

FLOW INDUCED IN A POROUS MEDIUM DUE TO A THREE DIMENSIONAL COUETTE FLOW ADJACENT TO IT

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ABSTRACT : The flow of a viscous fluid induced in a porous medium due to a uniform motion of a plate parallel to its surface has been discussed when there is a transverse sinusoidal injection of the fluid at the moving plate. The fluid fills the space between the plate and the porous medium which is fully saturated with the fluid. Due to this type of the injection velocity the flow in the clear fluid becomes three-dimensional. It is assumed that the flow in the clear fluid region is governed by Navier-Stokes equation and that in the porous medium by the Brinkman equation near the interface and by Darcy law far away from the interface. It is found that with the increase of the permeability of the porous medium the magnitude of velocity component increases in both the regions. In the porous region the velocity component parallel to the motion of the plate is maximum at the interface and then decreases exponentially forming a boundary layer at the interface. Also the velocity component parallel to the motion of the plate decreases with the increase of injection Reynolds number R_e and the magnitude of other velocity components increases with the increase of R_e .

Key Words : 3 D-Couette flow, sinusoidal injection and porous medium.

Mathematics Subject Classification (2000) : 76 S 05.

1. INTRODUCTION

Considerable work has been done when a viscous fluid flows over a porous surface because of its importance in many engineering problems such as flow of liquid in the porous bearing (Joseph and Tao [5]) and porous rollers, and its natural occurrence in the flow of rivers through porous banks and beds. The production of petroleum and natural gases and well drilling require many predictions based on the results to fluid flow through a porous medium. The flow of blood through lungs and arteries (Tang and Fung[15]) are also examples of flow through porous medium. In all these cases the fluid flows through two regions, viz. zone I where there is no porous medium and clear fluid flows freely and zone II where the fluid flows through the pores of a permeable solid. Navier-Stokes is taken as the equation governing the flow in the free flow region.

In region II, there is a boundary layer formation near the interface and the following equation proposed by Brinkman [2] is used near the interface

$$\nabla p = \mu \nabla^2 V - \frac{\mu}{k} V, \quad (1.1)$$

where V is velocity vector, μ is the coefficient of viscosity of the fluid, k is the permeability of the porous medium and p is the pressure. Far away from the interface or for the case when the diameter of the porous particles is vanishingly small, following Darcy law [3] is used :

$$\nabla p = -\frac{\mu}{k} V. \quad (1.2)$$

Singh [10] has analyzed the couette flow between two horizontal parallel flat plates with transverse sinusoidal injection of the fluid at the stationary plate and its corresponding removal by constant suction through the plate in uniform motion. In this paper we have discussed the above problem analyzed by Singh [10] when transverse sinusoidal injection is at the moving plate and the stationary disk is replaced by a porous medium. The fluid fills the space between the plate and the porous medium which is also fully saturated with the given fluid. It is assumed that the flow

in the clear fluid is governed by Navier-Stokes equation and that in the porous region by Brinkman equation [2] near the interface and by Darcy law [3] away from it. This type of flow analysis in the porous medium has been taken by Rudraiah et al. [7] in discussing the Hartmann flow over a permeable bed and by Rudraiah and Veerbhadraiah [8] in discussing the buoyancy effects on the plane couette flow past a permeable bed. Such types of coupled boundary value problems in the porous medium have been discussed by many workers [11-14].

2. FORMULATION OF THE PROBLEM

Consider the flow of a viscous incompressible fluid between a porous medium $y^* = 0$ and a flat plate $y^* = d$ moving with a constant velocity U parallel to itself. The porous medium is also fully saturated with the fluid. The direction of U is taken as x^* -axis and perpendicular to x^* and y^* is taken as axis of z^* . We assume that there is a transverse sinusoidal injection velocity distribution of the form

$$V^*(z^*) = V_0 \left(1 + \epsilon \cos \frac{\pi z^*}{d}\right) \quad (2.1)$$

at the moving plate (see fig. 1). All the physical quantities are taken as independent of x^* . The following non-dimensional quantities are defined :

$$y = \frac{y^*}{d}, \quad z = \frac{z^*}{d}, \quad u = \frac{u^*}{U}, \quad v = \frac{v^*}{V_0}, \quad w = \frac{w^*}{V_0}, \quad p = \frac{p^*}{\rho V_0^2}, \quad \nu = \frac{\mu}{\rho}, \quad R_e = \frac{V_0 d}{\nu}$$

where u, v, w are velocity components in the direction of x, y, z respectively, ρ is the density, ν is the kinematics coefficient of viscosity and R_e is the suction Reynolds number. Let the superscript in the bracket of an entity $\chi^{(i)}$ $i = 1, 2$, denote the zone to which the entity belongs. Denoting the velocity components $u^{(1)}, v^{(1)}, w^{(1)}$ in the x, y, z direction respectively, the flow of the clear fluid is governed by Navier-Stokes equation which gives the following non-dimensional equations :

$$\frac{\partial v^{(1)}}{\partial y} + \frac{\partial w^{(1)}}{\partial z} = 0, \quad (2.2)$$

$$R_e \left(v^{(1)} \frac{\partial u^{(1)}}{\partial y} + w^{(1)} \frac{\partial u^{(1)}}{\partial z} \right) = \frac{\partial^2 u^{(1)}}{\partial y^2} + \frac{\partial^2 u^{(1)}}{\partial z^2}, \quad (2.3)$$

$$R_e \left(v^{(1)} \frac{\partial v^{(1)}}{\partial y} + w^{(1)} \frac{\partial v^{(1)}}{\partial z} \right) = -R_e \frac{\partial p}{\partial y} + \left(\frac{\partial^2 v^{(1)}}{\partial y^2} + \frac{\partial^2 v^{(1)}}{\partial z^2} \right), \quad (2.4)$$

$$R_e \left(v^{(1)} \frac{\partial w^{(1)}}{\partial y} + w^{(1)} \frac{\partial w^{(1)}}{\partial z} \right) = -R_e \frac{\partial p}{\partial z} + \left(\frac{\partial^2 w^{(1)}}{\partial y^2} + \frac{\partial^2 w^{(1)}}{\partial z^2} \right). \quad (2.5)$$

In zone II where the fluid flows through the pores of the porous material and near the interface the flow is governed by the Brinkman equation (1.1) which gives the following differential equation for the velocity $u^{(2)}$:

$$\frac{\partial^2 u^{(2)}}{\partial y^2} + \frac{\partial^2 u^{(2)}}{\partial z^2} - \sigma^2 u^{(2)} = 0 \quad (2.6)$$

The boundary conditions of the problem are

$$u^{(1)} = 0, \quad v^{(1)} = 1 + \epsilon \cos \pi z, \quad w^{(1)} = 0, \quad \text{at } y = 1 \quad (2.7)$$

$$u^{(2)} = 0, \quad v^{(2)} = 0, \quad w^{(2)} = 0 \quad \text{as } y \rightarrow -\infty \quad (2.8)$$

When the flow in the porous medium is governed by Brinkman equation (1.1), Ochoa-Tapia and Whitaker [6] have proposed the conditions at the porous / clear fluid interface by applying volume average technique and have shown that the equation requires continuity in the velocity components and the normal stresses and discontinuity in the shearing stresses. In present notation their matching conditions can be written as

$$u^{(1)} = u^{(2)} \quad \text{at } y = 0. \quad (2.9)$$

$$\frac{\partial u^{(1)}}{\partial y} - \frac{\partial u^{(2)}}{\partial y} = \sigma \alpha u^{(1)}. \quad (2.10)$$

where α is the dimensionless constant depending on the surface of the porous material which is determined by experiments. The pressure and the normal velocity

$v^{(2)}$ and $w^{(2)}$ will continue to the same as in zone I but far away from the interface they are governed by the Darcy law (1.2). Hence at the interface, the velocity $v^{(2)}$ and $w^{(2)}$ and will satisfy Beavers and Joseph[1] conditions modified by Saffman [9] and Jones [4] as :

$$v^{(2)} = \frac{\sqrt{k}}{\alpha} \frac{dv^{(2)}}{dy} \quad (2.11)$$

$$w^{(2)} = \frac{\sqrt{k}}{\alpha} \frac{dw^{(2)}}{dy} \quad (2.12)$$

3. SOLUTION OF THE PROBLEM

Since the amplitude of the injection velocity $\epsilon (\leq 1)$ is very small, we now assume the solution of the following form

$$f(y, z) = f_0(y) + \epsilon f_1(y, z) + \epsilon^2 f_2(y, z) + \dots, \quad (3.1)$$

where f stands for any of $u^{(1)}$, $v^{(1)}$, $w^{(1)}$, $u^{(2)}$, $v^{(2)}$, $w^{(2)}$ and p . When , the problem is reduced to two-dimensional flow with constant injection. Substituting (3.1) into (2.3) and (2.6) and using the boundary conditions (2.7), (2.8) and matching conditions (2.9) and (2.10), we obtain the following solution:

$$u_0^{(1)} = 1 + \frac{\sigma(1+\alpha)}{R_e - \sigma(1+\alpha)(1-e^{R_e})} (e^{R_e y} - e^{R_e}). \quad (3.2)$$

$$u_0^{(2)} = \frac{R_e}{R_e - \sigma(1+\alpha)(1-e^{R_e})} e^{\sigma y}. \quad (3.3)$$

$$u_1^{(1)} = \frac{1}{A} \left[L e^{r_1 y} + M e^{r_2 y} + \frac{R_e \sigma(1+\alpha)}{R_e - \sigma(1+\alpha)(1-e^{R_e})} \right. \\ \left. + \left\{ A_1 e^{(R_e+r_1)y} + A_2 e^{(R_e+r_2)y} - A_2 e^{(R_e+\pi)y} + A_4 e^{(R_e-\pi)y} \right\} \right] \quad (3.4)$$

$$u_1^{(2)} = N e^{\sqrt{(\pi^2+\sigma^2)} y} \quad (3.5)$$

where

$$A_1 = \pi(r_2 - \sigma\alpha)(\pi - \sigma\alpha)(\pi - r_2)e^{-\pi} + \pi(r_2 - \sigma\alpha)(\pi + \sigma\alpha)(\pi + r_2)e^{\pi} \\ - 2\pi r_2(\pi + \sigma\alpha)(\pi - \sigma\alpha)e^{r_2} - (\pi + \sigma\alpha)(r_2 - \sigma\alpha)(\pi + r_1)e^{\pi + r_2 - r_1} \quad (3.6)$$

$$A_2 = 2\pi r_1(\pi + \sigma\alpha)(\pi - \sigma\alpha)e^{r_1} - \pi(\pi - r_1)(\pi - \sigma\alpha)(r_1 - \sigma\alpha)e^{-\pi} \\ - \pi(\pi + r_1)(\pi + \sigma\alpha)(r_1 - \sigma\alpha)e^{\pi} \quad (3.7)$$

$$A_3 = r_1(\pi + \sigma\alpha)(r_2 - \sigma\alpha)(\pi + r_2)e^{r_1} - \pi(r_1 - \sigma\alpha)(r_2 - r_1)(r_2 - \sigma\alpha)e^{-\pi} \\ - r_2(\pi + \sigma\alpha)(r_1 - \sigma\alpha)(\pi + r_1)e^{r_2} \quad (3.8)$$

$$A_4 = r_1(\pi - \sigma\alpha)(r_2 - \sigma\alpha)(\pi - r_2)e^{r_1} + \pi(r_1 - \sigma\alpha)(r_2 - r_1)(r_2 - \sigma\alpha)e^{\pi} \\ - r_2(\pi - r_1)(\pi - \sigma\alpha)(r_1 - \sigma\alpha)e^{r_2} \quad (3.9)$$

$$A = 2\pi(r_2 - r_1)(r_2 - \sigma\alpha)(r_1 - \sigma\alpha) + (\pi + r_1)(r_2 - \sigma\alpha)(\pi - \sigma\alpha)(\pi - r_2)e^{r_1 - \pi} \\ + (\pi - r_1)(\pi + \sigma\alpha)(r_2 - \sigma\alpha)(\pi + r_2)e^{\pi + r_1} - 2\pi(\pi + \sigma\alpha)(\pi - \sigma\alpha) \\ (r_2 - r_1)e^{r_1 + r_2} - (\pi - r_1)(\pi - \sigma\alpha)(r_1 - \sigma\alpha)(\pi + r_2)e^{r_2 - \pi} \\ - (\pi + r_1)(\pi + \sigma\alpha)(r_1 - \sigma\alpha)(\pi - r_2)e^{\pi + r_2} \quad (3.10)$$

$$L = \frac{R_0 \sigma \alpha (1 + \alpha)}{A \{ R_0 - \sigma (1 + \alpha) (1 - e^{R_0}) \} \{ e^{r_2} (r_1 - \sigma \alpha - \sqrt{\pi^2 + \sigma^2}) - e^{r_1} (r_2 - \sigma \alpha - \sqrt{\pi^2 + \sigma^2}) \}} \\ \left[(\sigma \alpha + \sqrt{\pi^2 + \sigma^2}) \left(\frac{A_1}{2r_1} + \frac{A_2}{2r_2} - \frac{A_3}{\pi} - \frac{A_4}{\pi} \right) e^{r_2} \right. \\ \left. - \left\{ \frac{A_1 (R_0 + r_1)}{2r_1} + \frac{A_2 (R_0 + r_2)}{2r_2} - \frac{A_3 (R_0 + \pi)}{\pi} - \frac{A_4 (R_0 - \pi)}{\pi} \right\} e^{r_1} \right. \\ \left. + e^{R_0} (r_2 - \sigma \alpha - \sqrt{\pi^2 + \sigma^2}) \left(\frac{A_1}{2r_1} e^{r_1} + \frac{A_2}{2r_2} e^{r_2} - \frac{A_3}{\pi} e^{\pi} - \frac{A_4}{\pi} e^{-\pi} \right) \right] \quad (3.11)$$

$$\begin{aligned}
M = & \frac{R_0 \sigma \alpha (1 + \alpha)}{A \{R_0 - \sigma(1 + \alpha)(1 - e^{R_0})\} \{e^{r_1}(r_2 - \sigma \alpha - \sqrt{\pi^2 + \sigma^2}) - e^{r_2}(r_1 - \sigma \alpha - \sqrt{\pi^2 + \sigma^2})\}} \\
& \left[(\sigma \alpha + \sqrt{\pi^2 + \sigma^2}) \left(\frac{A_1}{2r_1} + \frac{A_2}{2r_2} - \frac{A_3}{\pi} - \frac{A_4}{\pi} \right) e^{r_1} \right. \\
& \left. - \left\{ \frac{A_1(R_0 + r_1)}{2r_1} + \frac{A_2(R_0 + r_2)}{2r_2} - \frac{A_3(R_0 + \pi)}{\pi} - \frac{A_4(R_0 - \pi)}{\pi} \right\} e^{r_1} \right. \\
& \left. + e^{R_0}(r_1 - \sigma \alpha - \sqrt{\pi^2 + \sigma^2}) \left(\frac{A_1}{2r_1} e^{r_1} + \frac{A_2}{2r_2} e^{r_2} - \frac{A_3}{\pi} e^{\pi} - \frac{A_4}{\pi} e^{-\pi} \right) \right] \quad (3.12)
\end{aligned}$$

and N is given by

$$N = L + M + \frac{R_0 \sigma (1 + \alpha)}{A \{R_0 - \sigma(1 + \alpha)(1 - e^{R_0})\}} \left(\frac{A_1}{2r_1} + \frac{A_2}{2r_2} - \frac{A_3}{\pi} - \frac{A_4}{\pi} \right) \quad (3.13)$$

Substituting (3.1) into (2.4) and (2.5), and comparing the coefficients of identical powers of ϵ^0, ϵ^1 , neglecting those of ϵ^2, ϵ^3 , etc. we get the following solutions [see, Singh[10]]

$$v_0^{(1)} = 1, w_0^{(1)} = 0, p_0^{(1)} = \text{constant} \quad (3.14)$$

$$v_1^{(1)} = \frac{1}{A} (A_1 e^{r_1 y} + A_2 e^{r_2 y} - A_3 e^{\pi y} + A_4 e^{-\pi y}) \cos \pi z, \quad (3.15)$$

$$w_1^{(1)} = -\frac{1}{\pi A} (A_1 r_1 e^{r_1 y} + A_2 r_2 e^{r_2 y} - \pi A_3 e^{\pi y} - \pi A_4 e^{-\pi y}) \sin \pi z, \quad (3.16)$$

$$p_1^{(1)} = \frac{1}{A} (A_3 e^{\pi y} - A_4 e^{-\pi y}) \cos \pi z. \quad (3.17)$$

In zone II, far away from the interface, the flow is governed by Darcy's law (1.2) which gives the velocity components in the porous medium as

$$v^{(2)} = -\frac{\epsilon \pi R_e}{\sigma^2 A} (A_3 e^{\pi y} + A_4 e^{-\pi y}) \cos \pi z \quad (3.18)$$

$$w^{(2)} = \frac{\epsilon \pi R_e}{\sigma^2 A} (A_3 e^{\pi y} - A_4 e^{-\pi y}) \sin \pi z. \quad (3.19)$$

Since $v^{(2)}$, $w^{(2)}$ as $y \rightarrow -\infty$, hence solutions (3.18) and (3.19) will hold only if $A_4 = 0$. This gives the following condition

$$\begin{aligned} \sigma^2 \alpha^2 \{ & r_1 (\pi - r_2) e^{r_1} + \pi (r_2 - r_1) e^{\pi} - r_2 (\pi - r_1) e^{r_2} \} - \sigma \alpha \{ r_1 (\pi - r_2) (\pi + r_2) e^{r_1} \\ & + \pi (r_2 - r_1) (r_2 + r_1) e^{\pi} - r_2 (\pi - r_1) (\pi + r_1) e^{r_2} \} + \pi \{ r_1 r_2 (\pi - r_2) e^{r_1} \\ & + \pi r_1 (r_2 + r_1) e^{\pi} - r_1 r_2 (\pi + r_1) e^{r_2} \} = 0 \end{aligned} \quad (3.20)$$

This equation (3.20) has been solved by taking $R_e = 0.50, 1.0, 1.5, 2.0, 2.5, 3.0$ which gives two values of $\sigma \alpha$ for every R_e . The values of constants A_1, A_2, A_3, A, L, M and N are given in table -1 for these different values of R_e by taking $\alpha = 0.45$.

4. DISCUSSIONS AND CONCLUSIONS

The graph of the main flow velocity against the distance from the interface has been plotted in Figure - 2 for above mentioned different values of injection Reynolds number R_e by taking $\alpha = 0.45$. The graph shows that the main flow velocity decreases with the increase of R_e and decreases with the decrease of σ (for fixed α) i.e. with the increase of the permeability k . Also, the velocity in the porous medium is maximum at the interface and then decreases sharply to zero within a short distance indicating the boundary layer formation near the interface.

The magnitude of $v^{(2)}$, $w^{(2)}$ is given by

$$Q = \frac{\epsilon \pi A_3 R_e}{\sigma^2 A} e^{\pi y}$$

The values of magnitude of cross velocities at the interface Q is given in table- 2 for different values of injection parameter R_0 and σ by taking $\alpha = 0.45$. Table shows that the magnitude of cross velocities Q increases with the increase of R_0 and decreases with the increase of σ .

Now after determining the velocity field we calculate the skin friction components τ_x in the main flow direction and τ_z in the transverse direction on the moving plate and at the interface as

$$(\tau_x^{(1)})_{y=1} = \left(\frac{du_0^{(1)}}{dy} \right)_{y=1} + \epsilon \left(\frac{du_1^{(1)}}{dy} \right)_{y=1} \cos \pi z, \quad (4.1)$$

$$(\tau_x^{(2)})_{y=0} = \left(\frac{du_0^{(2)}}{dy} \right)_{y=0} + \epsilon \left(\frac{du_1^{(2)}}{dy} \right)_{y=0} \cos \pi z, \quad (4.2)$$

$$(\tau_z^{(1)})_{y=1} = \epsilon \left(\frac{dw_1^{(1)}}{dy} \right)_{y=1} \sin \pi z, \quad (4.3)$$

$$(\tau_z^{(2)})_{y=0} = \epsilon \left(\frac{dw_1^{(2)}}{dy} \right)_{y=0} \sin \pi z. \quad (4.4)$$

Substituting the velocity components into equations (4.1) - (4.4) we have given the values of the skin friction τ_x and τ_z in the main flow and transverse direction at the moving plate and at the interface in table - 2 by taking $\epsilon = 0.50$ and $z = 0.50$. Table shows that the skinfriction in the main flow direction increases with the increase of R_0 at the plate and decreases with the increase of R_0 at the interface. The component of the skin friction in the transverse direction increases with the increase of R_0 at the plate as well as at the interface.

Table-1

The values of constants A_1, A_2, A_3, A, L, M and N for different values of R_e by taking $\alpha = 0.45$.

| R_e | $\sigma\alpha$ | $-A_1$ | $-A_2$ | $-A_3$ | A | $-L$ | M | N |
|-------|----------------|---------|----------|----------|-----------|---------|---------|---------|
| 0.50 | 7.3730 | 1877.71 | 7357.36 | 2649.32 | 4473.09 | 461.90 | -898.30 | 10.58 |
| 1.00 | 5.5836 | 2411.95 | 11303.01 | 4846.98 | 15873.40 | 880.80 | -985.98 | 7.35 |
| 1.50 | 4.6289 | 2656.25 | 12829.35 | 7768.94 | 36886.60 | 1387.96 | -652.13 | -60.10 |
| 2.00 | 4.1052 | 2846.69 | 14582.20 | 12340.97 | 75277.03 | 2075.60 | -235.65 | -221.37 |
| 2.50 | 3.7830 | 3004.99 | 16574.16 | 19617.10 | 143849.56 | 2910.21 | 249.61 | -506.12 |
| 3.00 | 2.5735 | 3140.03 | 18801.24 | 31316.91 | 264603.67 | 3967.62 | 787.30 | -945.94 |

Table-2

The values of Q and $(\tau_x^{(1)})_{y=1}, (\tau_x^{(2)})_{y=0}, (\tau_z^{(1)})_{y=1}, (\tau_z^{(2)})_{y=0}$ for $z = 0.50$ and $\epsilon = -0.50$.

| R_e | $\sigma\alpha$ | Q | $(\tau_x^{(1)})_{y=1}$ | $(\tau_x^{(2)})_{y=0}$ | $(\tau_z^{(1)})_{y=1}$ | $(\tau_z^{(2)})_{y=0}$ |
|-------|----------------|-------|------------------------|------------------------|------------------------|------------------------|
| 0.50 | 7.3730 | .0017 | 1.2312 | 0.5176 | 1.7910 | 0.0054 |
| 1.00 | 5.5836 | .0031 | 1.5337 | 0.5078 | 1.9226 | 0.0098 |
| 1.50 | 4.6289 | .0047 | 1.8792 | 0.2956 | 2.0848 | 0.0147 |
| 2.00 | 4.1052 | .0062 | 2.2634 | 0.2183 | 2.2204 | 0.0194 |
| 2.50 | 3.7830 | .0076 | 2.6793 | 0.1585 | 2.3733 | 0.0238 |
| 3.00 | 3.5735 | .0088 | 3.1199 | 0.1131 | 2.5296 | 0.0278 |

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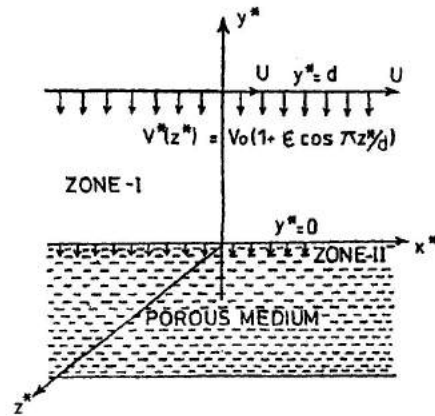
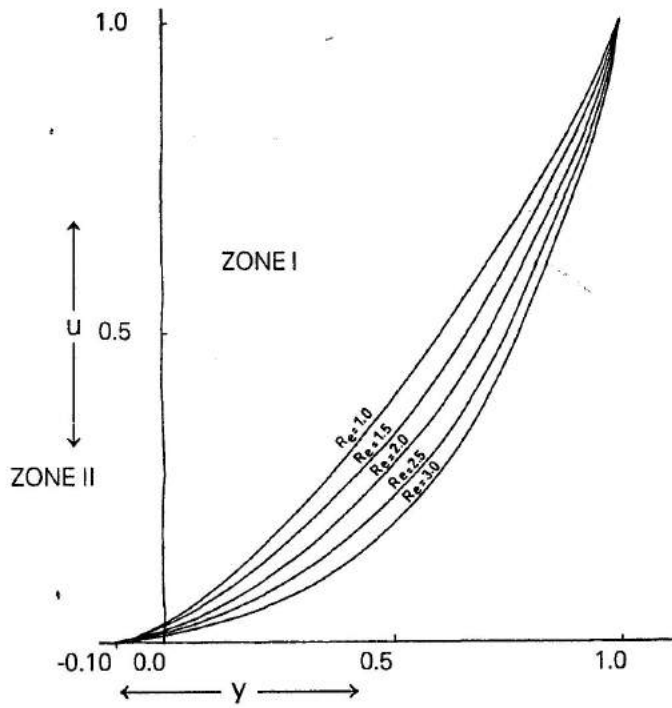


Figure 1 - Schematic of the problem.

Figure 2 - The graph of main flow velocity against y by taking $z = 0$, $\epsilon = 0.50$.

