J. T. S
Vol. 2 (2008), 101-113
https://doi.org/10.56424/jts.v2i00.9964

Finslerian Hypersurfaces of Special Matsumoto β- Change Metric

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(Received : January 2, 2008)

Abstract

In 1985 M. Matsumoto [8] studied the theory of Finslerian hypersurfaces and he has defined three kinds of hypersurfaces. After word many geometers [1], [2], [7], [9] etc. studied the geometry of Finslerian hypersurfaces given by β–change. The purpose of the present paper is to study the three kinds of the hyperplanes and Matsumoto change of Finsler metric.

1. Introduction

In 1984 C. Shibata introduced the transformation of Finsler metric [6] and he has also dealt with a change of Finsler metric which is called β-change. The differential one form β-play very important role. M. Matsumoto investigated three types of hyperplanes, they are called hyperplanes of the first kind, second kind and third kind. In the present paper using the field of linear frame ([2], [7], [9]), we shall consider Finslerian hypersurfaces given by a special Matsumoto change of Finsler metric.

Let $F^n = (M^n, L)$ be an n-dimensional Finsler space equipped with fundamental function $L(x, y)$ on M^n, where M^n be an n-dimensional smooth manifold and let $F^{*n} = (M^n, L^*)$ be another Finsler space, whose metric function $L^*(x, y)$ is defined as :

$$L^*(x, y) = f(L, \beta), \tag{1.1}$$

where f is positively homogeneous of degree one in L and β and $\beta = b_i(x)\, y^i$, $b_i(x)$ are components of a covariant vector in (M^n, L). A special type of Matsumoto metric is given by

$$L^*(x, y) = A_1 [L + (\beta^2/L)] + A_2[L^2/(L - \beta)], \tag{1.2}$$

where A_1 and A_2 are constants and $L^2 = a_{ij}(x)\, y^i y^j$. In which a_{ij} is a Riemannian metric.

If $A_1 = 0$ then the metric defined in (1.2) is homothetic to Matsumoto metric and if $A_2 = 0$, then metric (1.2) is homothetic to special (α, β) metric. Thus the Finsler space with metric (1.2) is the generalization of Matsumoto space as well as the Finsler space with a special (α, β) metric. If $L(x, y)$ reduces to the metric function of a Matsumoto space, then the transformation (1.2) has been called the special Matsumoto change of Finsler metric.

Our aim is to investigate to some relations between original Finslerian hypersurfaces and another Finslerian hypersurfaces given by the Matsumoto β–change of Finsler metric under certain conditions.

2. Preliminaries

Let $F^n = (M^n, L)$ be an n-dimensional Finsler space whose metric function is $L(x, y)$ on M^n, where M^n be an n-dimensional smooth manifold. The metric tensor $g_{ij}(x, y)$ and Cartan's C-tensor $C_{ijk}(x, y)$ on F^n are defined by :

$$g_{ij} = \frac{1}{2}\frac{\partial L^2}{\partial y^i \partial y^j}, \qquad C_{ijk} = \frac{1}{2}\frac{\partial g_{ij}}{\partial y^k},$$

respectively.

The connection $C\Gamma = (F^i_{jk}, N^i_j, C^i_{jk})$ introduced the Cartan's connection along F^n. A hypersurface M^{n-1} represented by the equation $x^i = x^i(u^\alpha)$, where u^α is Gaussian coordinates on M^{n-1} and greek indices vary from 1 to $n-1$. As for matrix we assumed that the projection factor $B^i_\alpha = \partial x^i / \partial y^\alpha$ is of rank $n-1$. It is to be noted that $B^i_{0\beta} = v^\alpha B^i_{0\beta}$ and y^i be the supporting element at a point (u^α) along M^{n-1} is tangential to M^{n-1} where $y^i = B^i_\alpha(u)\, v^\alpha$, i.e. v^α is the supporting element of M^{n-1} at the point u^α. We get a Finsler space $F^{n-1} = (M^{n-1}, \underline{L}(u, v))$ of $n-1$ dimension, where $\underline{L}(u, v) = L(x(u), y(u, v))$ along M^{n-1}.

The unit normal vector $N^i(u, v)$ at each point (u^α) of F^{n-1} is given by

$$g_{ij} B^i_\alpha N^i_j = 0, \qquad g_{ij} N^i N^j = 1. \tag{2.1}$$

If (B^{α}_{i}, N_{i}) is the inverse matrix of (B^{i}_{α}, N^{i}), then

$$B^{i}_{\alpha} B^{\beta}_{i} = \delta^{\beta}_{\alpha}, \qquad B^{i}_{\alpha} N_{i} = 0, \qquad N^{i} N_{j} = 1 \tag{2.2}$$

and $$B^{i}_{\alpha} B^{\alpha}_{j} + N^{i} N_{j} = \delta^{i}_{j}.$$

The inverse matrix $(g^{\alpha\beta})$ of $(g_{\alpha\beta})$, gives the following relations

$$B^{\alpha}_{i} = g^{\alpha\beta} g_{ij} B^{j}_{\beta}, \qquad N_{i} = g_{ij} N^{j}. \tag{2.3}$$

The second fundamental h-tensor $H_{\alpha\beta}$ and the normal curvature vector H_{α} of the induced Cartan's connection $C\Gamma = (F^{\alpha}_{\beta\gamma}, N^{\beta}_{\alpha}, C^{\alpha}_{\beta\gamma})$ on F^{n-1} are defined as [8]

$$H_{\alpha\beta} = N_{i} (B^{i}_{\alpha\beta} + F^{i}_{jk} B^{j}_{\alpha} B^{k}_{\beta}) + M_{\alpha} H_{\alpha}, \tag{2.4}$$

$$H_{\alpha} = N_{i} (B^{i}_{0\alpha} + N^{i}_{j} B^{j}_{\alpha}),$$

where

$$M_{\alpha} = C_{ijk} B^{i}_{\alpha} N^{j} N^{k}. \tag{2.5}$$

The contraction of $H_{\alpha\beta}$ by v^{α} is defined as $H_{\alpha\beta} v^{\alpha} = H_{\beta}$. The second fundamental v-tensor $M_{\alpha\beta}$ is given by [10]

$$M_{\alpha\beta} = C_{ijk} B^{i}_{\alpha} B^{j}_{\beta} N^{k}. \tag{2.6}$$

3. Matsumoto β–change of Finsler Space

Let $F^{n} = (M^{n}, L)$ be an n-dimensional Finsler space with a fundamental function $L(x, y)$ and $b_{i}(x)$ be one-form on M^{n}. We shall consider a function $L^{*}(x, y) > 0$ on M^{n} by the relation (1.2).

The following results are used to find the metric tensor g^{*}_{ij}, the angular metric tensor h^{*}_{ij} and the Cartan's C-tensor C^{*}_{ijk} along the $F^{*n} = (M^{n}, L)$.

$$\partial\beta / \partial y^{i} = b_{i}, \qquad \partial L / \partial y^{i} = l_{i}, \qquad \partial l_{j} / \partial y^{i} = L^{-1} h_{ij}, \tag{3.1}$$

where h_{ij} are components of angular metric tensor of F^n defined as :

$$h_{ij} = g_{ij} - l_i\, l_j = L\,(\partial^2 L / \partial y^i\, \partial y^j\,).$$

Differentiating (1.2) with respect to y^i, we get

$$l_i^* = [A_1\{1-(\beta^2/L^2)\} + A_2\,\{(L^2-2\beta)/(L-\beta^2)\}]\,l_i + [A_1\,(2\beta/L) + A_2\{L^2/(L-\beta)^2]\,b_i. \tag{3.2}$$

Remark 1. If $A_1 = 0$ and $A_2 = 1$, then the Finsler space with the metric (1.2) becomes Matsumoto space.

Remark 2. If $A_2 = 0$ and $A_1 = 1$, then the Finsler space with the metric (1.2) becomes special (α, β) metric.

In view of remark (1) and (2) the relation (3.2) takes the following form :

$$l_i^* = [(L^2-2\beta)/(L-\beta^2)]\,l_i + [L^2/(L-\beta)^2]\,b_i. \tag{3.2)(a}$$

and

$$l_i^* = [1-(\beta^2/L^2)]\,l_i + (2\beta/L)\,b_i. \tag{3.2)(b}$$

To obtain angular metric tensor, we differentiate (3.2) with respect to y^j which as follows :

$$h_{ij}^* = P_0\,h_{ij} + Q_0\,[l_i\,l_j - (L/\beta)\,(l_i\,b_j + l_j\,b_i) + (L^2/\beta^2)\,b_i\,b_j], \tag{3.3}$$

where

$$P_0 = P_1 + P_2 + P_3\,,$$

$$P_1 = A_1^2\,[(L^4-\beta^4)/L^4],$$

$$P_2 = A_2^2\,[L^2(L-2\beta)\,/(L-\beta)^3],$$

$$P_3 = A_1\,A_2\,[(2L^3-\beta^3-3L^2\beta)/L\,(L-\beta)^2],$$

$$Q_0 = Q_1 + Q_2 + Q_3\,,$$

$$Q_1 = A_1^2\,[2\beta^2\,(L^2+\beta^2)/L^4],$$

$$Q_2 = A_2^2\,[2\beta^2\,L^2/(L-\beta)^4],$$

$$Q_3 = A_1\, A_2\, [4\beta^2 (L^2 + \beta^2 - L\beta)/L\, (L - \beta)^3].$$

In view of remark (1) and (2) the angular metric tensor h^*_{ij} for Matsumoto space and special (α, β) metric space are respectively given by :

$$h^*_{ij} = P^*_2\, h_{ij} + Q^*_2\, [l_i\, l_j - (l/\beta)\,(l_i\, b_j + l_j\, b_i) + (L^2/\beta^2)\, b_i\, b_j], \qquad (3.3)(a)$$

where

$$P^*_2 = [L^2(L - 2\beta)\,/(L - \beta)^3],$$

$$Q^*_2 = [2\beta^2\, L^2/(L - \beta)^4]$$

and

$$h^*_{ij} = P^*_1\, h_{ij} + Q^*_1\, [l_i\, l_j - (l/\beta)\,(l_i\, b_j + l_j\, b_i) + (L^2/\beta^2)\, b_i\, b_j], \qquad (3.3)(b)$$

where

$$P^*_1 = [(L^4 - \beta^4)\,/L^4],$$

$$Q^*_1 = [2\beta^2\, (L^2 + \beta^2)/L^4].$$

From (3.2) and (3.3), we have

$$g^*_{ij} = P_0\, g_{ij} + B_0\, [(\beta/L)\, l_i\, l_j - (l_i\, b_j + l_j\, b_i)] + C_0\, b_i\, b_j \qquad (3.4)$$

where

$$B_0 = B_1 + B_2 + B_3\,,$$

$$B_1 = A_1^2\ (4\beta^3/L^3\,),$$

$$B_2 = A_2^2\, [L^3(4\beta - L)\,/(L - \beta)^4],$$

$$B_3 = A_1\, A_2\, [(3\beta L^2 + 3\,\beta^2 L - L^3 - \beta^3)/\,(L - \beta)^3],$$

$$C_0 = C_1 + C_2 + C_3\,,$$

$$C_1 = A_1^2\ [2\,(L^2 + 3\beta^2)/L^2],$$

$$C_2 = A_2^2\ [3L^4/(L - \beta)^4],$$

$$C_3 = A_1\, A_2\, [4L^3 /(L-\beta)^3].$$

According to remark (1) and (2) the metric tensor g^*_{ij} for Matsumoto space and special (α, β) space are respectively defined as :

$$g^*_{ij} = P^*_2\, g_{ij} + B^*_2\, [(\beta/L)\, l_i\, l_j - (l_i\, b_j + l_j\, b_i)] + C^*_2\, b_i\, b_j \qquad (3.4)(a)$$

where

$$B^*_2 = [L^3(4\beta - L) /(L-\beta)^4],$$

$$C^*_2 = [3L^4 /(L-\beta)^4]$$

and

$$g^*_{ij} = P^*_1\, g_{ij} + B^*_1\, [(\beta/L)\, l_i\, l_j - (l_i\, b_j + l_j\, b_i)] + C^*_1\, b_i\, b_j \qquad (3.4)(b)$$

where

$$B^*_1 = (4\beta^3 /L^3),$$

$$C^*_1 = [2\,(L^2 + 3\beta^2) /L^2].$$

Differentiating (3.4) with respect to y^k and using (3.1), we have the following result :

$$C^*_{ijk} = P_0\, C_{ijk} - R_0\, (h_{ij}\, m_k + h_{jk}\, m_i + h_{ki}\, m_j) + S_0\, (m_i\, m_j\, m_k), \qquad (3.5)$$

where

$$R_0 = R_1 + R_2 + R_3,$$

$$R_1 = A^2_1\, (2\beta/L^4),$$

$$R_2 = A^2_2\, [L^2(4\beta - L) /2(L-\beta)^4],$$

$$R_3 = A_1\, A_2\, [(3\beta L^2 + 3\,\beta^2 L - L^3 - \beta^3)/ 2L(L-\beta)^3],$$

$$S_0 = S_1 + S_2 + S_3,$$

$$S_1 = A^2_1\, (6\beta/L^2),$$

$$S_2 = A^2_2\, [6L^4/(L-\beta)^5],$$

$$S_3 = A_1 A_2 [6L^3 /(L - \beta)^4],$$

$$m_i = b_i - (\beta/L)\, l_i.$$

Beside these :

$$m_i\, l^i = 0, \qquad m_i\, b^i = b^2 - (\beta^2/L^2), \qquad h_{ij}\, l^j = 0, \qquad h_{ij}\, m^j = h_{ij}\, b^j = m_i$$

and
$$m^i = g^{ij}\, m_j = b^i - (\beta/L)\, l^i. \tag{3.6}$$

Inview of remark (1) and (2) the Cartan's C-tensor C_{ijk} for Matsumoto space and special (α, β) metric yields

$$C^*_{ijk} = P^*_2\, C_{ijk} - R^*_2\, (h_{ij}\, m_k + h_{jk}\, m_i + h_{ki}\, m_j) + S^*_2\, (m_i\, m_j\, m_k), \tag{3.7}$$

where

$$R^*_2 = [L^2(4\beta - L)/2(L - \beta)^4],$$

$$S^*_2 = [6L^4 /(L - \beta)^5]$$

and

$$C^*_{ijk} = P^*_1\, C_{ijk} - R^*_1\, (h_{ij}\, m_k + h_{jk}\, m_i + h_{ki}\, m_j) + S^*_1\, (m_i\, m_j\, m_k), \tag{3.8}$$

where

$$R^*_1 = (2\beta /L^4),$$

$$S^*_1 = (6\beta /L^2).$$

4. Hypersurfaces due to Special Matsumoto β–change

Let $F^{n-1} = (M^{n-1}, \underline{L}(u, v))$ be a Finslerian hypersurface along the F^n and $F^{*n-1} = (M^{n-1}, \underline{L}^*(u, v))$ be another Finslerian hypersurface along the F^{*n} introduced by the Matsumoto change. Let (B^α_i, N_i) be the inverse matrix of (B^i_α, N^i), where N^i be the unit normal vector at each point of F^{*n-1}. The function B^i_α be tangent vector as component of $(n - 1)$ linearly independent along F^{n-1} and B^i_α are invariant function under Matsumoto change. Thus we shall show that a unit normal vector $N^*(u, v)$ of F^{*n-1} is uniquely determined by :

$$g^*_{ij}\ B^i_\alpha\ N^{*j} = 0 \qquad \text{and} \qquad g^*_{ij}\ N^{*i} N^{*j} = 1.$$

On account of (2.1) and $l_i N^i = 0$, the multiplication of (3.4) by $N^i N^j$ yields

$$g^*_{ij}\ N^i N^j = P_0 + C_0 (b_i N^i)^2. \tag{4.1)(a}$$

From above, we have

$$g^*_{ij} [\pm N^i/\surd\{P_0 + C_0 (b_i N^i)^2\}] \times [\pm N^i/\surd\{P_0 + C_0 (b_i N^i)^2\}] = 1. \tag{4.1)(b}$$

Therefore, we can put

$$N^{*i} = N^i/\surd\{P_0 + C_0 (b_i N^i)^2\}, \tag{4.2}$$

we have choosen only positive sign.

In view of equation (3.1), (3.4), (4.2) and from (4.1), we have

$$[C_0 b_i B^i_\alpha - B_0 l_i B^i_\alpha] \times [b_i N^i/\surd\{P_0 + C_0 (b_i N^i)^2\}] = 0. \tag{4.3}$$

If $[C_0 b_i B^i_\alpha - B_0 l_i B^i_\alpha]$, then contracting it by v^α and using $y^i = B^i_\alpha\ v^\alpha$, we get $L = 0$, which is a contradiction with assumption that $L > 0$. Hence $b_i N^i = 0$. Therefore (4.2) becomes

$$N^{*i} = (N^i/\surd P_0). \tag{4.4}$$

Note : Where P_0 and C_0 are already defined in the section (3.3). Summarising the above, we have

Proposition 4.1. For a field of linear frame $(B^i_1, B^i_2, \ldots\ldots, B^i_{n-1}, N^i)$ of F^n there exists a field of linear frame $(B^i_1, B^i_2, \ldots\ldots, B^i_{n-1}, N^{*i} = (N^i/\surd P_0))$ of F^{*n} such that (4.1) is satisfied along F^{*n-1} and then b_i is tangential to both the hypersurfaces F^{n-1} and F^{*n-1}.

In view of remark (1), the relation (4.1)(a), (4.1)(b), (4.2), (4.3) and (4.4) become

$$g^*_{ij}\ N^i N^j = P^*_2 + C^*_2\ (b_i N^i)^2,$$

$$g^*_{ij}\,[\pm N^i/\surd\{P^*_2 + C^*_2\,(b_i N^i)^2\}] \times [\pm N^i/\surd\{P^*_2 + C^*_2\,(b_i N^i)^2\}] = 1,$$

$$N^{*i} = N^i/\surd\{P^*_2 + C^*_2\,(b_i N^i)^2\}, \tag{4.5}$$

$$[C^*_2\, b_i B^i_\alpha - B^*_2\, l_i B^i_\alpha] \times [b_i N^i/\surd\{P^*_2 + C^*_2\,(b_i N^i)^2\}] = 0 \tag{4.6}$$

and

$$N^{*i} = (N^i/\surd P^*_2). \tag{4.7}$$

Note : P^*_2 and C^*_2 are already defined in the section (3.3)(a).

From the above the proposition (4.1) inverted for Matsumoto space which is as follows :

Proposition 4.2. For a field of linear frame $(B^i_1, B^i_2, \ldots\ldots, B^i_{n-1}, N^i)$ of F^n there exists a field of linear frame $(B^i_1, B^i_2, \ldots\ldots, B^i_{n-1}, N^{*i} = (N^i/\surd P^*_2))$ of F^{*n} such that (4.1) is satisfied along F^{*n-1} and then b_i is tangential to both the hypersurfaces F^{n-1} and F^{*n-1}.

Similarly in view of remark (2), the relation (4.1)(a), (4.1)(b), (4.2), (4.3) and (4.4) become

$$g^*_{ij}\, N^i N^j = P^*_1 + C^*_1\,(b_i N^i)^2,$$

$$g^*_{ij}\,[\pm N^i/\surd\{P^*_1 + C^*_1\,(b_i N^i)^2\}] \times [\pm N^i/\surd\{P^*_1 + C^*_1\,(b_i N^i)^2\}] = 1,$$

$$N^{*i} = N^i/\surd\{P^*_1 + C^*_1\,(b_i N^i)^2\}, \tag{4.8}$$

$$[C^*_1\, b_i B^i_\alpha - B^*_1\, l_i B^i_\alpha] \times [b_i N^i/\surd\{P^*_1 + C^*_1\,(b_i N^i)^2\}] = 0 \tag{4.9}$$

and

$$N^{*i} = (N^i/\surd P^*_1). \tag{4.10}$$

Note : P^*_1 and C^*_1 are already defined in the section (3.3)(b).

From the above the proposition (4.1) becomes Finsler space with special (α, β) metric which as follows :

Proposition 4.3. For a field of linear frame $(B_1^i, B_2^i, \ldots\ldots, B_{n-1}^i, N^i)$ of F^n there exists a field of linear frame $(B_1^i, B_2^i, \ldots\ldots, B_{n-1}^i, N^{*i} = (N^i/\sqrt{P_1^*}))$ of F^{*n} such that (4.1) is satisfied along F^{*n-1} and then b_i is tangential to both the hypersurfaces F^{n-1} and F^{*n-1}.

Making use of inverse matrix $(g^{*\alpha\beta})$ of $(g^*_{\alpha\beta})$, we may write the quantities $B_i^{*\alpha}$ of F^{n-1} by

$$B_i^{*\alpha} = g^{*\alpha\beta} g^*_{ij} B^j_\beta .$$

If $(B_i^{*\alpha}, N_i^*)$ be the inverse matrix of (B^i_α, N^i), then we get $B^i_\alpha B_i^{*\beta} = \delta^\beta_\alpha$, $B^i_\alpha N_i^* = 0$, $N^{*i} N_i^* = 1$ and $B^i_\alpha B_j^{*\alpha} + N^{*i} N_j^* = \delta^i_j$, we also get $N_i^* = g^*_{ij} N^{*j}$ which is on account of (3.2), (3.4) and (4.4) gives

$$N_i^* = (\sqrt{P_0}) N_i. \tag{4.11}$$

We define the Cartan's connection by $(F^i_{jk}, N^i_j, C^i_{jk})$ along F^n and $(F^{*i}_{jk}, N^{*i}_j, C^{*i}_{jk})$ along F^{*n} and let D^i_{jk} called the difference tensor which is defined as

$$D^i_{jk} = F^{*i}_{jk} - F^i_{jk},$$

$$D^i_{jk} = A_{jk} b^i - B_{jk} l^i, \tag{4.12}$$

where b_i is the vector field along F^n and A_{jk} and B_{jk} are components of a symmetric covariant tensors of second order. Since $N_i b^i = 0$ and $N_i l^i = 0$, contracting (4.12) by N_i, we get $N_i D^i_{jk} = 0$ and $N_i D^i_{0k} = 0$.

From (2.4) and (4.11), we have

$$H^*_\alpha = (\sqrt{P_0}) H_\alpha. \tag{4.13}$$

According to remark (1) and by using (3.2)(a), (3.4)(a) and (4.7), we have

$$N_i^* = (\sqrt{P_2^*}) N_i. \tag{4.14}$$

From (2.4) and (4.14), we have

$$H^*_\alpha = (\sqrt{P^*_2})\, H_\alpha. \tag{4.15}$$

Again in view of remark (2) and by using (3.2)(b), (3.4)(b) and (4.10), we have

$$N^*_i = (\sqrt{P^*_1})\, N_i. \tag{4.16}$$

From (2.4) and (4.14), we have

$$H^*_\alpha = (\sqrt{P^*_1})\, H_\alpha. \tag{4.17}$$

If each path of a hypersurface F^{n-1} with respect to the induced connection is also a path of enveloping space F^n, then F^{n-1} is called a hyperplane of the first kind [8]. A hyperplane of the first kind is characterized by $H_\alpha = 0$. From (4.13), (4.15) and (4.17), we have $H^*_\alpha = 0$ when $H_\alpha = 0$. Hence, we have

Theorem 4.1. If $b_i(x)$ be a vector field $F^n = (M^n, L)$ satisfying (4.6), then a hypersurface F^{n-1} is a hyperplane of the first kind if and only if the hypersurface F^{*n-1} is a hyperplane of the first kind.

This theorem holds good for Matsumoto space as well as Finsler space with special (α, β) metric.

From (2.5), (3.5), (4.4) and by using $m_i N^i = 0$, $h_{jk} N^j N^k = 1$ and $h_{ij} B^i_\alpha N^j = 0$, we have

$$M^*_\alpha = M_\alpha - [B_0 (m_i B^i_\alpha)/P_0 (2L)]. \tag{4.18}$$

Again from (2.4), (4.11), (4.12), (4.13) and (4.18), we get

$$H^*_{\alpha\beta} = \sqrt{P_0}\, [H_{\alpha\beta} - \{B_0 (m_i B^i_\alpha\, H_\beta)/P_0 (2L)\}]. \tag{4.19}$$

In view of remark (1), from (2.5), (3.7), (4.7) and by using $m_i N^i = 0$, $h_{jk} N^j N^k = 1$ and $h_{ij} B^i_\alpha N^j = 0$, we have

$$M^*_\alpha = M_\alpha - [B^*_2 \ (m_i B^i_\alpha)/P^*_2 \ (2L)]. \qquad (4.20)$$

Again from (2.4), (4.12), (4.14), (4.15) and (4.20), we get

$$H^*_{\alpha\beta} = \sqrt{P^*_2} \ [H_{\alpha\beta} - \{B^*_2 \ (m_i B^i_\alpha \ H_\beta)/P^*_2 \ (2L)\}]. \qquad (4.21)$$

In view of remark (2), from (2.5), (3.8), (4.10) and by using $m_i N^i = 0$, $h_{jk} N^j N^k$ and $h_{ij} B^i_\alpha N^j = 0$, we have

$$M^*_\alpha = M_\alpha - [B^*_1 \ (m_i B^i_\alpha)/P^*_1 \ (2L)]. \qquad (4.22)$$

On solving (2.4), (4.12), (4.16), (4.17) and (4.22), we get

$$H^*_{\alpha\beta} = \sqrt{P^*_1} \ [H_{\alpha\beta} - \{B^*_1 \ (m_i B^i_\alpha \ H_\beta)/P^*_1 \ (2L)\}]. \qquad (4.23)$$

If each h-path of a hypersurface F^{n-1} with respect to the induced connection is also a h-path of enveloping space F^n, then F^{n-1} is called a hyperplane of the second kind [8]. A hyperplane of the second kind is characterized by $H_{\alpha\beta} = 0$. Since $H_{\alpha\beta} = 0$ implies that $H_\alpha = 0$, from (4.13), (4.15), (4.17) and (4.19), (4.21), (4.23) we have the following :

Theorem 4.2. If $b_i(x)$ be a vector field in $F^n = (M^n, L)$ satisfying (4.6), then a hypersurface F^{n-1} is a hyperplane of the second kind if and only if the hypersurface F^{*n-1} is a hyperplane of the second kind.

The above theorem holds good for Matsumoto space as well as Finsler space with special (α, β) metric.

From (2.6), (3.5) and (4.4), we have

$$M^*_{\alpha\beta} = (\sqrt{P_0}) \ M_{\alpha\beta}. \qquad (4.24)$$

According to remark (1) and by using (2.6), (3.7) and (4.7), we have

$$M^*_{\alpha\beta} = (\sqrt{P^*_2}) \ M_{\alpha\beta}. \qquad (4.25)$$

Similarly in view of remark (2) and by using (2.6), (3.8) and (4.10), we have

$$M^*_{\alpha\beta} = (\sqrt{P^*_1})\ M_{\alpha\beta}. \tag{4.26}$$

If the unit normal vector of F^{n-1} is parallel along each curve of F^{n-1}, the F^{n-1} is called a hyperplane of the third kind [8]. A hyperplane of the third kind is characterized by $H_{\alpha\beta} = 0$, $M_{\alpha\beta} = 0$. Hence from (4.13), (4.15), (4.17), (4.19), (4.21), (4.23) and (4.24), (4.25), (4.26), we have

Theorem 4.3. If $b_i(x)$ be a vector field in $F^n = (M^n, L)$ satisfying (4.6), then a hypersurface F^{n-1} is a hyperplane of the third kind if and only if the hypersurface F^{*n-1} is a hyperplane of the third kind.

Thus we have shown that a special Matsumoto change make three types of hypersurfaces invariant under certain conditions.

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