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A classification of totally umbilical proper slant and hemi-slant submanifolds of (k,μ)-contact manifolds

M.S. Siddesha[1], M.M. Praveena[2] and C.S. Bagewadi[3]

[1] *Department of Mathematics, Jain University, Bengaluru-562112, Karnataka, INDIA. E-mail: mssiddesha@gmail.com*

[2] *Department of Mathematics, M.S. Ramaiah Institute of Technology, Bangalore-54, Affiliated to VTU, Belagavi, Karnataka, INDIA. E-mail: mmpraveenamaths@gmail.com*

[3] *Department of Mathematics, Kuvempu University, Shankaraghatta - 577 451, Shimoga, Karnataka, INDIA. E-mail: prof$_b$agewadi@yahoo.co.in*

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The object of the present paper is to study slant and hemi-slant submanifolds of (k,μ)-contact manifolds which are totally umbilical. We prove that every totally umbilical proper slant submanifold M of a (k,μ)-contact manifold $\tilde{M}$ is either totally geodesic or if M is not totally geodesic then we derive a formula for slant angle. Also necessary and sufficient conditions for distributions of hemi-slant submanifolds to be integrable are worked out. Further we give a characterization theorem.

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1. Introduction

As a natural generalization to the holomorphic and totally real submanifolds, Chen [7], introduced and studied slant submanifolds of an almost Hermitian manifolds. The contact version of slant submanifolds was introduced by Lotta [16]. Later, the study of slant submanifolds was enriched by the authors of [5 9 10 12 19 21 28] and many others. As a generalization to the slant submanifolds Papaghiuc [17] introduced the notion of semi-slant submanifolds of almost Hermitian manifolds. Later, Carriazo [4] defined generalized version of semi-slant submanifolds known as Bi-slant submanifolds. One of the classes of bi-slant submanifolds is that of anti-slant submanifolds which are studied by Carriazo [4], but the name anti-slant seems to refer that it has no slant factor, so Sahin [20] gave the name of hemi-slant submanifolds instead of anti-slant submanifolds. Later on many research articles on hemi-slant submanifolds of ambient manifold in the setting of complex as well as contact manifolds [13 15 25 26 27] and references therein.

In 1995, Blair, Koufogiorgos and Papantoniou [2] introduced the notion of (k,μ)-

contact manifold with an example, which are the generalization of Sasakian manifold and the case $R(X,Y)\xi = 0$, where R is the curvature tensor. Moreover (k,μ)-contact manifolds become Sasakian for $k = 1$ or $h = 0$, non-Sasakian for $k \neq 1$ and N(k)-contact manifold for $\mu = 0$. For more details, we refer to [2] [3] [18] [24]

Recently, we have defined and studied the slant and semi-slant submanifolds of (k,μ)-contact manifolds and prove the existence by giving counter example [22] [23]. Motivated by these aspects, in the present paper we study totally umbilical slant submanifolds and hemi-slant submanifolds of (k,μ)-contact manifold and is organized as follows: In section-2, we recall the notion of (k,μ)-contact manifold and some basic results of submanifolds, which are used for further study. In section-3, we consider totally umbilical slant submanifolds and find the classification result. Section 4 is devoted to study hemi-slant submanifolds. We prove the integrability of the distributions involved in the definition of hemi-slant submanifolds.

2. Preliminaries

A contact manifold is a $C^\infty - (2n+1)$ manifold $\tilde{M}^{2n+1}$ equipped with a global 1-form η such that $\eta \wedge (d\eta)^n \neq 0$ everywhere on $\tilde{M}^{2n+1}$. Given a contact form η it is well known that there exists a unique vector field ξ, called the characteristic vector field of η, such that $\eta(\xi) = 1$ and $d\eta(X,\xi) = 0$ for every vector field X on $\tilde{M}^{2n+1}$. A Riemannian metric is said to be associated metric if there exists a tensor field ϕ of type (1,1) such that

$$\phi^2 = -I + \eta \otimes \xi, \quad \eta(\xi) = 1, \quad \phi\xi = 0, \quad \eta \cdot \phi = 0, \tag{1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \quad g(X,\xi) = \eta(X), \quad g(X,\phi Y) = -g(\phi X, Y) \tag{2}$$

for all vector fields $X, Y \in T\tilde{M}$. Then the structure (ϕ,ξ,η,g) on $\tilde{M}^{2n+1}$ is called a contact metric structure and the manifold $\tilde{M}^{2n+1}$ equipped with such a structure is called a contact metric manifold [1].

We now define a $(1,1)$ tensor field h by $h = \frac{1}{2}\mathcal{L}_\xi \phi$, where $\mathcal{L}$ denotes the Lie differentiation, then h is symmetric and satisfies $h\phi = -\phi h$. Further, a q-dimensional distribution on a manifold M is defined as a mapping D on M which assigns to each point $p \in M$, a q-dimensional subspace D_p of T_pM.

As a generalization of both $R(X,Y)\xi = 0$ and the Sasakian case: Blair, Koufogiorgos and Papantoniou [2] considered the (k,μ)-nullity condition on a contact metric manifold and gave several reasons for studying it. The (k,μ)-nullity distribution $N(k,\mu)$ of a contact metric manifold $\tilde{M}$ is defined by

$$\begin{aligned} N(k,\mu) : p \to N_p(k,\mu) = \{&Z \in T_pM : \tilde{R}(X,Y)Z \\ &= k[g(Y,Z)X - g(X,Z)Y] + \mu[g(Y,Z)hX - g(X,Z)hY]\}, \end{aligned}$$

for all $X, Y \in T\tilde{M}$. Hence if the characteristic vector field ξ belongs to the (k,μ) nullity distribution, then we have

$$\tilde{R}(X,Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)hX - \eta(X)hY]. \tag{3}$$

The contact metric manifold satisfying the relation (3) is called (k, μ) contact metric manifold [2]. It consists of both k-nullity distribution for $\mu = 0$ and Sasakian for $k = 1$. A (k, μ)-contact metric manifold $\tilde{M}(\phi, \xi, \eta, g)$ satisfies

$$(\tilde{\nabla}_X \phi)Y = g(X + hX, Y)\xi - \eta(Y)(X + hX), \tag{4}$$

for all $X, Y \in T\tilde{M}$, where $\tilde{\nabla}$ denotes the Riemannian connection with respect to g. From (4), we have

$$\tilde{\nabla}_X \xi = -\phi X - \phi hX, \tag{5}$$

for all $X, Y \in T\tilde{M}$.

Let M be a submanifold of a (k, μ)-contact manifold $\tilde{M}$, we denote by the same symbol g the induced metric on M. Let TM be the set of all vector fields tangent to M and $T^{\perp}M$ is the set of all vector fields normal to M. Then, the Gauss and Weingarten formulae are given by

$$\tilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \quad \tilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V, \tag{6}$$

for any $X, Y \in TM$, $V \in T^{\perp}M$, where ∇ (resp. $\nabla^{\perp}$) is the induced connection on the tangent bundle TM (resp. normal bundle $T^{\perp}M$) [8]. The shape operator A is related to the second fundamental form σ of M by

$$g(A_V X, Y) = g(\sigma(X, Y), V), \tag{7}$$

Now, for any $x \in M$, $X \in T_x M$ and $V \in T^{\perp}_x M$, we put

$$\phi X = TX + FX, \quad \phi V = tV + fV, \tag{8}$$

where TX (resp. FX) is the tangential (resp. normal) component of ϕX, and tV (resp. fV) is the tangential (resp. normal) component of ϕV. From (4) and (8)

$$g(TX, Y) + g(X, TY) = 0, \tag{9}$$

for each $X, Y \in TM$ and $V \in T^{\perp}M$. The covariant derivatives of the tensor fields T, F, t and f are defined as

$$(\tilde{\nabla}_X \phi)Y = \tilde{\nabla}_X \phi Y - T\tilde{\nabla}_X Y, \tag{10}$$
$$(\tilde{\nabla}_X T)Y = \nabla_X TY - T\nabla_X Y, \tag{11}$$
$$(\tilde{\nabla}_X F)Y = \nabla_X FY - F(\nabla_X Y). \tag{12}$$
$$(\tilde{\nabla}_X t)V = \nabla_X tV - t(\nabla_X V). \tag{13}$$
$$(\tilde{\nabla}_X f)V = \nabla_X fV - f(\nabla_X V). \tag{14}$$

Now, on a submanifold of a (k, μ)-contact manifold by equations (6) and (7) we get

$$\nabla_X \xi = -TX - ThX \tag{15}$$

and

$$\sigma(X, \xi) = -FX - FhX, \tag{16}$$

for each $X \in TM$. Further from equation (15)

$$A_V\xi = 0, \quad \eta(A_V X) = 0, \tag{17}$$

for each $V \in T^{\perp}M$. On using equations (4), (6), (8), (11) and (13), we obtain

$$(\tilde{\nabla}_X T)Y = A_{FY}X + t\sigma(X,Y) + g(X + hX, Y)\xi - \eta(Y)(X + hX), \tag{18}$$

$$(\tilde{\nabla}_X F)Y = -\sigma(X, TY) + f\sigma(X,Y). \tag{19}$$

A submanifold M of an almost contact metric manifold $\tilde{M}$ is said to be totally umbilical if

$$\sigma(X,Y) = g(X,Y)H, \tag{20}$$

where H is the mean curvature vector of M. Furthermore, a submanifold M is called totally geodesic, if $\sigma(X,Y) = 0$ for all $X, Y \in \Gamma(TM)$ and if $H = 0$, then M is minimal in $\tilde{M}$.

3. Slant submanifolds of a (k, μ)-contact manifold

In the present section, we consider M is a proper slant submanifold of a (k, μ)-contact manifold $\tilde{M}$. We always consider such submanifolds tangent to the structure vector fields ξ.

An immersed submanifold M of a (k, μ)-contact manifold $\tilde{M}$ is slant in $\tilde{M}$ if for any $x \in M$ and any $X \in T_xM$ such that X, ξ are linearly independent, the angle $\theta(x) \in [0, \frac{\pi}{2}]$ between ϕX and T_xM is a constant θ, i.e., θ does not depend on the choice of X and $x \in M$, θ is called the slant angle of M in $\tilde{M}$. Invariant and anti-invariant submanifolds are slant submanifolds with slant angle $\theta = 0$ and $\theta = \frac{\pi}{2}$ respectively [16].

We have the following theorem which characterize slant submanifolds of a contact manifold

Theorem 1. [5] *Let M be a submanifold of an almost contact metric manifold $\tilde{M}$ such that $\xi \in TM$. Then, M is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that*

$$T^2 = -\lambda(I - \eta \otimes \xi). \tag{21}$$

Further more, if θ is the slant angle of M, then $\lambda = cos^2\theta$.

From [5], for any X, Y tangent to M, we can easily obtain the results for a (k, μ)-contact manifold $\tilde{M}$,

$$g(TX, TY) = cos^2\theta\{g(X,Y) - \eta(X)\eta(Y)\} \tag{22}$$

$$g(FX, FY) = sin^2\theta\{g(X,Y) - \eta(X)\eta(Y)\} \tag{23}$$

Theorem 2. *Let M be a totally umbilical slant submanifold of a (k, μ)-contact manifold $\tilde{M}$, then the following statements are equivalent:*

(i) $H \in \nu$;

(ii) either M is trivial or invariant submanifold of $\tilde{M}$.

Proof: For any $X, Y \in TM$, then from equation (18), we have

$$(\tilde{\nabla}_X T)Y = A_{FY}X + t\sigma(X,Y) + g(X + hX, Y)\xi - \eta(Y)(X + hX). \tag{24}$$

Taking inner product with ξ, we get

$$g(\nabla_X TY, \xi) = g(\sigma(X,\xi), FY) + g(t\sigma(X,Y),\xi) + g(X + hX, Y) - \eta(Y)\eta(X).$$

As M is totally umbilical slant submanifold of $\tilde{M}$, then from equation (20)

$$-g(TY, \nabla_X \xi) = g(H, FY)\eta(X) + g(X,Y)g(tH,\xi) + g(X + hX, Y) - \eta(Y)\eta(X).$$

using equation (15), (8) and (22), we obtain

$$cos^2\theta\{g(X,Y) - \eta(X)\eta(Y)\} + cos^2\theta g(Y, hX) = g(H, FY)\eta(X) + g(X + hX, Y) - \eta(Y)\eta(X)$$

The above equation can be written as

$$sin^2\theta\{g(X + hX, Y) - \eta(X)\eta(Y)\} = -g(H, FY)\eta(X). \tag{25}$$

If $H \in \nu$, then right hand side of the equation (25) is identically zero. Hence statement (ii) holds. Conversely, if (ii) holds then from (25) we get $H \in \nu$. This completes the proof of the theorem. □

Theorem 3. *Let M be a totally umbilical proper slant submanifold of a (k,μ)-contact manifold $\tilde{M}$, such that $H, \nabla^{\perp}_U H \in \nu$, for all $U \in TM$. Then,*
(i) either M is totally geodesic;
(ii) or the slant angle $\theta = tan^{-1}\left(\sqrt{\frac{g(X,Y)}{\eta(X)\eta(Y)}}\right)$

Proof: For $X, Y \in TM$, we have

$$\tilde{\nabla}_X \phi Y - \phi\tilde{\nabla}_X Y = g(X + hX, Y)\xi - \eta(Y)(X + hX).$$

Using (6), (8) and the fact that M is totally umbilical proper slant submanifold, we obtain

$$\begin{aligned} &\nabla_X TY + g(X, TY)H - A_{FY}X + \nabla^{\perp}_X FY - T\nabla_X Y - F\nabla_X Y - g(X,Y)\phi H \\ &= g(X + hX, Y)\xi - \eta(Y)(X + hX). \end{aligned} \tag{26}$$

Taking inner product with ϕH in (26) yields

$$g(X, TY)g(H, \phi H) + g(\nabla^{\perp}_X FY, \phi H) = g(F\nabla_X Y, \phi H) + g(X,Y)g(\phi H, \phi H).$$

Using equation (2) and the fact that $H \in \nu$, we get

$$g(\nabla^{\perp}_X FY, \phi H) = g(X,Y)\|H\|^2.$$

Then, from (6), we derive

$$g(\tilde{\nabla}_X FY, \phi H) = g(X,Y)\|H\|^2. \tag{27}$$

Now, for any $X \in TM$, we have

$$(\tilde{\nabla}_X \phi)H = \tilde{\nabla}_X \phi H - \phi\tilde{\nabla}_X H.$$

Using (4) and the fact that $H \in \nu$, we obtain

$$0 = \tilde{\nabla}_X \phi H - \phi \tilde{\nabla}_X H.$$

Using equations (6) and (8), we obtain

$$-A_{\phi H}X + \nabla^{\perp}_X \phi H = -TA_H X - FA_H X + t\nabla^{\perp}_X H + f\nabla^{\perp}_X H. \tag{28}$$

Taking inner product with FY in (28) for any $Y \in TM$ and using the fact that $n\nabla^{\perp}_X H \in \nu$, (28) yields

$$g(\nabla^{\perp}_X \phi H, FY) = -g(FA_H X, FY)$$

Applying (6) and (23), we get

$$g(\tilde{\nabla}_X FY, \phi H) = sin^2\theta[g(X,Y) + \eta(X)\eta(Y)]\|H\|^2. \tag{29}$$

In view of (27) and (29), we obtain

$$\{cos^2\theta g(X,Y) - sin^2\theta\eta(X)\eta(Y)\}\|H\|^2 = 0. \tag{30}$$

Since M is proper slant submanifold, then from (30) it follows that either $H = 0$, that is M is totally geodesic in $\tilde{M}$ or θ is acute angle, then $\theta = tan^{-1}\left(\sqrt{\frac{g(X,Y)}{\eta(X)\eta(Y)}}\right)$. This completes the proof of the theorem. □

4. Hemi-slant submanifolds of a (k, μ)-contact manifold

Definition 4. A submanifold M of $\tilde{M}$ is said to be hemi-slant submanifold of an almost contact metric manifold $\tilde{M}$ if there exists two orthogonal complementary distribution D^θ and $D^\perp$ on M such that
(i) $TM = D^\theta \oplus D^\perp \oplus <\xi>$;
(ii) the distribution D^θ is slant with slant angle $\theta \neq \frac{\pi}{2}$;
(iii) the distribution $D^\perp$ is totally real i.e., $\phi D^\perp \subseteq T^\perp M$.

It is clear from above that CR-submanifolds and slant submanifolds are hemi-slant submanifolds with slant angle $\theta = \frac{\pi}{2}$ and $D^\theta = 0$, respectively.
Let M be a hemi-slant submanifold of an almost contact metric manifold $\tilde{M}$, and $X \in TM$. Then as $TM = D^\theta \oplus D^\perp \oplus <\xi>$, we write

$$X = P_1X + P_2X + \eta(X)\xi, \tag{31}$$

where $P_1X \in D^\theta$ and $P_2X \in D^\perp$. Now by equations (8) and (31)

$$\phi X = TP_1X + FP_1X + \phi P_2X.$$

It is easy to see that

$$\phi P_2X = FP_2X,\ TP_2X = 0,\ TP_1X \in D^\theta.$$

Thus

$$TX = TP_1X,\ FX = FP_1X + FP_2X.$$

Proof: Let M be a hemi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$, then the anti-invariant distribution $D^{\perp}\oplus<\xi>$ is integrable if and only if

$$A_{\phi Z}W - A_{\phi W}Z + \nabla^{\perp}_{Z}\phi W - \nabla^{\perp}_{W}\phi Z = 0 \in D^{\perp}\oplus<\xi>, \tag{32}$$

for all $Z, W \in D^{\perp}\oplus<\xi>$. □

Proof: For any $Z, W \in D^{\perp}\oplus<\xi>$, we have

$$\tilde{\nabla}_Z\phi W = (\tilde{\nabla}_Z\phi)W + \phi\tilde{\nabla}_Z W = (\tilde{\nabla}_Z\phi)W + \phi\nabla_Z W + \phi\sigma(Z,W).$$

Using (6), we obtain

$$-A_{\phi W}Z + \nabla^{\perp}_{Z}\phi W = (\tilde{\nabla}_Z\phi)W + \phi\nabla_Z W + \phi\sigma(Z,W). \tag{33}$$

Interchanging Z and W, and subtract, we get

$$A_{\phi Z}W - A_{\phi W}Z + \nabla^{\perp}_{Z}\phi W - \nabla^{\perp}_{W}\phi Z = (\tilde{\nabla}_Z\phi)W - (\tilde{\nabla}_W\phi)Z + \phi[Z,W]. \tag{34}$$

Taking inner product with ϕX, for any $X \in D^{\theta}$ and by applying (4), we obtain

$$g(A_{\phi Z}W - A_{\phi W}Z + \nabla^{\perp}_{Z}\phi W - \nabla^{\perp}_{W}\phi Z, \phi X) = g(\phi[Z,W], \phi X).$$

Thus from (2), the above equation takes the form

$$g([Z,W],X) = g(A_{\phi Z}W - A_{\phi W}Z + \nabla^{\perp}_{Z}\phi W - \nabla^{\perp}_{W}\phi Z, \phi X).$$

The distribution $D^{\perp}\oplus<\xi>$ is integrable if and only if the right hand side of the above equation is zero. Hence the result follows from (32). □

Proof: Let M be a hemi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$, then the distribution $D^{\theta}\oplus<\xi>$ is integrable if and only if

$$\sigma(X,TY) + \nabla^{\perp}_{X}FY - \sigma(Y,TX) - \nabla^{\perp}_{Y}FX \in \nu, \ \forall X,Y \in D^{\theta}\oplus<\xi>. \tag{35}$$
□

Proof: For any $X, Y \in D^{\theta}\oplus<\xi>$, we have

$$\phi[X,Y] = \phi\tilde{\nabla}_X Y - \phi\tilde{\nabla}_Y X = \tilde{\nabla}_X\phi Y - (\tilde{\nabla}_X\phi)Y - \tilde{\nabla}_Y\phi X + (\tilde{\nabla}_Y\phi)X.$$

Then from (4) and (8), we obtain

$$\begin{aligned}\phi[X,Y] = {} & \tilde{\nabla}_X TY + \tilde{\nabla}_X FY - g(X+hX,Y)\xi + \eta(Y)(X+hX) \\ & -\tilde{\nabla}_Y TX - \tilde{\nabla}_Y FX + g(Y+hY,X)\xi - \eta(X)(Y+hY).\end{aligned}$$

Applying (6), we get

$$\begin{aligned}\phi[X,Y] = {} & \nabla_X TY + \sigma(X,TY) - A_{FY}X + \nabla^{\perp}_{X}FY - g(X+hX,Y)\xi \\ & +\eta(Y)(X+hX) - \nabla_Y TX - \sigma(Y,TX) + A_{FX}Y - \nabla^{\perp}_{Y}FX \\ & +g(Y+hY,X)\xi - \eta(X)(Y+hY).\end{aligned} \tag{36}$$

Taking the inner product in (36) with ϕZ, for any $Z \in D^{\perp}$, we derive

$$g(\phi[X,Y],\phi Z) = g(\sigma(X,TY) + \nabla^{\perp}_{X}FY - \sigma(Y,TX) - \nabla^{\perp}_{Y}FX, \phi Z).$$

In view of (2), the above equation yields

$$g([X,Y],Z) = g(\sigma(X,TY) + \nabla^{\perp}_{X}FY - \sigma(Y,TX) - \nabla^{\perp}_{Y}FX, \phi Z).$$

Thus the assertion follows from (35). □

Theorem 5. *Let M be a hemi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$, then at least one of the following statement is true:*
(i) $dim D^{\perp} = 1$;
(ii)$H \in \nu$;
(iii) M is proper slant.

Proof: For any $U, V \in TM$, we have

$$(\tilde{\nabla}_U \phi)V = g(U + hU, V)\xi - \eta(V)(U + hU).$$

If we take the vector fields $Z, W \in D^{\perp}$, then the above equation will be

$$(\tilde{\nabla}_Z \phi)W = g(Z + hZ, W)\xi.$$

In particular, if we take the above equation for one vector $Z \in D^{\perp}$ i.e.,

$$(\tilde{\nabla}_Z \phi)Z = g(Z + hZ, Z)\xi. \tag{37}$$

Using (6) and (8) in (37), we obtain

$$-A_{FZ}Z - T\nabla_Z Z - F\nabla_Z Z - t\sigma(Z,Z) - f\sigma(Z,Z) = g(Z + hZ, Z)\xi.$$

Comparing the tangential component of the above, we get

$$-T\nabla_Z Z = A_{FZ}Z + t\sigma(Z,Z) + g(Z + hZ, Z)\xi.$$

Taking the inner product with $W \in D^{\perp}$ and in view of (7), we obtain

$$g(T\nabla_Z Z, W) = g(\sigma(Z,W), FZ) + g(t\sigma(Z,Z), W).$$

Using the fact that M is totally umbilical submanifold and $TW = 0$ for any $W \in D^{\perp}$, then the above equation takes the form

$$0 = g(H, FZ)g(Z,W) + \|Z\|^2 g(tH, W). \tag{38}$$

Thus the equation (38) has a solution if either $\dim D^{\perp} = 1$ or $H \in \nu$ or $D^{\perp} = < 0 >$ i.e., M is proper slant. □

Theorem 6. *Let M be a totally umbilical hemi-slant submanifold of a (k,μ)-contact manifold $\tilde{M}$. Then at least one of the following statement is true.*
(i) M is totally geodesic submanifold;
(ii)$\theta = tan^{-1}\left(\sqrt{\frac{g(X,Y)}{\eta(X)\eta(Y)}}\right)$;
(iii) dim $D^{\perp}$=1;
(iv) M is a proper slant submanifold.

Proof: If $H \in \nu$, then the statements (i) and (ii) are followed from the Theorem 3.3. directly. Finally, if $H \notin \nu$, then the equation (38) has a solution if either dim $D^{\perp} = 1$ or $D^{\perp} = 0$ which are cases (iii) and (iv) respectively. This completes the proof of the theorem. □

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