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Higher Dimensional Cosmological Model of the Universe Dominated by Extended Chaplygin Gas

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Abstract

In this paper, we consider the extended Chaplygin gas equation of state as a model of dark energy which recovers quadratic barotropic fluids equation of state. We obtain an analytical expression of the energy density, Hubble parameter in terms of the scale factor in the framework of Kaluza Klein type FRW cosmological model.

Keywords: Extended Chaplygin gas, Dark energy.

1. Introduction

It is observed that major portion of the universe is filled with dark matter and dark energy. Therefore dark energy is one of the important discipline in theoretical physics and cosmology [1]. The most important question is describing the nature of dark energy. For searching the best possible solutions of this problems number of models are proposed during last few decades for example new exotic forms of matter i.e. quintessence [2], [3]. Another candidate is Einsteins cosmological constant which has two crucial problems so called fine tuning and cosmic coincidence [4]. There are also other interesting models to describe the dark energy such as k-essence model [5], tachyonic model [6], holographic models [7], inhomogeneous spacetime [8], [9] and higher dimensional space time. While the above mentioned alternatives to explain the observed acceleration of

the current phase have both positive and negative aspects, the one that caught the attention of large number of workers is the introduction of a Chaplygin type of gas [10]-[12] as new matter field to stimulate a sort of dark energy.

Kaluza [13] and Klein [14] independently were the first who initiated the study of unify gravity with electromagnetic interaction an extra dimension. Kaluza-Klein theory is essentially an extension of Einstein general theory of relativity in five dimensions which is of much interest in partical physics and cosmology. Modern discoveries shows that higher dimensional gravity theories may provide deep insight to understand the interaction of particle and it plays remarkable role to explain main problem of astrophysics particularly dark energy. In view of this many authors investigated Kaluza-Klein cosmological models with different dark energy and dark matter. The original Kaluza-Klein theory becomes base for other extra dimensional theories in different perspectives like brane models [15], string theory [16] and super gravity [17]. Wanjari and Khadekar [18], considered the cosmological application in the framework of Kaluza-Klein theory, by taking the Newtonian and cosmological constant G and Λ to be a function of t. Therefore, it would be very interesting to develop the Chaplygin gas model in the Kaluza-Klein universe. Many researchers studied Chaplygin gas model in the frame work of Kaluza-Klein theory, for example, Khadekar and Ramtekkar [19] presented Kaluza Klein type FRW cosmological model of dark energy filled with extended Chaplygin gas and studied the behaviour of cosmological parameters for particular and arbitrary values of n, m and α. Salti [20] describe unified dark matter-energy scenario in the context of Kaluza-Klein cosmology by investigating cosmological features of the variable Chaplygin gas and shows that, the variable Chaplygin gas evolves from the dust-like phase to the phantom or the quintessence phases.

Motivated by the above discussion and investigations in Kaluza-Klein cosmology we have extended the work obtained earlier by Kahya and Pourhassan [21] and obtained physical parameter in the framework of Kaluza Klein theory of gravitation. This paper is organized as follows: In section 2, we introduced our model and write Modified Chaplygin gas. In section 3, we consider Friedmann-Robertson-Walker (FRW) universe and obtain field equations. Also we obtained expression for energy density in terms of scale factor in section 4. Concluding remarks is given in section 5.

2. Modified Chaplygin Gas

The Chaplygin gas equation of state is given by [22]

$$p_{CG} = -\frac{B}{\rho_{CG}}, \tag{1}$$

where B is positive constant, p_{CG} is pressure and ρ_{CG} is energy density.
The equation of state for generalized Chaplygin gas (GCG) is given by

$$p_{GCG} = \frac{B}{\rho^{a}_{GCG}} \tag{2}$$

where α and B are free parameters. It is clear that $\alpha = 1$ reproduces the pure Chaplygin gas model. This model is called the generalized Chaplygin gas model. At high energy generalized Chaplygin gas behaves almost like a pressure less dust whereas at low energy regime it behaves like a dark energy, its pressure being negative and almost constant. Thus generalized Chaplygin gas smoothly interpolates between a non relativistic matter dominated phase in the early universe to a dark energy dominated phase in the late universe. This interesting property of generalized Chaplygin gas has motivated cosmologists to consider it as a candidate for dark matter and dark enery models. It is also possible to study viscosity in generalized Chaplygin gas [23]-[25].
Although the Chaplygin gas models were introduced to explain late time acceleration without dark energy, the primordial acceleration which is believed to be driven by a scalar field, Inflaton, can also be described by the same modified Chaplygin gas model. Therefore we would like to describe both the primordial and recent inflationary phase with a scalar field, that we can still call inflaton, for which the equation of state is the one that the modified Chaplygin gas (MCG)obeys. modified Chaplygin gas obeys equation of state of the form [26],

$$p_{MCG} = A\rho_{MCG} - \frac{B}{\rho^{\alpha}_{MCG}}, \tag{3}$$

where A, α, and B are parameters of the model. $A = 0$ recovers generalized Chaplygin gas equation of state by setting $A = 0$ together with $\alpha = 1$ recovers the original Chaplygin gas equation of state. The modified Chaplygin gas equation of state has two parts, the first term gives an ordinary fluid obeying a linear barotropic equation of state, while there are some models with quadratic equation of state [27]

$$p = \rho_0 + \omega_1\rho + \omega_2\rho^2 \tag{4}$$

where ρ_0, ω_1 and ω_2 are constants. We set $\rho_0 = \omega_2 = 0$ which recover linear barotropic equation of state.

However, it is possible to consider barotropic fluid with quadratic equation of state or even with higher order equation of state [28], [29]. Therefore, it is interesting to extend modified Chaplygin gas equation of state which recovers at least barotropic fluid with quadratic equation of state. This is called extended Chaplygin gas (ECG), which was first proposed by Pourhassan and Kahya [30]. The equation of state for extended Chaplygin gas is defined as,

$$p_{ECG} = \Sigma A_n \rho_{ECG}^n - \frac{B}{\rho_{ECG}^a}, \tag{5}$$

It is obvious that the $n = 1$ reduced to modified Chaplygin gas. In this paper, we would focus on the second-order term which recovers quadratic barotropic equation of state,

$$p_{ECG} = A_1 \rho_{ECG} + A_2 \rho_{ECG}^2 - \frac{B}{\rho_{ECG}^a}, \tag{6}$$

where a, A_1, A_2 and B are free parameters of the model.

3. Field equations and their solutions

The Kaluza-Klein type Friedmann-Robertson-Walker (FRW) model is given by

$$ds^2 = dt^2 - a^2[dr^2 - r^2(d\theta^2 + sin^2\theta d\phi^2) + d\Psi^2], \tag{7}$$

where $a(t)$ is the scale factor.
The Einstein field equations given by,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{8}$$

where the energy momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (p + \rho) u_\mu u_\nu - p g_{\mu\nu}. \tag{9}$$

The Einstein field equations for the model (7) with the help of Eq. (9) becomes

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{ECG}}{6}, \tag{10}$$

$$\dot{H} = \frac{1}{6}(2p_{ECG} + \rho_{ECG}), \tag{11}$$

where H is the Hubble expansion parameter and a is the scale factor.
The energy conservation equation is as follows,

$$\dot{\rho}_{ECG} + 4\frac{\dot{a}}{a}(p_{ECG} + \rho_{ECG}) = 0, \tag{12}$$

here we put $8\pi G = 1$.

4. Extended Chaplygin Gas Cosmology

Now to simplify the calculations and to reduce the number of free parameters of the model and also to obtain an analytical expression of the energy density, Hubble expansion parameter in terms of the scale factor, we assume the following conditions,

$$\begin{aligned} \alpha &= 1, \\ A_1 &= A_2 - 1, \\ B &= 2A_2 \end{aligned} \tag{13}$$

So, the Eq. (6) is pure Chaplygin gas plus barotropic fluid equation of state. While it is a very special case of the extended Chaplygin gas. Having an explicit expression of the energy density in terms of the cosmological parameter is our motivation to choose the above condition. Therefore, only free parameter of the model is A_2, and we can solve the conservation Eq.(12) using (6) and (13) we obtain the following ralation,

$$\begin{aligned} lna = &\frac{ln(\rho_{ECG}^2 + 2\rho_{ECG} + 2)}{40A_2} - \frac{ln(\rho_{ECG} - 1)}{20A_2} \\ &- \frac{3arctan(\rho_{ECG} + 1)}{20A_2} + C \end{aligned} \tag{14}$$

where C is an integration constant. To obtain an analytical solution we use $tan^{-1}(\rho_{ECG} + 1) \approx \frac{\pi}{2}$ approximation which is exact for $\rho_{ECG} \leq 1$. So, Eq. (14) can be rewrite as follows:

$$\rho_{ECG} = 1 + \frac{2 + \sqrt{5a^{40A_2}e^{3\pi} - 1}}{a^{40A_2}e^{3\pi} - 1}. \tag{15}$$

$tan^{-1}(\rho_{ECG} + 1) \approx \frac{\pi}{2}$ approximation is valid when $\rho_{ECG} \leq 1$ corresponds to the early universe. However,our solution will be valid at all times and our approximate solution is very close to the late time behavior with $\rho_{ECG} \leq 1$. This is due to the fact that $tan^{-1}(\rho_{ECG} + 1) \approx \frac{\pi}{4}$, for $\rho_{ECG} \leq 1$ therefore small compared to the logarithm term. Behavior of energy density versus scale factor is shown in fig. 1 for different values of A_2. From fig. 1 it is observed that energy density increases with increasing value of A_2.

Hence, the Hubble parameter can be written as follows

$$H = \sqrt{\frac{1}{6} + \frac{2 + \sqrt{5a^{30A_2}e^{3\pi} - 1}}{6a^{40A_2}e^{3\pi} - 6}} \tag{16}$$

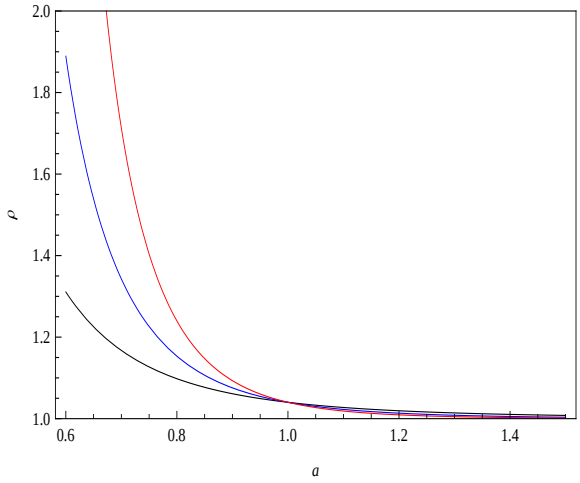

FIGURE 1. Energy density versus scale factor for $A_2 = 0.2$ (black line), $A_2 = 0.3$ (blue line), $A_2 = 0.5$ (red line).

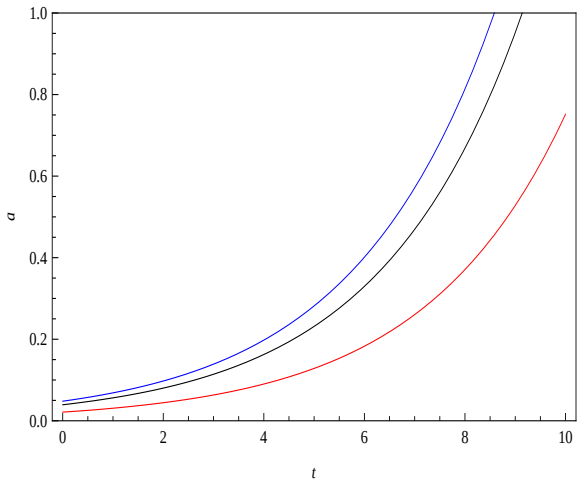

FIGURE 2. Scale factor versus time for $A_2 = 0.2$ (red line), $A_2 = 0.4$ (black line), $A_2 = 0.6$ (blue line).

By solving the above equation we get the scale factor in term of time and obtain the following relation.

$$t - f(a) + g(a) + C = 0 \tag{17}$$

where C is an arbitrary integration constant and $f(a)$ and $g(a)$ defined as follows,

$$f(a) \equiv \frac{\sqrt{3}ln(2\sqrt{-1+5a^{30A_2}e^{3\pi}}+1)}{15A_2} \tag{18}$$

$$g(a) \equiv \frac{2\sqrt{3}arctan(\sqrt{-1+5a^{30A_2}e^{3\pi}})}{15A_2}. \tag{19}$$

We remove the second function $g(a)$ as the contribution of it is small as compared to the logarithm term $f(a)$ and fix the constant C to capture the effect of $g(a)$.

By putting $g(a) + C = t_0$, we obtain the time-dependence scale factor,

$$a = \frac{e^{-\frac{3\pi}{40A_2}}}{20}[X(t)^2 - 2X(t) + 5]^{\frac{1}{40A_2}} \tag{20}$$

where

$$X(t) \equiv e^{5\sqrt{2}A_2(t+t_0)}. \tag{21}$$

To study the nature of scale factor we have represented it graphically in Fig. (2). From the figure it is clear that value of scale factor increases with increasing A_2.

The spatial volume of the model is given by [31]

$$V = a^4 = \left[\frac{e^{-\frac{3\pi}{40A_2}}}{20}[X(t)^2 - 2X(t) + 5]^{\frac{1}{40A_2}}\right]^4 \tag{22}$$

Behavior of volume is represented graphically in Fig. (3).

4.1. Deceleration Parameter

The deceleration parameter is important parameter in cosmology which describes cosmic dynamics for the late time acceleration. The deceleration parameter is given by

$$q = -\left(\frac{\dot{a}}{a}\right)^{-2}\frac{\ddot{a}}{a} = -1 - \frac{\dot{H}}{H^2}. \tag{23}$$

The value of q may be negative, positive or zero. $q = 1$ represents decelerating universe, $q = -1$ represents accelerating universe and $q = 0$ shows expansion of universe with constant rate.

By using Eqs. (10) and (11) in Eq. (23) we get,

$$q = -1 - \left[2(A_2 - 1) + 2A_2\left(1 + \frac{2 + \sqrt{5a^{40A_2}e^{3\pi} - 1}}{a^{40A_2}e^{3\pi} - 1}\right)\right] +$$

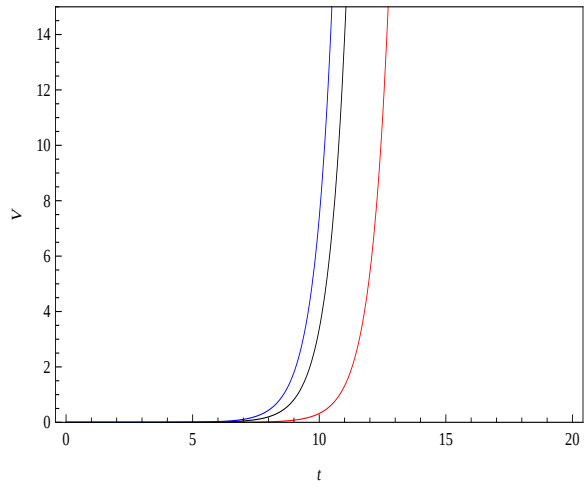

FIGURE 3. Volume versus time for $A_2 = 0.2$ (red line), $A_2 = 0.4$ (black line), $A_2 = 0.6$ (blue line).

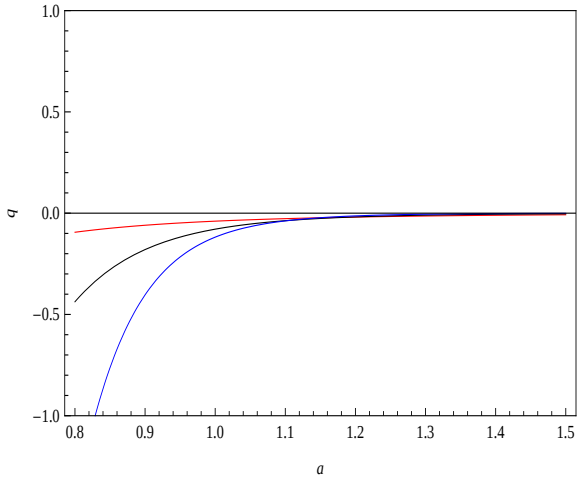

FIGURE 4. Deceleration parameter versus scale factor for $A_2 = 0.2$ (red line), $A_2 = 0.4$ (black line), $A_2 = 0.6$ (blue line).

$$4A_2\left[1+\frac{2+\sqrt{5a^{40A_2}e^{3\pi}-1}}{a^{40A_2}e^{3\pi}-1}\right]^{-1}. \tag{24}$$

Behavior of deceleration parameter is represented graphically in Fig. (4). From the graph it is clear that as the value of A_2 increases, the value of deceleration

parameter increase at reaches to $q = -1$. Therefore we can say that model under consideration represents accelerating universe.

5. Discussion and the concluding remarks

In this paper, we have presented FRW cosmological model dominated by the extended Chaplygin gas at the second order for which it recovers barotropic fluids with quadratic equation of state. Here, we obtained energy density and Hubble expansion parameter in terms of scale factor. Also we obtained the analytical expression for the scale factor in the framework of Kaluza-Klein type FRW cosmological model. From the expression of scale factor given in Eq. (21) it is clear that the scale factor is an increasing function of time and the nature of scale factor have represented graphically in Fig. (2). Numerical analysis for energy density in terms of scale factor is represented in Fig. 1, from this figure, we observe that energy density is also a increasing function of time. Behavior of volume of the universe and deceleration parameter are represented graphically in Fig. (3) and Fig. (4) respectively. From Fig. 3, it is clear that volume of the universe is a increasing function of time. Also from Fig. 4, we observe that as the value of A_2 increases, the value of deceleration parameter increase at reaches to $q = -1$. Therefore we can say that model under consideration represents accelerating universe.

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