t^{\frac{-2(2\eta+1-\alpha)}{(3-\alpha)}},\ \alpha \neq 3. \tag{57}$$

$$\sigma^2 \approx t^{\frac{-4(\eta+1)}{(3-\alpha)}},\ \alpha \neq 3. \tag{58}$$

$$q = \frac{3}{2}(\alpha - 1) + 2. \tag{59}$$

The deceleration parameter is always positive when $\alpha > -1/3$ and negative when $\alpha < -1/3$.

4. Conclusion

In this paper we have generalized the work of [44] for KS model in the framework of VSL theory. We have obtained the solutions for KS model in a small time limit in which variables G, c and Λ are functions of the volume scale factor. For extremely small value of V and $\gamma = 2$, with the condition $\alpha_0V^{2-\beta} >> B_1$, $F(V) \propto V^{2-\beta}$ with $t_0 = 0$, it is observed that $V \propto t^{\frac{2}{2\eta+\beta}}$ and hence, the expansion of the early Universe is of the form of power law of the expansion. The mean anisotropy will increase for $\beta > 2$, and KS space-time will not end in isotropic state in large time limit. However, for $\beta < 2$, the mean anisotropy tends to 0 in the large time limits, thus the KS type VSL cosmology is providing an effective mechanism for making the universe isotropic. The evolution of the anisotropic flat Universe is generally non-inflationary, with the deceleration parameter $q > 0$, and tending, for large times, to a constant value given in the Eq. (52). The deceleration parameter is given by $q = 3(\eta+\beta/2)-1$, which is always positive when $\beta > (2/3-2\eta)$ and negative when $\beta < (2/3-2\eta)$.

Similarly, for the case $\gamma = 1$ with $\Lambda_1 = 0$ we have shown that $V \propto t^{\frac{2}{3-\alpha}}$ as $t \to 0$. Hence again the expansion of the early Universe is of the form of power law expansion. For $\alpha > 3$, the mean anisotropy will increase for large times limit and KS space time will not end in an isotropic state, whereas for $\alpha < 3$, the mean anisotropy tends to 0, thus the KS type VSL cosmology becomes

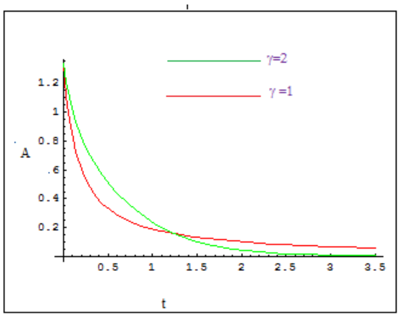

FIGURE 1. Variation of the mean anisotropy parameter A with respect to time, for KS model with perfect cosmological fluid for $\gamma = 2$ (green line) and $\gamma = 1$ (red line), in the small time limit. The constants are chosen as: $\beta = 1, c_1 = 1, C = 1, \Lambda_1 = 1$

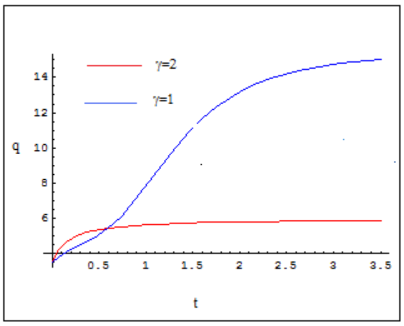

FIGURE 2. Variation of the deceleration parameter q with respect to the time, for stiff fluid $\gamma = 2$ (Red line), $\gamma = 1$ (blue line) and in the small V limit. The constants are chosen as: $\beta = 1, \eta = 1/2, c_1 = 1, C = 1, \Lambda_1 = 1, 8\pi G_1\rho_0 = 1, 4c_1^2 a_1 = 1$.

isotropic. Moreover, the deceleration parameter is given by $q = \frac{3}{2}(\alpha - 1) + 2$, which is always positive for $\alpha > 1$.

The time variation of mean anisotropic parameter A and deceleration parameter q for KS space-time are plotted with respect to the time as shown in the Figs. 1 and 2.

It have stated by [47] that the existence of cosmological constant Λ or variable G is not in conflict with observational determination of the age of the Universe or with some astrophysical data. The dynamics and evolution of the Universe is essentially determined by the values of the constants α, β, and η

describing the time variations of G, c and Λ and which are arbitrary in this model.

However, we would like to conclude here with a comment that [48] investigated the possible variation of c in the context of the present accelerating Universe as discovered through SN Ia observations and showed that variability of c is not permitted under the variable Λ models. This obviously impose an observational constraint on the theoretical speculation of VSL theory, however at the same time allows variation of Λ in terms of *dark energy* via the background platform of LNH, as put forward by [1].

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