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PERFORMAMCE EVALUATION OF A MILK PACKING SYSTEM

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ABSTRACT : This paper deals with the performance evaluation of a Milk Packing System involving two essential components viz. one main unit and two associate units. Associate units depend upon main for functioning. Only one repairman is used for repairing the failed components of both the units. Taking exponential failure rates and arbitrary repair rates, various system effectiveness measures such as transition probabilities, mean time to system failure, availability, busy period of repairman are calculated. At last profit analysis is done on the basis of above measures.

1. INTRODUCTION

This chapter deals with the analysis of a milk packing system having three independently functioning units namely main unit and two associate units. The processing on any typical milk packing system is carried out using these units. In the past, Mohammad El-Moniem Soleha et al have done reliability and availability characteristics of a two dissimilar unit cold standby system with three modes using linear first order differential equation. Arora et-al have done reliability analysis of two unit standby redundant system with constrained repair time. Gupta et-al have worked on a compound redundant system involving human failure. Rander et-al have evaluated the cost analysis of two dissimilar cold standby system with preventive maintenance and replacement of standby units. This model particularly differs from the other models in the sense that the concept used in this model is based on the real situation. This kind of analysis is of immense help to the system managers. Also the involvement of preventive maintenance in the model increases the reliability of the functioning units.

2. ABOUT THE MODEL

The milk packing system consists of three independently functioning units namely Guide roller, Sealing unit, Nozzle unit. Guide roller serves the purpose of Main unit in this model and Sealing and Nozzle unit work as associate units. Initially, all the three units work without load. As soon as a job arrives, all the units start working together. First the job is processed in the main unit called the Guide Roller. After it is processed in the main unit, other two units do their work simultaneously i.e. both the sealing and the nozzle units are invoked instantaneously. It is assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, mean time to system failure, availability and busy period of the repairman are evaluated. In the end the expected profit is also calculated.

3. ASSUMPTIONS USED IN THE MODEL

a. Guide roller unit is given preference over other unit for repair.
b. Sealing section is preference over nozzle unit .
c. There is a single repairman which repairs the failed units on priority basis.
d. After random period of time the whole system goes to preventive maintenance.
e. All units work as new after repair.
f. The failure rates of all the units are taken to be exponential whereas the repair

time distributions are arbitrary.

4. SYMBOLS AND NOTATIONS

E_0 = State of the system at epoch $t = 0$

E = set of regenerative states $S_0 - S_8$

λ = Job arrival Rate

$q_{ij}(t)$ = Probability density function of transition time from S_i to S_i

$Q_{ij}(t)$ = Cumulative distribution function of transition time from S_i to S_i

$\pi_i(t)$ = Cumulative distribution function of time to system failure when starting from state $E_0 = S_i \subset E$

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \subset E$

$B_i(t)$ = Repairman is busy in the repair at time $t / E_0 = S_i \subset E$

$r_1 / r_2 / r_3$ = Constant repair rate of Guide roller/ Sealing unit/ Nozzle unit

$\alpha / \beta / \gamma$ = Failure rate of Guide roller/ Sealing unit/ Nozzle unit

$g_1(t) / g_2(t) / g_3(t)$ = Probability density function of repair time of Guide roller / Sealing unit / Nozzle unit

$G_1(t) / G_2(t) / G_3(t)$ = Cumulative distribution function of repair time of Guide roller/ Sealing unit/ Nozzle unit

a(t) = Probability density function of preventive maintenance

b(t) = Probability density function of preventive maintenance completion time

A(t) = Cumulative distribution function of preventive maintenance

B(t) = Cumulative distribution function of preventive maintenance completion time

$\boxed{S}$ = Symbol for Laplace -stieltjes transform

$\boxed{C}$ = Symbol for Laplace-convolution

5. SYMBOLS USED FOR STATES OF THE SYSTEM

$G_o / G_g / G_r$ – Guide roller under operation/good and non-operative mode / repair

$N_o / N_g / N_r / N_{wr}$ – Nozzle unit under operation/ good and non-operative-mode / repair/ waiting for repair

$S_o / S_g / S_r / S_{wr}$ – Sealing unit under operation / good and non-operative mode / repair/ waiting for repair

P.M. – System under preventive maintenance.

Up States – $S_0 = (G_0, N_0, S_0)$; $S_2 = (G_o, N_r, S_o); S_3 = (G_o, N_g, S_r); S_5 = (G_o, N_{wr}, S_r)$

Down States –

$S_1 = (G_r, N_g, S_g); S_4 = (G_r, N_g, S_{wr}); S_6 = (G_r, N_{wr}, S_g); S_7 = (P.M.), S_8 = (S.D.)$

6. TRANSITION PROBABILITIES

Simple probabilistic consideration yields the following expression for non-zero transition probabilities

1. $p_{01} = \dfrac{\alpha}{\alpha+\beta+\gamma}\{1-a^*(\alpha+\beta+\gamma)\}$, 2. $p_{02} = \dfrac{\gamma}{\alpha+\beta+\gamma}\{1-a^*(\alpha+\beta+\gamma)\}$,

3. $p_{03} = \dfrac{\beta}{\alpha+\beta+\gamma}\{1-a^*(\alpha+\beta+\gamma)\}$, 4. $p_{07} = a^*(\alpha+\beta+\gamma)$,

5. $p_{20} = g_3^*(\alpha+\beta)$, 6. $p_{25} = \dfrac{\beta}{\alpha+\beta}[1-g^*{}_3(\alpha+\beta)]$,

7. $p_{26} = \dfrac{\alpha}{\alpha+\beta}[1-g^*{}_3(\alpha+\beta)]$, 8. $p_{30} = g^*{}_2(\alpha) = p_{52}$,

9. $p_{34} = 1-g^*{}_2(\alpha) = p_{58}$, 10. $p_{10} = p_{43} = p_{62} = p_{70} = p_{80} = 1$

It is easy to see that $p_{01}+p_{02}+p_{03}+p_{07}=1,\ p_{20}+p_{25}+p_{26}=1,$

$$p_{30}+p_{34}=1, \qquad p_{50}+p_{58}=1 \qquad [11\text{-}14]$$

and the mean sojourn times are given by ,

15. $\mu_0(t)=\dfrac{1}{x_1}[1-a^*(x_1)],$

16. $\mu_1(t)=\int_0^\infty \overline{G_1}(t)dt,$

17. $\mu_2(t)=\dfrac{1}{\alpha+\beta}[1-g^*{}_3(\alpha+\beta)],$

18. $\mu_3(t)=\dfrac{1}{\alpha}[1-g_2^*(\alpha)],$

19. $\mu_4(t)=\int_0^\infty \overline{G_1}(t)dt,$

20. $\mu_5(t)=\dfrac{1}{\alpha}[1-g_2^*(\alpha)],$

21. $\mu_6(t)=\int_0^\infty \overline{G_2}(t)dt,$

22. $\mu_7(t)=\int_0^\infty \overline{B}(t)dt,$

23. $\mu_{80}(t)=\int_0^\infty \overline{G_4}(t)dt$

We define m_{ij} as follows :—

$$m_{ij}=-\frac{d}{ds}\tilde{Q}_{ij}(s)\ {}_{s=0}=Q_{ij}'(0)$$

We can see that $m_{12}+m_{13}+m_{14}+m_{17}=\mu_1;\ m_{41}+m_{45}+m_{46}=\mu_4$ [24-26]

7. MEAN TIME TO SYSTEM FAILURE

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing states. Using the arguments as for the regenerative process, we obtain the following recursive relations for $\pi_i(t)$:

$$\pi_0(t)=Q_{01}(t)+Q_{02}(t)\ \boxed{S}\ \pi_2(t)+Q_{03}(t)\ \boxed{S}\ \pi_3(t)+Q_{07}(t)$$

$$\pi_2(t) = Q_{20}(t) \boxed{S} \pi_0(t) + Q_{25}(t) \boxed{S} \pi_5(t) + Q_{26}(t)$$

$$\pi_3(t) = Q_{30}(t) \boxed{S} \pi_0(t) + Q_{34}(t)$$

$$\pi_5(t) = Q_{52}(t) \boxed{S} \pi_2(t) + Q_{58}(t) \qquad [7.1\text{-}7.4]$$

Taking Laplace -stiltjes transform of equation 7.1 to 7.4 and writing in matrix form, we obtained

$$\begin{bmatrix} 1 & -\tilde{Q}_{02} & -\tilde{Q}_{03} & 0 \\ -\tilde{Q}_{20} & 1 & 0 & -\tilde{Q}_{25} \\ -\tilde{Q}_{30} & 0 & 1 & 0 \\ 0 & -\tilde{Q}_{52} & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_0 \\ \tilde{\pi}_2 \\ \tilde{\pi}_3 \\ \tilde{\pi}_5 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{01} + \tilde{Q}_{07} \\ \tilde{Q}_{26} \\ \tilde{Q}_{34} \\ \tilde{Q}_{58} \end{bmatrix}$$

where

$$D_1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{02} & -\tilde{Q}_{03} & 0 \\ -\tilde{Q}_{20} & 1 & 0 & -\tilde{Q}_{25} \\ -\tilde{Q}_{30} & 0 & 1 & 0 \\ 0 & -\tilde{Q}_{52} & 0 & 1 \end{vmatrix}$$

$$= 1 - \tilde{Q}_{02}\tilde{Q}_{20} - \tilde{Q}_{03}\tilde{Q}_{30} - \tilde{Q}_{25}\tilde{Q}_{52} + \tilde{Q}_{03}\tilde{Q}_{30}\tilde{Q}_{25}\tilde{Q}_{52},$$

$$N_1(s) = \begin{vmatrix} \tilde{Q}_{01} + \tilde{Q}_{07} & -\tilde{Q}_{02} & -\tilde{Q}_{03} & 0 \\ \tilde{Q}_{26} & 1 & 0 & -\tilde{Q}_{25} \\ \tilde{Q}_{34} & 0 & 1 & 0 \\ \tilde{Q}_{58} & -\tilde{Q}_{52} & 0 & 1 \end{vmatrix}$$

$$= (\tilde{Q}_{01} + \tilde{Q}_{07} + \tilde{Q}_{03}\tilde{Q}_{34})(1 - \tilde{Q}_{25}\tilde{Q}_{52}) + \tilde{Q}_{02}\tilde{Q}_{26} + \tilde{Q}_{02}\tilde{Q}_{25}\tilde{Q}_{58}$$

Now letting $s \to 0$

$$D_1(0) = (1 - p_{25}p_{52})(1 - p_{03}p_{30}) - p_{02}p_{20} = N_1(0)$$

The mean time to system failure when the system starts from the state is given by

$$E(T) = -\frac{d}{ds}\tilde{\pi}_0(s)\Big|_{s=0} = \frac{D_1'(0) - N_1'(0)}{D_1(0)}$$

To obtain the numerator of the above equation, we collect the coefficients of relevant of m_{ij} in $D_1'(0) - N_1'(0)$.

Coefficient of $(m_{01} = m_{02} = m_{03} = m_{07}) = 1 - p_{25}p_{52}$

Coefficient of $(m_{20} = m_{25} = m_{26}) = p_{02}$

Coefficient of $(m_{30} = m_{34}) = p_{03}(1 - p_{25}p_{52})$

Coefficient of $(m_{52} = m_{58}) = p_{02}p_{25}$

$$\text{MTSF} = \frac{\mu_0[1 - p_{25}p_{52}] + \mu_2 p_{02} + \mu_3 p_{03}[1 - p_{25}p_{52}] + \mu_6 p_{02}p_{25}}{(1 - p_{25}p_{52})(1 - p_{03}p_{30}) - p_{02}p_{20}} \quad [7.5]$$

8. AVAILABILITY ANALYSIS

Let $M_i(t)$ denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states .i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself. By probabilistic arguments, we have the following recursive relations giving point wise availability given as follows :

$$A_0(t) = M_0(t) + q_{01}(t) \boxed{C} A_1(t) + q_{02}(t) \boxed{C} A_2(t) + q_{03}(t) \boxed{C} A_3(t)$$
$$+ q_{07}(t) \boxed{C} A_7(t)$$

$$A_1(t) = q_{10}(t) \boxed{C} A_0(t)$$

$$A_2(t) = M_2(t) + q_{20}(t) \boxed{C} A_0(t) + q_{25}(t) \boxed{C} A_5(t) + q_{26}(t) \boxed{C} A_6(t)$$

$$A_3(t) = M_3(t) + q_{30}(t) \boxed{C} A_0(t) + q_{34}(t) \boxed{C} A_4(t)$$

$$A_4(t) = q_{43}(t) \boxed{C} A_3(t)$$

$$A_5(t) = M_5(t) + q_{52}(t) \boxed{C} A_2(t) + q_{58}(t) \boxed{C} A_8(t)$$

$$A_6(t) = q_{62}(t) \boxed{C} A_2(t)$$

$$A_7(t) = q_{70}(t) \boxed{C} A_0(t)$$

$$A_8(t) = q_{80}(t) \boxed{C} A_0(t) \qquad [8.1\text{-}8.9]$$

Taking laplace stieltjes transformation of above equations and writing in matrix form

$$q(A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*, A_6^*, A_7^*, A^*_8)' = (M_0^*, 0, M_2^*, M_3^*, 0, M_5^*, 0, 0, 0)'$$

Where

$$D_2(s) = (1 - q_{34}^* q_{43}^*)[(1 - q_{01}^* q_{10}^* - q_{07}^* q_{70}^*)(1 - q_{25}^* q_{52}^* - q_{26}^* q_{62}^*)$$
$$- q_{02}^* q_{20}^* - q_{02}^* q_{25}^* q_{58}^* q_{80}^*] - q_{03}^* q_{30}^* [1 - q_{25}^* q_{52}^* + q_{26}^* q_{62}^*]$$

$$N_2(s) = M_0^*(1 - q_{25}^* q_{52}^* - q_{26}^* q_{62}^*)(1 - q_{34}^* q_{43}^*) + M_2^* q_{02}^* (1 - q_{34}^* q_{43}^*)$$
$$+ M_3^* q_{03}^* (1 - q_{25}^* q_{52}^* - q_{26}^* q_{62}^*) + M_5^* q_{02}^* q_{25}^* (1 - q_{34}^* q_{43}^*)$$

Now letting $s \to 0$ we get $D_2(0) = 0$ and

$$N_2(0) = \mu_0 p_{30}(1 - p_{25} p_{52} - p_{26}) + \mu_2 p_{02} p_{30}$$
$$+ \mu_3 p_{03}(1 - p_{25} p_{52} - p_{26}) + \mu_5 p_{02} p_{25} p_{30}$$

To find the steady -state availability we first calculate

$$M_0^*(0) = \int_0^\infty e^{-x_1 t} \overline{A}(t) dt = \mu_0, \quad M_2^*(0) = \int_0^\infty e^{-(\alpha+\beta)t} \overline{G_3}(t)\, dt = \mu_1,$$

$$M_3^*(0) = \int_0^\infty e^{-\alpha t} \overline{G_2}(t)\, dt = \mu_3 = \mu_5$$

Using relations :

$$q_{ij}^{*}(0)=\int_{0}^{\infty} q_{ij}(t)\,dt=p_{ij} \qquad \text{We have}$$

Thus the steady-state availability of the system when the system starts from $S_i \in E$ is obtained as follows :

$$A_0(\infty)=\lim_{s\to 0} A_0^{*}(s)=\frac{N_2(0)}{D_2^{\prime}(0)}$$

To obtain the value of $D_2^{1}(0)$ we collect the coefficients of relevant m_{ij} in $D_2(s)$

Coefficient of $(m_{01}=m_{02}=m_{03}=m_{07}) = p_{30}(1-p_{26}-p_{25}p_{52})$

Coefficient of $m_{10} = p_{01}p_{30}(1-p_{26}-p_{25}p_{52})$

Coefficient of $(m_{20}=m_{25}=m_{26}) = p_{02}p_{30}$

Coefficient of $(m_{30}=m_{34}) = p_{03}(1-p_{26}-p_{25}p_{52})$

Coefficient of $m_{43} = p_{03}p_{34}(1-p_{26}-p_{25}p_{52})$

Coefficient of $(m_{52}=m_{58}) = p_{02}p_{25}p_{30}$

Coefficient of $m_{62} = p_{02}p_{26}p_{30}$

Coefficient of $m_{70} = p_{07}p_{30}(1-p_{26}-p_{25}p_{52})$

Coefficient of $m_{80} = p_{30}p_{02}p_{25}p_{58}$

$$\begin{aligned}\therefore D_2^{\prime}(0)=&\ \mu_0 p_{30}(1-p_{26}-p_{25}p_{52})+\mu_1 p_{01}p_{30}(1-p_{26}-p_{25}p_{52})\\ &+\mu_2 p_{02}p_{30}+\mu_3 p_{03}(1-p_{26}-p_{25}p_{52})\\ &+\mu_4 p_{03}p_{34}(1-p_{26}-p_{25}p_{52})+\mu_5 p_{02}p_{25}p_{30}+\mu_6 p_{02}p_{26}p_{30}\\ &+\mu_7 p_{07}p_{30}(1-p_{26}-p_{25}p_{52})+\mu_8 p_{30}p_{02}p_{25}p_{58}\end{aligned}$$

$$\therefore A_0(\infty)=\frac{N_2(0)}{D_2^{\prime}(0)} \qquad [8.10]$$

9. BUSY PERIOD ANALYSIS

(a) Let $W_i(t)$ denote the probability that the repairman is busy initially with repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

$$W_1(t)=W_4(t)=W_6(t)=\overline{G}_1(t),\ W_3(t)=\overline{G}_2(t)=W_5(t),\ W_2(t)=\overline{G}_3(t)$$

By probabilistic arguments, we have

Developing similar relationship in availability, we have

$$B_0(t)=q_{01}(t)\boxed{C}B_1(t)+q_{02}(t)\boxed{C}B_2(t)+q_{03}(t)\boxed{C}B_3(t)$$
$$+q_{07}(t)\boxed{C}B_7(t)$$
$$B_1(t)=W_1(t)+q_{10}(t)\boxed{C}B_0(t)$$
$$B_2(t)=W_2(t)\boxed{C}+q_{20}(t)\boxed{C}B_0(t)+q_{25}(t)\boxed{C}B_5(t)+q_{26}(t)\boxed{C}B_6(t)$$
$$B_3(t)=W_3(t)+q_{30}(t)\boxed{C}B_0(t)+q_{34}(t)\boxed{C}B_4(t)$$
$$B_4(t)=W_4(t)+q_{43}(t)\boxed{C}B_3(t)$$
$$B_5(t)=W_5(t)+q_{52}(t)\boxed{C}B_2(t)+q_{58}(t)\boxed{C}B_8(t)$$
$$B_6(t)=W_6(t)+q_{62}(t)\boxed{C}B_2(t)$$
$$B_7(t)=q_{70}(t)\boxed{C}B_0(t)$$
$$B_8(t)=q_{80}(t)\boxed{C}B_0(t) \qquad [9.1\text{-}9.9]$$

Taking laplace- stieltjes transformation of above equations and writing in matrix form

$$q(B_0^*,B_1^*,B_2^*,B_3^*,B_4^*,B_5^*,B_6^*,B_7^*,B_8^*)'=(0,W_1^*,W_2^*,W_3^*,W_4^*,W_5^*,W_6^*,0,0)'$$

Where

$$N_3(s) = W_1^* q_{01}^* (1 - q_{25}^* q_{52}^* - q_{26}^* q_{62}^*)(1 - q_{34}^* q_{43}^*) + W_2^* q_{02}^* (1 - q_{34}^* q_{43}^*)$$
$$+ W_3^* q_{03}^* (1 - q_{25}^* q_{52}^* - q_{26}^* q_{62}^*) + W_4^* q_{03}^* q_{34}^* (1 - q_{25}^* q_{52}^* - q_{26}^* q_{62}^*)$$
$$+ W_5^* q_{02}^* q_{25}^* (1 - q_{34}^* q_{43}^*) + W_6^* q_{02}^* q_{26}^* (1 - q_{34}^* q_{43}^*)$$

Now letting $s \to 0$ we get

$$N_3(0) = \mu_1 p_{01} p_{30} (1 - p_{26} - p_{25} p_{52}) + \mu_2 p_{02} p_{30}$$
$$+ \mu_3 p_{03} (1 - p_{26} - p_{25} p_{52}) + \mu_4 p_{03} p_{34} (1 - p_{26} - p_{25} p_{52})$$
$$+ \mu_5 p_{02} p_{25} p_{30} + \mu_6 p_{02} p_{26} p_{30}$$

Therefore $B_0^{1*}(\infty) = \dfrac{N_3(0)}{D_2'(0)}$ [9.10]

(b) Busy period of the repairman in the performing preventive maintenance

Here $W_7(t) = \overline{B}(t)$, using similar argument as in 8(a), in the long run the fraction of time for which the repairman is busy in preventive maintenance of the Guide roller we get

$$N_4(s) = W_7^* q_{07}^* (1 - q_{25}^* q_{52}^* - q_{26}^*)(1 - q_{34}^* q_{43}^*)$$

Now letting $s \to 0$ we get

$$N_4(0) = \mu_7 p_{07} p_{30} (1 - p_{26} - q_{25}^* q_{52}^*). \text{ Therefore } B_0^{2*}(\infty) = \frac{N_4(0)}{D_2'(0)} \quad [9.11]$$

(c) Busy period of the repairman in preparation of the shut down condition.

Here $W_8(t) = \overline{G}_4(t)$. Using similar argument as in 8(a), in the long run the fraction of time for which the repairman is busy with shut down repair of the Guide roller we get

$$N_5(s) = W_8^* q_{02}^* q_{25}^* q_{58}^* (1 - q_{34}^* q_{43}^*)$$

Now letting $s \to 0$ we get

$$N_5(0) = \mu_8 q_{30} p_{02} p_{25}' p_{58} \text{ Therefore } B_0^{3*}(\infty) = \frac{N_5(0)}{D_2'(0)} \quad [9.12]$$

10. PARTICULAR CASES

When all repair time distribution are n-phase Erlangian distribution i.e.

Density function $g_i(t) = \sum \frac{nr_i (nr_i t)^{n-1} e^{-nr_i t}}{n-1!}$

and Survival function $\overline{G}_i(t) = \sum_{j=0}^{n-1} \frac{(nr_i t)^j e^{-nr_i t}}{j!}$

and other distribution are negative exponential

$$a(t) = \theta e^{-\theta t},\ b(t) = \eta e^{-\eta t},\ \overline{A}(t) = e^{-\theta t},\ \overline{B}(t) = e^{-\eta t}$$

For n=1 $g_i(t) = r_i e^{-r_i t}$, $\overline{G}_i(t) = e^{-r_i t}$

If i=1,2,3,4 $\quad g_1(t) = r_1 e^{-r_1 t},\quad g_2(t) = r_2 e^{-r_2 t},\ g_3(t) = r_3 e^{-r\,t}\ , g_4(t) = r_4 e^{-r_4 t}$

$$\overline{G}_1(t) = e^{-r_1 t},\quad \overline{G}_2(t) = e^{-r_2 t},\quad \overline{G}_3(t) = e^{-r_3 t}\quad ,\quad \overline{G}_4(t) = e^{-r_4 t}$$

Also $\quad p_{10} = 1 = p_{43} = p_{62} = p_{70} = p_{80}$

$$p_{01} = \int_0^\infty \alpha e^{-x_1 t} e^{-\theta t}\, dt = \frac{\alpha}{x_1+\theta},\ p_{02} = \frac{\gamma}{x_1+\theta},\ p_{03} = \frac{\beta}{x_1+\theta},\ p_{07} = \frac{\theta}{x_1+\theta}$$

$$p_{20} = \int_0^\infty e^{-(\alpha+\beta)t} r_3 e^{-r_3 t}\, dt \ = \ \frac{r_3}{x_2+r_3},\ p_{25} = \frac{\beta}{x_2+r_3},\ p_{26} = \frac{\alpha}{x_2+r_3}$$

$$p_{30} = \frac{r_2}{\alpha+r_2}, p_{34} = \frac{\alpha}{\alpha+r_2}, p_{52} = \frac{r_2}{\alpha+r_2}, p_{58} = \frac{\alpha}{\alpha+r_2}$$

Here also $p_{01} + p_{02} + p_{03} + p_{07} = 1,\ p_{20} + p_{25} + p_{26} = 1,$

$$p_{30} + p_{34} = 1,\ p_{52} + p_{58} = 1$$

$$\mu_0 = \frac{1}{x_1+\theta},\ \ \mu_1 = \frac{1}{r_1},\ \ \mu_2 = \frac{1}{x_2+r_3},\ \ \mu_3 = \frac{1}{\alpha+r_2},\ \ \mu_4 = \frac{1}{r_1},$$

$$\mu_5 = \frac{1}{\alpha + r_2}, \quad \mu_6 = \frac{1}{r_1}, \quad \mu_7 = \frac{1}{\eta}, \quad \mu_8 = \frac{1}{r_4}$$

Where $x_1 = \alpha + \beta + \gamma,\ x_2 = \alpha + \beta$

$$\text{MTSF} = E(T) = \frac{K_0 + K_2 + K_3 + K_5}{K_0(x_1 + \theta)}$$

$$\text{Availability} = A^*_0(\infty) = \frac{L_0 + L_2 + L_3 + L_5}{L_0 + L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8}$$

$$\text{Busy Period } B_0^{1*}(\infty) = \frac{L_1 + L_2 + L_3 + L_4 + L_5 + L_6}{\sum_{i=1}^{8} L_i},$$

$$B_0^{2*}(\infty) = \frac{L_7}{\sum_{i=1}^{8} L_i}, \qquad B_0^{3*}(\infty) = \frac{L_8}{\sum_{i=1}^{8} L_i}$$

Where $K_0 = \frac{1}{x_1 + \theta}\left[1 - \frac{\beta}{(x_2 + r_3)}\frac{r_2}{(\alpha + r_2)}\right],\ K_2 = \frac{\gamma}{(x_2 + r_3)(x_1 + \theta)},$

$$K_3 = \frac{1}{\alpha + r_2}\frac{\beta}{(x_1 + \theta)}\left[1 - \frac{\beta}{x_2 + r_3}\frac{r_2}{\alpha + r_2}\right],\ K_5 = \frac{\beta\gamma}{(\alpha + r_2)(x_2 + r_3)(x_1 + \theta)},$$

$$L_0 = \frac{1}{x_1 + \theta}\frac{r_2}{(\alpha + r_2)}\left[1 - \frac{\beta}{(x_2 + r_3)}\frac{r_2}{(\alpha + r_2)} - \frac{\alpha}{(x_2 + r_3)}\right],$$

$$L_1 = \frac{1}{r_1}\frac{\alpha}{x_1 + \theta}\frac{r_2}{(\alpha + r_2)}\left[1 - \frac{\beta}{(x_2 + r_3)}\frac{r_2}{(\alpha + r_2)} - \frac{\alpha}{(x_2 + r_3)}\right],$$

$$L_2 = \frac{1}{x_2 + r_3}\frac{r_2}{(\alpha + r_2)}\frac{\gamma}{(\theta + x_1)}$$

$$L_3 = \frac{\beta}{x_1+\theta}\frac{1}{(\alpha+r_2)}\left[1-\frac{\beta}{(x_2+r_3)}\frac{r_2}{(\alpha+r_2)}-\frac{\alpha}{(x_2+r_3)}\right],$$

$$L_4 = \frac{1}{r_1}\frac{\beta}{x_1+\theta}\frac{\alpha}{(\alpha+r_2)}\left[1-\frac{\beta}{(x_2+r_3)}\frac{r_2}{(\alpha+r_2)}-\frac{\alpha}{(x_2+r_3)}\right],$$

$$L_5 = \frac{1}{\alpha+r_2}\frac{\gamma}{x_1+\theta}\frac{\beta}{(x_2+r_3)}\frac{r_2}{(\alpha+r_2)},$$

$$L_6 = \frac{1}{r_2}\frac{\gamma}{x_1+\theta}\frac{\alpha}{(x_2+r_3)}\frac{r_2}{(\alpha+r_2)},$$

$$L_7 = \frac{1}{\eta}\frac{\theta}{x_1+\theta}\frac{r_2}{(\alpha+r_2)}\left[1-\frac{\beta}{(x_2+r_3)}\frac{r_2}{(\alpha+r_2)}-\frac{\alpha}{(x_2+r_3)}\right],$$

$$L_8 = \frac{1}{r_4}\frac{\gamma}{x_1+\theta}\frac{\beta}{(x_2+r_3)}\frac{\alpha}{(\alpha+r_2)}\frac{r_2}{(\alpha+r_2)}$$

11. PROFIT ANALYSIS

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t]. Therefore,

G(t) = Expected total revenue earned by the system in (0,t]-Expected repair cost of the failed units-Expected repair cost of the repairman in preventive maintenance.

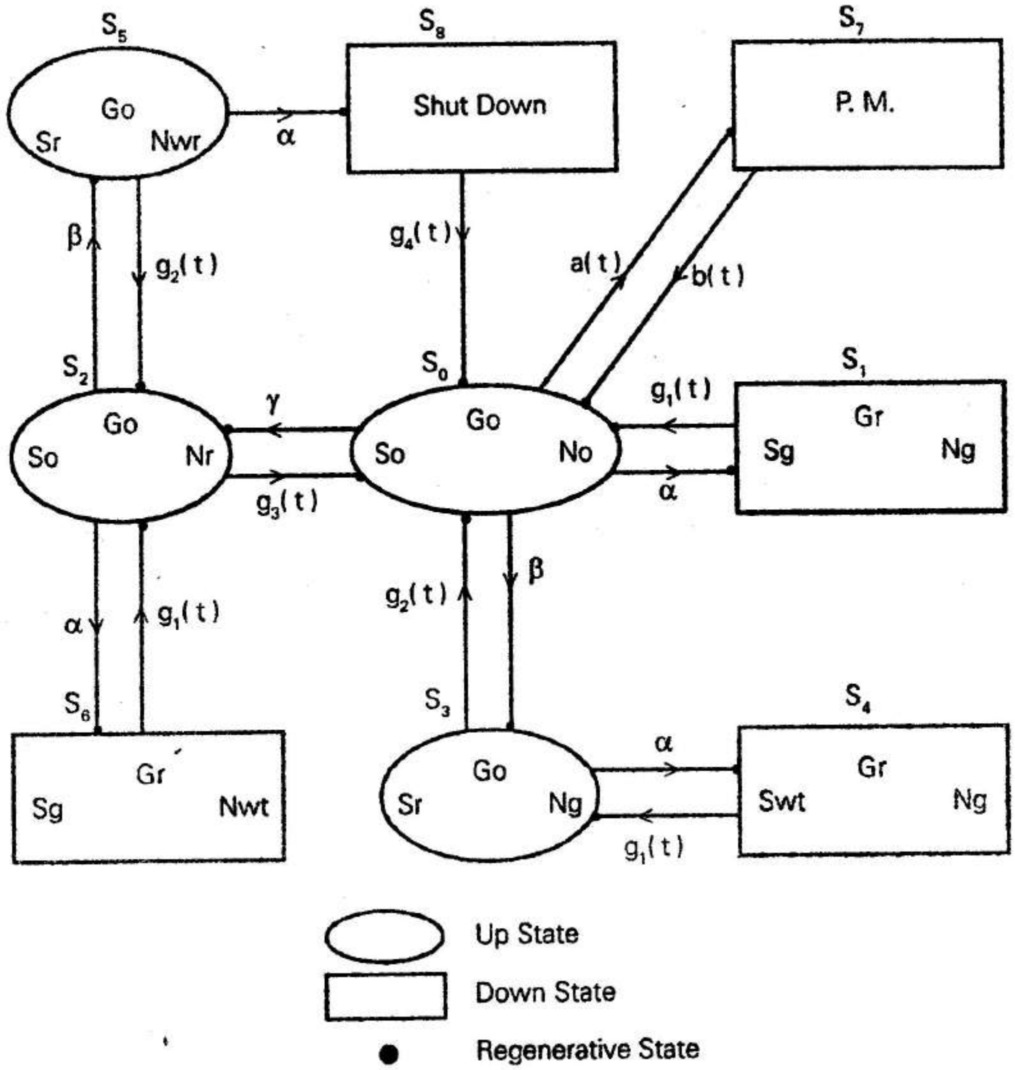
$$= C_1\mu_{up}(t)-C_2\mu_{b1}-C_3\mu_{b2}-C_4\mu_{b3}$$

Where,

$$\mu_{up}(t)=\int_0^t A_0(t)dt;\quad \mu_{b1}(t)=\int_0^t B_0^1(t)dt;$$

$$\mu_{b2}(t)=\int_0^t B_0^2(t)dt;\quad \mu_{b3}(t)=\int_0^t B_0^3(t)dt$$

STATE TRANSITION DIAGRAM



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