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ON RECURRENCE TENSOR FIELD

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ABSTRACT : In this paper, some properties of recurrence parameters in birecurrent GF - Manifold have been studied.

1. INTRODUCTION :

Consider a diffrentiable manifold Mn of differtiability class C^∞. Let there be in Mn, a vector valued linear function F of class C^∞ satisfying the algebraic equation

$$\bar{\bar{X}} \overset{\text{def}}{=} a^2 X, \tag{1.1)a$$

for arbitrary vector field X and complex number a, where

$$\bar{X} \overset{\text{def}}{=} FX . \tag{1.1)b$$

Then {F} is said to give to Mn a general differentiable structure briefly known as GF-structure defined by equations (1.1)a and the manifold Mn is called GF-manifold[1]. If the GF-structure is endowed with Hermite metric tensor g, such that

$$g(\bar{X},\bar{Y}) + a^2 g(X,Y) = 0 \tag{1.1)c$$

Then {F,g} gives to Mn, a Hermite structure or H-structure subordinate to GF-structure.

AGREEMENT 1.1 : In what follows and above, the equations containing X, Y, Z, ..., etc. hold for arbitrary vectors X, Y, Z, ..., etc. in Mn.

Let K denote the curvature tensor in Mn. Then the following equations hold[6]:

$$K(X,Y,\overline{Z})=\overline{K(X,Y,Z)} \tag{1.2)a}$$

$$\overline{K(X,Y,\overline{Z})}=a^2K(X,Y,Z) \tag{1.2)b}$$

$$Ric(Y,Z)=g(r(y),Z),\ Ric(Y,Z)=(C_1^1K)(Y,Z) \tag{1.2)c}$$

$$Ric(\overline{X},\overline{Y})=-a^2Ric(X,Y),\ (C_1^1r)=R \tag{1.2)d}$$

$$Ric(\overline{X},Y)=-Ric(X,\overline{Y}), \tag{1.2)e}$$

where r is the self-adjoint Ricci map and C_1^1 is the contraction with respect to first slot.

The manifold M_n is said to be recurrent[2], if

$$(\nabla K)(X,Y,Z,U)=A_1(U)K(X,Y,Z), \tag{1.3)a}$$

it is said to be Ricci-recurrent,if

$$(\nabla Ric)(Y,Z,U)=A_1(U)Ric(Y,Z), \tag{1.3)b}$$

where A_1 is a non-vanishing C^∞ 1-form.

The manifols is said to be birecurrent[5], if

$$(\nabla\nabla K)(X,Y,Z,T_1,T_2)=A_2(T_1,T_2)K(X,Y,Z), \tag{1.4)a}$$

it is said to be Ricci-birecurrent, if

$$(\nabla\nabla Ric)(Y,Z,T_1,T_2)=A_2(T_1,T_2)Ric(Y,Z), \tag{1.4)b}$$

where A_2 is a non-vanishing C^∞ 2-form, such that

$$A_2(T_1,T_2)=(\nabla A_1)(T_1,T_2)+A_1(T_1)A_2(T_2). \tag{1.4)c}$$

The curvature tensor K satisfies the following Ricci-identities :

$$(\nabla\nabla K)(X,Y,Z,T,V)-(\nabla\nabla K)(X,Y,Z,V,T)=K(V,T,K(X,Y,Z,))$$
$$-K(K(V,T,X),Y,Z)-K(X,K(V,T,Y),Z)-K(X,YK(V,T,Z)). \quad (1.5)$$

LetQ, a vector - valued trilinear function, be any one of the curvature tensors K, W, C, L or V.

A GF-manifold is said to be (1)-recurrent in Q^3, if

$$a^2((\nabla Q)(X,Y,Z,T))+Q(\nabla F(\overline{X},T),Y,Z)=a^2 A_1(T)Q(X,Y,Z), \quad (1.6)$$

where $A_1(T)$is a non-vanishing C^∞ function.

A GF-structure manifold is said to be (1)-birecurrent in Q^4, if

$$a^2(\nabla\nabla Q)(X,Y,Z,T,S)+(\nabla Q)((\nabla F)(\overline{X},T),Y,Z,S)+(\nabla Q)((\nabla F)(\overline{X},S),Y,Z,T)$$
$$+Q((\nabla\nabla F)(\overline{X},T,S),Y,Z)=a^2 A_2(T,S)Q(X,Y,Z), \quad (1.7)$$

where $A_2(T,S)$ is a non-vanishing C^∞ function.

2. RECURRENCE TENSOR FIELD

THEOREM 2.1 : In a (1)-birecurrent GF-manifold the recurrence tensor field $A_2(T,S)$ is non-symmetric.

PROOF : Interchanging T and S in equation (1.7) and then subtracting the resulting equation from equation (1.7), we get

$$a^2[(\nabla\nabla K)(X,Y,Z,T,S)-(\nabla\nabla K)(X,Y,Z,S,T)]+K[(\nabla\nabla F)(\overline{X},T,S)$$
$$-(\nabla\nabla F)(\overline{X},S,T),Y,Z]=a^2[A_2(T,S)-A_2(S,T)]K(X,Y,Z). \quad (2.1)$$

Using Ricci-identities for K(X,Y,Z) in equation (2.1) and then using equation (1.1)a in the resulting equation, we get

$$a^2K(S,T,K(X,Y,Z))-a^2K(X,K(S,T,Y),Z)-a^2K(X,Y,K(S,T,Z))$$
$$-K(\overline{K(S,T,\overline{X})}Y,Z)=a^2[A_2(T,S)-A_2(S,T)]K(X,Y,Z). \quad (2.2)$$

The equation (2.2) proves the statement.

NOTE 2.1 : Theorems of type (2.1) can also be proved taking (2), (3), (12), (13), (23) and (123)-birecurrent GF-manifold instead of (1)-birecurrent GF-manifold.

THEOREM 2.2 : In a (1)-birecurrent GF-manifold, the recurrence tensor field $A_2(T,S)$ satisfies the relation

$$a^2A_1(U)(A_2(T,S)-A_2(S,T))K(X,Y,Z)-K((\nabla F)K(S,T,\overline{X}),U),Y,Z)$$
$$-K(\overline{K(S,T,(\nabla F)(X,U)}),Y,Z)=a^2((\nabla A_2)(T,S,U)$$
$$-(\nabla A_2)(S,T,U))K(X,Y,Z). \qquad (2.3)$$

PROOF : Differentiating equation (2.2) covariantly and then using equations (1.3)a and (2.2) in the resulting equation, we get the eqaution (2.3).

THEOREM 2.3 : If the GF-manifold is birecurrent and (1)-birecurrent for the same recurrence parameter, then we have

$$a^2A_1(U)(A_2(T,S)-A_2(S,T))=a^2((\nabla A_2(T,S,U)-(\nabla A_2)(S,T,U)). \qquad (2.4)$$

PROOF : Interchanging T and S in equation (1.4)a and then subtracting the resulting equation from equation (1.4)a, we get

$$(\nabla\nabla K)(X,Y,Z,T,S)-(\nabla\nabla K)(X,Y,Z,S,T)$$
$$=(A_2(T,S)-A_2(S,T))K(X,Y,Z). \qquad (2.5)$$

Using Ricci-identities in equation (2.5)and then comparing the resulting equation with the equation (2.2), we get

$$K(\overline{K(S,T,\overline{X})}Y,Z)=a^2K(K(S,T,X),Y,Z). \qquad (2.6)$$

Using equation (2.6) in equation (2.2), we get

$$a^2K(S,T,K(X,Y,Z))-a^2K(K(S,T,X),Y,Z)-a^2K(X,K(S,T,Y),Z)$$
$$-a^2K(X,Y,K(S,T,Z))=a^2(A_2(T,S)-A_2(S,T))K(X,Y,Z). \qquad (2.7)$$

Differentiating equation (2.7) covariantly and using equations (1.3) and (2.7) in the resulting equation, we have

$$a^2A_1(U)(A_2(T,S)-A_2(S,T))K(X,Y,Z)$$
$$=a^2((\nabla A_2)(T,S,U)-(\nabla A_2)(S,T,U))K(X,Y,Z),$$

which gives the equation (2.4).

THEOREM 2.4 : If the GF-manifold is birecurrent and (1)-birecurrent for the same recurrence parameter, then

$$a^2(A_2(K(U,V,S),T)+A_2(S,K(U,V,T)-A_2(K(U,V,T),S)-A_2(T,K(U,V,S)$$
$$=a^2(A_2(T,S)-A_2(S,T))((\nabla A_1)(U,V)-(\nabla A_1)(V,U)). \qquad (2.8)$$

PROOF : Differentiating equation (2.4) covariantly and using equation (2.4), we get

$$a^2(\nabla\nabla A_2)(T,S,U,V)-a^2(\nabla\nabla A_2)(S,T,U,V)=a^2(A_2(T,S)$$
$$-A_2(S,T))(\nabla A_1)(U,V)+a^2(A_2(T,S)-A_2(S,T))A_1(U)A_1(V). \qquad (2.9)$$

Interchanging U and V in equation (2.9) and then subtracting the resulting equation obtained from equation (2.9), we get

$$a^2(\nabla\nabla A_2)(T,S,U,V)-a^2(\nabla\nabla A_2)(S,T,U,V)=a^2(A_2(T,S)$$
$$-A_2(S,T))\{(\nabla A_1)(U,V)-(\nabla A_1)(V,U)\}$$
$$+a^2(\nabla\nabla A_2)(T,S,V,U)-a^2(\nabla\nabla A_2)(S,T,V,U). \qquad (2.10)$$

Also, from Ricci-identities for $A_2(T,S)$, we have

$$(\nabla\nabla A_2)(T,S,U,V)-(\nabla\nabla A_2)(T,S,V,U)$$
$$=-A_2(K(U,V,T),S)-A_2(T,K(U,V,S)). \qquad (2.11)$$

Interchanging T and S in equation (2.11) and then subtracting the resulting equation obtained from equation (2.11), we get

$$(\nabla\nabla A_2)(T,S,U,V)-(\nabla\nabla A_2)(S,T,U,V)=-A_2(K(U,V,T),S)$$
$$-A_2(T,K(U,V,S))+A_2(K(U,V,S),T)+A_2(S,K(U,V,T))$$
$$+(\nabla\nabla A_2)(T,S,V,U)-(\nabla\nabla A_2)(S,T,V,U). \qquad (2.12)$$

From the equations (2.10) and (2.12), we get the equation (2.8).

NOTE 2.2 : Theorems of type (2.2), (2.3) and (2.4) can also be proved taking (2) or (3)-birecurrent GF-manifold instead of (1)-birecurrent GF-manifold.

REFERENCES

1. Duggal K. L. : On differentiable structures defined by algebraic equations I, Nijenhuis tensor Tensor, N. S. , 22 (1971), 238-242.

2. Mishra R. S. : A course in tensors with applications to Riemannian Geometry, Pothishala Private Limited, Allahabad, India (1965).

3. Panday S. B. and Dasila Lata : On General Differentiable Manifold, U. Scientist of Phyl. Science Vol. 7. No. 2, 147-152 (1995).

4. Takano K. : On space with birecurrent curvature Tensor, N.S. 22, 329-32.

5, Yano K. : Differential Geometry on Complex and almost complex spaces, Pergamon Press, New York, (1965).