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Emmission of Energy from Schwarzschild Black Holes

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Abstract

Induced energy is obtained in the exterior Schwarzschild space-time through one-loop quantum correction to scalar field. It is discussed that energy tunnels quantum mechanically through the event horizon contrary to results of classical mechanics that black-holes cannot emit anything. Three decades ago, Hawking had obtained this type of result through the process of creation of scalar particles. Evaporation of energy from black-hole results into loss of mass. It is found that a primordial black-hole of mass 10^{15} gm might have evaporated by now. Moreover, luminosity of hole also increases gradually. It is suggested that decreasing area of the event horizon due to loss of mass causes increase in entropy in the outer space of the hole. It is speculated that black-holes in the extreme past might be glowing bright now and will leave behind dark naked singularities.

Keywords : Black-holes, one-loop quantum correction, vacuum energy, luminosity.

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1. Introduction

After exhausting their nuclear fuel, stars pass through the phase of gravitational collapse, if certain initial conditions are satisfied. When collapse is complete, there remains a singularity hidden from the view of an external observer. This hidden object is called black-hole. There are two school of thoughts about the existence of black holes. One believes that black holes do not exist as so far attempts to observe them have yielded null results and these are only mathematical solutions of Einstein's field equations having spatial singularities. Physicists belonging to other school say that black holes are not mathematical solutions only, but they do exist physically. We are unable to observe them because they trap particles going near to them and absorb, but they do not emit anything.

If one believes in the second school of thought, it is natural to investigate the effect of their existence in the universe. One such attempt was made by S. W. Hawking. In 1974, he published his result shaking the old belief based on the classical mechanics that black holes absorb everything but emit nothing [1].

According to Hawking [1-2], black holes are not completely black, but emit radiation with thermal spectrum due to quantum effects. He obtained that virtual pairs of particles and anti-particles are created in the external space-time of the black hole with frequency $\omega \leq (GM)^{-1}$ (G is the Newtonian gravitational constant and M is the mass of the black hole). The gravitational tidal force between particles and anti-particles prevent them from re-annihilation. Particles with positive energy escape to infinity contributing to Hawking's flux and particles with negative energy (anti-particles with positive energy) are trapped in the black hole. Thus black holes radiate quanta of energy with frequency ω.

In what follows, it is obtained that energy is radiated out of the black hole through quantum process, but the approach to obtain this result is different from Hawking's one i.e. creation of particles in the external space-time of the hole. The line-element for the Swartzschild space-time is taken as

$$dS^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\psi^2\right), \qquad (1.1)$$

where $2GM < r < \infty$, $0 \leq \theta \leq \pi$ and $0 \leq \psi \leq 2\pi$. Like Hawking, here also a massless hermitan scalar field ϕ is considered in the manifold with the distance function, given by equation (1.1). The action for ϕ is written as

$$S_\phi = \frac{1}{2} \int d^4 x \sqrt{|g(4)|} \; g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi, \qquad (1.2)$$

where $\mu, \nu \equiv (0, 1, 2, 3)$, $g_{\mu\nu}$ are components of the metric tensor and g(4) their determinant in 4-dim. spacetime.

Further the entire problem is reduced to 3-dimensional space-time and a spectrum of infinite number of massive scalar fields is obtained. One-loop quantum correction is calculated and summed up over all scalar fields yielding 3-dim. induced gravity. It is possible to do so, because all 3-dim. scalar fields in the spectrum are identical in nature. Through this process, finite results are obtained without renormalization, it is because divergences do not appear in odd-dimensional space-times. It is explained below that the 3-dim. induced action for gravity can be converted into 4-dim. induced action for gravity in the space-time given by equation (1.1). As a result, one obtains induced vacuum energy. It is interpreted that induced vacuum energy in the exterior Swartzschild space-time is radiated from the black-hole itself as there exists no other source of energy in this space-time. It is important to mention that results, obtained here, do not contradict Hawking's results. Section 2 contains one-loop quantum correction to scalar fields in 3-dim. hypersurface. 4-dim. induced Einstein-Hilbert action and vacuum energy are obtained in section 3. Section 4 is the concluding section, where results are discussed and compared with Hawking.

Natural units $k_B = \hbar = c = 1$ (where k_B is Boltzman's constant, $\hbar$ is Plank's constant divided by 2π and c is the speed of light), are used here. In this system, GeV is the fundamental unit, but sometimes c.g.s. system is needed to get better understanding. So conversion scales are given as $1\text{GeV} = 1.16 \times 10^{13}\ {}^{o}\text{K} = 1.78 \times 10^{-24}$ gm. and $1\text{GeV}^{-1} = 1.97 \times 10^{-14}$ cm. $= 6.58 \times 10^{-25}$ sec.

2. Scalar field and one-loop quantum correction

The equation (1.2) can also be written as

$$S_\phi = -\frac{1}{2}\int d^4x \sqrt{|g(4)|}\ \phi^* \Box_{(4)} \phi, \tag{2.1}$$

where $\Box_{(4)} = (\sqrt{|g(4)|}\,)^{-1} \frac{\partial}{\partial x^\mu}(\sqrt{|g(4)|}\, g^{\mu\nu} \frac{\partial}{\partial x^\nu})$.

The conformal transformations

$$g_{\mu\nu} \to \bar{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \tag{2.2a}$$

with $\Omega = (r \sin\theta)^{-1}$ yield

$$\bar{g}_{\mu\nu} \equiv \left[(r \sin\theta)^{-2}\left(1 - \frac{2GM}{r}\right), -(r\sin\theta)^{-2}\left(1-\frac{2GM}{r}\right)^{-1}, -(\sin\theta)^{-2}, -1 \right] \tag{2.2b}$$

Using transformations (2.2), equation (2.1) is re-written as

$$S_\phi = -\frac{1}{2}\int d^4x \sqrt{|g(4)|}\; \bar{\phi}^* \,\Box'_{(4)} + m_0^2)\,\bar{\phi}, \tag{2.3a}$$

where

$$m_0^2 = 6\sin^2\theta - \frac{18\,GM}{r}\sin^2\theta - 3\cos\theta - \frac{3\cos\theta}{r} + \frac{18}{r^4}\sin^2\theta\left(1-\frac{2GM}{r}\right) + 18\cot^2\theta\,\mathrm{cosec}^2\theta$$

and $\Box'_{(4)}$ is $\Box_{(4)}$ with $g_{\mu\nu}$ replaced by $\bar{g}_{\mu\nu}$.

Using m_0, given by equation (2.3b), one obtains

$$\frac{m_0^2}{\sin^2\theta} = \frac{|m_0|^2}{\sin^2\theta} = \frac{|m_0^2|}{\sin^2\theta}$$

$$= \left| 6 - \frac{18GM}{r} - \left(3+\frac{3}{r}\right)\frac{\cos\theta}{\sin^2\theta} + \frac{18}{r^4}\left(1-\frac{2GM}{r}\right) + 18\frac{\cos^2\theta}{\sin^4\theta} \right|$$

$$\geq 6 - \frac{18GM}{r} + \frac{18}{r^4}\left(1-\frac{2GM}{r}\right), \tag{2.3c}$$

which shows a strong possibility for m_0 to be imaginary when $r < 3GM$ which is inconsistent as m_0 being mass of 3-dim. scalar field can not be imaginary. For example, eq. (2.3b) shows that m_0 is definitely imaginary when $r < 3GM$ at the hyper-surface $\theta = \pi/2$. So to be on the safe side $r < 3GM$ should be avoided while using these results.

Now $\bar{\phi}$ is decomposed as

$$\bar{\phi} = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \bar{\phi}_{(3)n}(t, r, \theta)\, e^{in\psi}. \tag{2.4}$$

Connecting equations (2.3) and (2.4)

$$S_\phi = -\frac{1}{4\pi}\int d^4x\sqrt{|\bar{g}(4)|}\sum_{n,n'=-\infty}^{\infty}\bar{\phi}^*(3)\,n'$$

$$[\Box_{(3)} + n^2 + m_0^2]\,\bar{\phi}_{(3)n} \times \exp\{-i(n-n')\psi\}. \tag{2.5}$$

Integration over ψ and summation over n' leads equation (2.5) to

$$S_\phi = -\frac{1}{2}\sum_{n=-\infty}^{\infty}\int d^3x\sqrt{|g(3)|}\ \ \bar{\phi}^*_{(3)n}\,[\Box_{(3)} + m_n^2]\,\bar{\phi}_{(3)n} \tag{2.6}$$

with $m_n^2 = n^2 + m_0^2$. Scalar fields $\phi_{(3)n}$ live in the ψ = constant hypersurface with the line-element

$$dS^2_{(3)} = (r\sin\theta)^{-2}\left(1-\frac{2GM}{r}\right)dt^2 - (r\sin\theta)^{-2}\left(1-\frac{2GM}{r}\right)^{-1}dr^2 - (\sin\theta)^{-2}\,d\theta^2 \tag{2.7}$$

To compute one-loop quantum correction to scalar fields $\phi_{(3)n}$, operator regularization method [4] is employed. One-loop effective action upto adiabatic order 4 is given as

$$\Gamma = (4\pi)^{-3/2}\sum_{n=-\infty}^{\infty}\frac{d}{ds}\left[\frac{\Gamma\left(s+\frac{1}{2}\right)}{\Gamma(s)}\left(\frac{\mu^2}{m_n^2}\right)^s\int d^3x\sqrt{|\bar{g}(3)|}\times\left\{\frac{m_n^3}{\left(s-\frac{3}{2}\right)\left(s-\frac{1}{2}\right)} + \frac{m_n}{G\left(s-\frac{1}{2}\right)} + m_n^{-1}\left(\frac{1}{30}\Box_{(3)}\bar{R}_{(3)} + \frac{1}{180}\bar{R}^{ijkl}_{(3)}\bar{R}_{(3)ijkl} - \frac{1}{180}\bar{R}^{ijkl}_{(3)}\bar{R}_{(3)ij} + \frac{1}{72}\bar{R}_{(3)}\right)\right\}\right]\Bigg|_{s=0}, \tag{2.8}$$

where i, j, k, $l \equiv (0, 1, 2)$. In equation (2.8) $\Gamma(s)$ can be expanded as

$$\Gamma(s) = \frac{1}{s} - \gamma + O(s), \tag{2.9}$$

with γ as Euler's constant. Using eq. (2.9) in eq. (2.8)

$$\Gamma = (24\pi)^{-1}\sum_{n=-\infty}^{\infty}\int d^3x\sqrt{|\bar{g}(3)|}\ \{4m_n^3 - m_n\bar{R}_{(3)} +$$

$$+ m_n^{-1}\left(\frac{1}{30}\Box_{(3)}\overline{R}_{(3)} + \frac{1}{180}\overline{R}^{ijkl}_{(3)}\overline{R}_{(3)ijkl} - \frac{1}{180}\overline{R}^{ij}_{(3)}\overline{R}_{(3)ij} + \frac{1}{72}\overline{R}^2_{(3)}\right)\}. \qquad (2.10)$$

In eq. (2.10),

$$\sum_{n=-\infty}^{\infty} m_n^2 = \sum_{n=-\infty}^{\infty} (n^2 + |m_0|^2)^{3/2} = \sum_{n=-\infty}^{\infty} |m_0|^3 \left(1 + \sum_{q=1}^{\infty} {}^{(3/2)}C_q (m_0^{-2})^q n^{2q}\right)$$

$$= 2\,\zeta(0)\,|m_0|^3 + 2\sum_{q=1}^{\infty} {}^{(3/2)}C_q (m_0)^{(3-2q)}\,\zeta(-2q) = -|m_0|^3,$$

where $\zeta(w) = \sum_{n=1}^{\infty} n^{-w}$ (Re$w > 0$) is the Riemann zeta function $\zeta(0)$ is divergent, but through analytic continuation it is evaluated equal to $-1/2$ and $\zeta(-2q) = 0$ for positive integers q. In a similar manner, one obtains

$$\sum_{n=-\infty}^{\infty} m_n = |m_0| \qquad \text{and} \qquad \sum_{n=-\infty}^{\infty} m_n^{-1} = |m_0|^{-1}.$$

Thus

$$\Gamma = -(24\pi)^{-1}\int d^3x \sqrt{|\overline{g}(3)|}\;\{4\,|m_0|^3 - |m_0|\,\overline{R}_{(3)} +$$

$$+ |m_0|^{-1}\left(\frac{1}{10}\Box_{(3)}\overline{R}_{(3)} + \frac{1}{60}\overline{R}^{ijkl}_{(3)}\overline{R}_{(3)ijkl} - \frac{1}{60}\overline{R}^{ij}_{(3)}\overline{R}_{(3)ij} + \frac{1}{24}\overline{R}^2_{(3)}\right)\}. \qquad (2.11)$$

3. 4-Dim. induced gravity and vacuum energy

From equation (2.11), 3-dim. induced Einstein_Hilbert action of gravity is obtained as

$$S^{ind}_{(3)g} = -(24\pi)^{-1}\int d^3x \sqrt{|\overline{g}(3)|}\;\{4\,|m_0|^3 - |m_0|\,\overline{R}_{(3)}\}. \qquad (3.1)$$

To neutralize the effect of conformal transformations (2.2), another conformal transformation

$$\overline{g}_{ij} \to g_{ij} = \overline{\Omega}^2\,\overline{g}_{ij} \qquad (3.2)$$

with $\overline{\Omega} = (r \sin\theta)$, is used. As a result, eq. (3.1) is written as

$$S^{ind}_{(3)g} = -(24\pi)^{-1}\int d^3x \sqrt{|\overline{g}(3)|}\left[\frac{|m_0|}{r\sin\theta}\left(\frac{4\,|m_0|^2}{r^2\sin^2\theta} + \frac{4}{r^2} - \frac{12\,GM}{r^3}\right.\right.$$

$$+\frac{4(1+\cos^2\theta)}{r^2\sin^2\theta}-\frac{2}{r^3}-\frac{2\cot^2\theta}{r^2}-R_{(3)}], \tag{3.3}$$

where

$$g_{ij}\equiv\left\{\left(1-\frac{2GM}{r}\right),-\left(1-\frac{2GM}{r}\right)^{-1},-r^2\right\}.$$

Putting $R_{(3)}=R_{(4)}-f(r)-R^3_{(4)3}$ with f(r) and $R^3_{(4)3}$ computed in the background geometry of Swartzchild space time in eq. (3.3) and integrating over ψ, one obtains

$$S^{ind}_{(3)g}=-(24\pi)^{-1}\int d^4x\sqrt{|g(4)|}\left[\frac{|m_0|}{r^2\sin^2\theta}\left(\frac{4|m_0|^2}{r^2\sin^2\theta}+\frac{4}{r^2}-\frac{12\,GM}{r^3}\right.\right.$$

$$+\frac{4(1+\cos^2\theta)}{r^2\sin^2\theta}-\frac{2}{r^3}-\frac{2\cot^2\theta}{r^2}$$

$$\left.\left.-\frac{2}{r^2}\left(1-\frac{2GM}{r}\right)^{-1}\left(1-\frac{3GM}{r}+\frac{4(GM)^2}{r^2}\right)-R_{(4)}\right.\right. . \tag{3.4}$$

The line element for Schwarzschild space-time, given by eq. (1.1) is obtained solving field equations derived from Einstein-Hilbert action

$$S_{(4)\,g}=\int d^4x\sqrt{|g(4)|}\;\frac{R_{(4)}}{16\,\pi G} \tag{3.5}$$

using invariance under transformations $g_{\mu\nu}\to g_{\mu\nu}+\delta g_{\mu\nu}$. So,

$$S^{eff}_{(4)\,g}=S_{(4)\,g}+S^{ind}_{(4)\,g}\,. \tag{3.6}$$

Eqs. (3.4) - (3.6) yield

$$\frac{1}{16\,\pi G_{eff}}=\frac{1}{16\,\pi G}+\frac{|m_0|}{24\,\pi r^2\sin^2\theta}$$

which implies

$$G_{eff}=G\left[1+\frac{2|m_0|G}{3r^2}\operatorname{cosec}^2\theta\right]^{-1} \tag{3.7}$$

giving that $G_{eff}\approx G$ when $>3GM$.

The general form of $S^{ind}_{(3)\,g}$ is supposed to be

$$\int d^4 x \sqrt{|g(4)|}\left(\frac{R_{(4)}}{16\,\pi G}-\frac{\lambda_{ind}}{8\,\pi G_{ind}}\right).$$

So eqs. (3.4)-(3.7) yield

$$\frac{\lambda_{eff}}{8\,\pi G_{eff}}=\frac{\lambda_{ind}}{8\,\pi G}=\frac{1}{24\,\pi}+\frac{|m_0|}{r^2\sin^2\theta}\left(\frac{4\,m_0^2}{r^2\sin^2\theta}+\frac{4}{r^2}-\frac{12\,GM}{r^3}\right.$$

$$+\frac{4\,(1+\cos^2\theta)}{r^2\sin^2\theta}-\frac{2}{r^3}-\frac{2\cot^2\theta}{r^2}$$

$$\left.-\frac{2}{r^2}\left(1-\frac{2GM}{r}\right)^{-1}\left(1-\frac{3GM}{r}+\frac{4\,G^2M^2}{r^2}\right)\right)$$

$$\geq\frac{1}{24\,\pi r^2}\sqrt{G-\frac{18\,GM}{r}+\frac{18}{r^4}\left(1-\frac{2GM}{r}\right)\times}$$

$$\left[\frac{24}{r^2}-\frac{72\,GM}{r^3}-\frac{72}{r^5}\left(1-\frac{2\,GM}{r}\right)+\frac{8}{r^2}-\frac{12\,GM}{r^3}-\frac{2}{r^3}-\frac{2}{r^2}\left(1-\frac{2\,GM}{r}\right)\times\right.$$

$$\left(1-\frac{3\,GM}{r}+\frac{4\,G^2M^2}{r^2}\right), \qquad (3.8)$$

using the inequality (2.3c).

$\frac{\lambda_{ind}}{8\,\pi G}$ is the vacuum energy density in the exterior space-time of the black-hole. So vacuum energy around the black-hole is computed as

$$E=2\int_0^{\pi/2}\int_0^{2\pi}\int_{3GM}^{r}\frac{\lambda_{ind}}{8\,\pi G}(r')\frac{r'^2}{\sqrt{1-\frac{2GM}{r'}}}\sin\theta\,d\theta\,d\psi\,dr'$$

$$\geq\frac{1}{\sqrt{6}}\left[30\left(\frac{1}{2GM}-\frac{1}{r}\right)-\frac{97\,GM+2}{2}\left(\frac{1}{(2GM)^2}-\frac{1}{r^2}\right)+\frac{GM}{3}\left(\frac{1}{(2GM)^3}-\frac{1}{r^3}\right)+\ldots\right].$$

$$\approx \frac{293}{\sqrt{6}} (3GM)^{-1}. \tag{3.9}$$

The Schwarzschild exterior solution given by eq. (1.1), is obtained in empty space-time. It means that the energy, given by eq. (3.9), is radiated from the black-hole by the process of quantum tunnelling through the layer r = 2 GM, as there exists no other source of energy except a massless scalar field activating the black-hole.

4. Implications and comparison with Hawking's results

Physical parameters such as temperature, life-time, entropy and luminosity can be calculated from results obtained above.

(a) Temperature

If black-hole is treated as a black-body radiant, one can use Plank's formula for the energy distribution given as [5]

$$E = (2/\pi^2) \int_0^{\pi/2} \int_0^{2\pi} \int_{3GM}^{r} \frac{r'^2}{\sqrt{1 - \frac{2GM}{r'}}} \sin\theta \, d\theta \, d\psi \, dr' \times \int_0^{\infty} \frac{\nu^2 \, d\nu}{e^{(\nu/T)} - 1}$$

$$= \frac{\pi^3}{360} (2GM)^3 [\sqrt{x(x-1)} \{2(4x-1)(4x-3) + 36(2x-1) + 90 + \frac{60}{\sqrt{x(x-1)}} \cosh^{-1}(\sqrt{x})\} - (\sqrt{3/2}) \{192 + \frac{120}{\sqrt{3}} \cosh^{-1}(\sqrt{3/2})\}] \; T^4, \tag{4.1}$$

where r = x (2GM) gives different orbits around the black-hole with real numbers $x \geq 1$.

Comparing equations (3.9) and (4.1), temperature is obtained as

$$T \geq B (2GM)^{-1}, \tag{4.2}$$

where

$$B \equiv 5.516 [\sqrt{x(x-1)} \{(4x-3) + 36(2x-1) + 90 + \frac{60}{\sqrt{x(x-1)}} \cosh^{-1}(\sqrt{x})\}$$

$$-(\sqrt{3/2})\{192+\frac{120}{\sqrt{3}}\cosh^{-1}(\sqrt{3/2})\}]^{-1/4}. \qquad (4.3)$$

It is interesting to note that as r increases, temperature of the black-hole decreases. It means that observed temperature will be less for distant observers than the temperature observed by those who are comparatively closer to the black-hole. For a black-hole of mass 10^{14} gm, temperature at r = 200 GM is $\geq 2.06 \times 10^{-1}$ GeV = $2.387 \times 10^{12}\ {}^0$K. For a black-hole of one solar mass, temperature at r = 200 GM is $\geq 2.058 \times 10^{-20}$ GeV = $2.387 \times 10^{-7}\ {}^0$K. These results obtained here at r = 200 GM agree with Hawking's results [1.2]. Temperature, obtained by Hawking, has no radial dependence.

(b) Life-time of the black-hole

As energy is radiated from the black-hole, its mass will decrease. So, the rate of loss of mass can be equated with luminosity L which is given as [3]

$$L \sim T^4 (GM)^2 \qquad (4.4)$$

in the case of black-hole implying its life-time

$$T = \int_0^r dt = -\int_M^0 dM T^{-4} (GM)^{-2} \qquad (4.5)$$

T is the time period during which entire mass of the black-hole evaporates after its formation.

The result for temperature, given by eqs. (4.2) and (4.3), is very crucial here due to its dependence on r. If T, given by eq. (4.2) is directly introduced in eq. (4.5), T will also depend on r. It means that for distant observers life-time will be more than that for those sitting comparatively closer to the black-hole. It may happen so due to time-dilation in a strong gravitational field. Due to this reason, for calculation of the correct life-time it is better to obtain temperature near the event-horizon (given by the surface r = 2 GM), say at r = 2.1 GM.

But as discussed above, the result for temperature [given by eq. (4.2)] can not be used for evaluation at r < 3 GM. So, to be on safe side, one can calculate T at r = 200 GM using the eq. (4.2) and extrapolate T to r = 2.1 GM through the formula given by the phenomena of gravitational red-shift/blue-shift [6]

$$T_2 = T_1 \, [g_{00}(r_2)/g_{00}(r_1)]^{1/2} \tag{4.6}$$

where $g_{00}(r) = 1 - \frac{2GM}{r}$, $T_1 = T(r = r_2)$ and $T_2 = T(r = r_1)$ observing from the point $r = r_1 = 200$ GM. It is calculated above that $T_2 \geq 0.231\,(2GM)^{-1}$. So putting $r_2 = 2.1$ GM, T_1 is obtained from the eq. (4.6) as

$$T_1 \geq 1.054 \times (2GM)^{-1}. \tag{4.6}$$

Now eqs. (4.5) and (4.6) yield

$$T \leq 4.8 \times G^{-1}\,(GM)^3, \tag{4.7}$$

which implies that life-time of a black-hole of mass 10^{15} gm is less than or approximately equal to 5.6×10^{16} sec. According to this result, a primordial black-hole of mass 10^{15} gm might have radiated by now. Hawking also has obtained similar result [1, 2].

(c) Entropy of the black-hole

Entropy of the black-hole can also be calculated using the temperature at r = 2.1 GM, given by eq. (4.6). It is given by

$$S = -\int_M^0 \frac{dm}{T}, \tag{4.8}$$

where M is the initial mass.

As area of the event horizon is directly proportional to square of mass, using the definition of entropy [given by eq. (4.8)] and temperature at r = 2.1 GM, one obtains

$$S \leq -(0.015/\pi G) \int_{A(0)}^0 dA = (0.015/\pi G)\; A(0), \tag{4.9}$$

where $A(0) = 16\,\pi(GM)^2$ is the initial area of the event horizon of the black-hole, which is a constant. So, in the light of the result, given by eq. (4.9), with a non-negative multiple λ one can write

$$S + \chi^A = (0.015/\pi G)\; A(0) = \text{constant} \tag{4.10}$$

implying that $\chi = 0$ if $S \cong (0.015/\pi G)\, A(0)$ and $\chi \neq 0$ if $S < (0.015/\pi G)\, A(0)$. Moreover, eq. (4.10) suggests that entropy increases as area of the event-horizon decreases due to loss of mass. Thus this results confirms the Generalized Second Law of black-hole physics proposed by Beckenstein which states that sum of entropy and some multiple of area of the event horizon never decreases [2, 7]. Hawking too has confirmed this law and obtained $\chi = 1/4$.

(d) Luminosity of the black-hole

Equating the rate of loss of mass with luminosity as above, one also obtains

$$-\int_{M}^{m(t)} m'^2\, dm' \geq (B^4/16G^2) \int_0^t d\bar{t} \tag{4.11a}$$

yielding time-dependence of mass as

$$m^3 \leq M^3 - (3B^4/16G^2)\, t, \tag{4.11b}$$

where B is given by eq. (4.3) and $0 \leq t \leq T$. The equation (4.11b) shows how mass of the black-hole will decrease with time.

Replacing M in eq. (3.9) by m(t), given by eq. (4.11b), one obtains

$$E \geq \frac{293}{3G\sqrt{6}} \left[M^3 - \frac{3B^4}{16G^2} t \right]^{-1/3}, \tag{4.12}$$

which shows that initially black-hole losses energy of the amount $\geq \frac{293}{3G\sqrt{6}}$ and gradually it increases till $t = T$. The equation (4.12) also shows that at $t = (16/3B^4G)\,(GM)^3$, E is divergent and when $t > (16/3B^4G)\,(GM)^3$, E will be either negative or imaginary. These are unphysical situations. It leads to a natural conclusion that there will be no emission of energy when $t \geq (16/3B^4G)\,(GM)^3$, because $T < (16/3B^4G)\,(GM)^3$.

Similarly, time dependence of temperature is obtained as

$$T \geq \frac{B}{2G} \left[M^3 - \frac{3B^4}{16G^2} t \right]^{-1/3}, \tag{4.13}$$

implying that a stationary observer sitting at a particular space point will find that black-hole is gradually getting hotter and hotter.

Moreover, eqs. (4.2), (4.3) and (4.4) imply

$$\frac{dL}{dt} \geq \frac{B^8}{128G^4} m^{-5}. \tag{4.14}$$

Connecting inequalities (4.11b) and (4.14), it is obtained after integration that

$$L \geq \frac{B^4}{16G^2}\left[\left(M^3 - \frac{3B^4}{16G^2}t\right)^{-2/3} - M^{-2}\right], \tag{4.15}$$

which shows that luminosity also increases with time. For an observer sitting at r = 200 GM, luminosity is given as

$$L \geq 1.78 \times 10^{-4}G^{-2}[(M^3 - 5.34 \times 10^{-4}\, G^{-2}t)^{-2/3} - M^{-2}]. \tag{4.16}$$

After t = T (life – time), black-hole reduces to a dark naked singularities as luminosity will vanish due to loss of total mass.

It is interesting to infer from these results that black-hole may not remain black for ever as it becomes more and more luminous with time. So, it is tempting to speculate that some of the luminous celestial objects today might have been black-hole in the past and black-holes today may glow in future.

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