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On P2-Like Finsler Spaces

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Abstract

In the present paper we have discussed a special form of (v) hv-torsion tensor P_{ijk} given by $P_{ijk} = \lambda X_i X_j X_k$, where X_i are covariant components of unit vector, λ is a scalar function of x, y in a Finsler space. Since P_{ijk} of every two dimensional Finsler space may be written in this form, we shall say an n-dimensional Finsler space F^n $(n \geq 2)$ as P2-like Finsler space whose P_{ijk} is of this form. The values of λ and X_i are obtained in terms of main scalars and h-connection vectors with respect to orthonormal frame in three and four dimensional P2-like Finsler spaces.

Keywords. P2-like three and four dimensional Finsler spaces.

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1. Introduction

There are three kinds of torsion tensors in Cartan's theory of Finsler spaces. Two of them are (h) hv-torsion tensor C_{ijk} and (v) hv-torsion tensor P_{ijk} which are symmetric in all their indices and both are indicatory tensors. Various interesting special forms of these torsion tensors have been studied by mathematicians. For example C-reducible [2], semi-C-reducible [3], [4], C2-like [4] and C3-like [7] Finsler spaces are based on the special forms of C_{ijk} where as P-reducible [1].[5] and P-symmetric [5] Finsler spaces are based on the special forms of P_{ijk}.

In the present paper we shall discuss another special form P_{ijk} given by

$$P_{ijk} = \lambda X_i X_j X_k, \tag{1.1}$$

where X_i are covariant components of unit vector λ is a scalar function of x and y.

It should be remarked here that a P2-like Finsler space is a Landsberg space if and only if $\lambda = 0$.

2. n-Dimensional P2-like Finsler space.

Let F^n be an n-dimensional Finsler space with the metric function $L(x, y)$. We denote $g_{ij}(x, y)$ as the fundamental metric tensor : $g_{ij} = \frac{1}{2} \dot{\partial}_i \dot{\partial}_j L^2$; $C_{ijk}(x, y)$, the (h) hv-torsion tensor : $C_{ijk} = \frac{1}{2} \dot{\partial}_k g_{ij}$. The h-covariant derivative of C_{ijk} with respect to Cartan's connection $C\Gamma$, is used in defining (v) hv-torsion tensor P_{ijk} :

$$P_{ijk} = C_{ijk|h} y^h = C_{ijk|0}.$$

Let X_i are components of a covariant unit vector in F^n. Then we consider the special form of P_{ijk} given by

$$P_{ijk} = \lambda X_i X_j X_k, \tag{2.1}$$

where λ is a scalar function of x, y and we call F^n as P2-like Finsler space if its P_{ijk} is of this form. Since P_{ijk} is a symmetric tensor and $P_{ijk} y^i = 0$ from (2.1) ot follows that $X_i y^i = 0$. The v-covariant derivative of this equation with respect to $C\Gamma$ gives

$$X_i|_j y^i + X_j = 0. \tag{2.2}$$

If the vector field X_i is v-covariantly constant then from (2.2) it follows that $X_j = 0$, which is a contradiction. Hence we have the following :

Proposition (2.1). The vector field X_i occuring in P2-like Finsler space is not v-covariantly constant.

The concurrent vector field has been defined by Matsumoto [6] and Tachibana [12]. The semi-parallel vector field, the semi-parallel h-vector field and the concircular vector fields have been defined by Singh and Prasad [10], Singh and Kumari [11] and Prasad et al. [8] respectively. In all these works the vector field are v-covariantly constant. Hence, we may state that

Proposition (2.2). The vector field X_i occuring in P2-like Finsler space is none of the following :

(a) a concurrent vector field (b) a semi-parallel vector field,

(b) a semi-parallel h-vector field (d) a concircular vector field.

Contracting (2.1) with g^{jk}, using the fact that X_i are components of unit covariant vector and $C_{i\,|\,0} = P_i = P_{ijk}\, g^{jk}$, we get $\lambda X_i = C_{i\,|\,0}$. This shows that λ is the magnitude of the vector $C_{i\,|\,0}$, i.e.,

$$\lambda^2 = g^{ij}\, C_{i\,|\,0}\, C_{j\,|\,0}.$$

3. Two and three-dimensional P2-like Finsler space.

First of all we consider the two dimensional Finsler space in which the Berwald's frame (l_i, m_i) plays an important role. With respect to his frame, the metric tensor and (h) hv-torsion tensor are respectively given by [3]

$$g_{ij} = l_i\, l_j + m_i\, m_j \qquad \text{and} \qquad LC_{ijk} = I\, m_i\, m_j\, m_k,$$

where I is main scalar of two dimensional Finsler space F^2. Since $m_{i\,|\,j} = 0$ in F^2, we have $LC_{ijk\,|\,h} = I_{|\,h}\, m_i\, m_j\, m_k$, which gives $P_{ijk} = C_{ijk\,|\,0} = L^{-1}\, I_{|\,0}\, m_i\, m_j\, m_k$. Thus we have the following :

Theorem (3.1). Every two dimensional Finsler space is P2-like with $\lambda = L^{-1}\, I_{|\,0}$ and $X_i = m_i$.

Now we consider the three-dimensional Finsler space F^3 in which the Moor's frame (l_i, m_i, n_i) plays the important role. With respect to his frame, the metric tensor and (h) hv-torsion tensor are respectively given by

$$g_{ij} = l_i\, l_j + m_i\, m_j + n_i\, n_j$$

and

$$LC_{ijk} = H\, m_i\, m_j\, m_k - J\, \pi_{(ijk)}\, \{m_i\, m_j\, n_k\} + I\pi_{(ijk)}\{m_i\, n_j\, n_k\} + J\, m_i\, n_j\, n_k \qquad (3.1)$$

where H, I, J are main scalar of three-dimensional Finsler space F^3 such that H + I = LC [3] and $\pi_{(i\,j\,k)}$ { } stands for cyclic permutation of i, j, k and summation. Taking h-covariant derivative of (3.1), using [3]

$$m_i\,|_j = n_i\, h_j\,, \qquad n_i\,|_j = -\, m_i\, h_j,$$

and then contracting the resulting equation with y^h, we get

$$P_{ijk} = (H,_1 + 3J\,h_1)\, m_i\, m_j\, m_k + (J,_1 + 3I\,h_1)\, n_i\, n_j\, n_k + \{- J,_1 +$$

$$(H - 2I)\, h_1\}\, \pi_{(ijk)}\, \{m_i\, m_j\, n_k\} + (I,1 \text{ -3Jh1})\, \pi_{(ijk)}\, \{m_i\, n_j\, n_k\}, \qquad (3.2)$$

where h_i are components of h connection vector and

$$h_1 = h_i\, l^i,\ H,_1 = \left(\frac{\partial H}{\partial x^h} - \frac{\partial H}{\partial y^r} G^r_h\right) I^h \text{ etc.}$$

If X_α (α = 1, 2, 3) are scalar components of X_i with respect to Moor's frame (l_i, m_i, n_i) , i.e.

$$X_i = X_1\, l_i + X_2\, m_i + X_3\, n_i\,. \qquad (3.3)$$

Then $X_i\, y^i = 0$ implies that $X_1 = 0$ and $g^{ij}\, X_i\, X_j = 1$ implies that

$$X_2^2 + X_3^2 = 1. \qquad (3.4)$$

If F^3 is P2-like Finsler space, then from (2.1) and (3.3), we get

$$P_{ijk} = \lambda\, [X_2^3\ m_i\, m_j\, m_k + X_3^3\ n_i\, n_j\, n_k + X_2^2\, X_3\, \pi_{(ijk)}\, \{m_i\, m_j\, n_k\}$$

$$+ X_2\, X_3^2\ \pi_{(ijk)}\, \{m_i\, n_j\, n_k\}]. \qquad (3.5)$$

Comparing (3.2) with (3.5), we get

(a) $\lambda X_2^3 = H,_1 + 3J\,h_1$, (b) $\lambda X_3^3 = J,_1 + 3I\,h_1$,

(c) $\lambda X_2^2\ X_3 = - J,_1 + (H - 2I)\,h_1$, (d) $\lambda X_2 X_3^2 = I,_1 - 3J\,h_1$. (3.6)

Adding (a) and (d) of equation (3.6), using (3.4) and the fact that H + I = LC, we get $(LC),_1 = \lambda\, X_2$. Similarly adding (b) and (c) of equation (3.6), using (3.4) and the fact that H + I = LC, we get $LCh_1 = \lambda\, X_3$. Therefore

$$\lambda X_i = (LC),_1\, m_i + LC\, h_1\, n_i, \qquad (3.7)$$

and

$$\lambda^2 = [(LC),_1]^2 + L^2\, C^2\, h_1{}^2. \qquad (3.8)$$

Theorem (3.2). In a three dimensional P2-like Finsler space the unit vector field X_i and the scalar λ are given by (3.7) and (3.8).

Now we consider the particular case of P2-like three dimensional Finsler space for which $X_i = m_i$. Then we have $X_2 = 1$, $X_3 = 0$. Therefore (3.6) reduces to

(a) $H,_1 + 3J\,h_1 = \lambda$, (b) $J,_1 + 3I\,h_1 = 0$,

(c) $-J,_1 + (H - 2I)\,h_1 = 0$, (d) $I,_1 - 3J\,h_1 = 0$. (3.9)

Solving these equations, we get

$$h_1 = 0, \qquad H,_1 = \lambda, \qquad J,_1 = I,_1 = 0,$$

provided unified main scalar LC is non-zero.

Theorem (3.3). If the vector field X_i in three dimensional P2-like Finsler space is along one of the vector m_i of the Moor's frame (l_i, m_i, n_i) then $h_1 = 0$, $H,_1 = \lambda$, $J,_1 = I,_1 = 0$, provided unified main scalar LC is non-zero.

Next suppose that $X_i = n_i$. Then $X_2 = 0$, $X_3 = 1$ and hence (3.6) reduces to

(a) $H,_1 + 3J\,h_1 = 0$, (b) $J,_1 + 3I\,h_1 = \lambda$,

(c) $-J,_1 + (H - 2I)\,h_1 = 0$, (d) $I,_1 - 3J\,h_1 = 0$.

Solving these equations and using the fact that $H + I = LC$ we get $(LC),_1 = 0$, $LCh_1 = \lambda$.

Theorem (3.4). If the vector field X_i in three dimensional P2-like Finsler space is along one of the vector n_i of the Moor's frame (l_i, m_i, n_i) then $(LC),_1 = 0$ and $\lambda = LCh_1$.

4. Four dimensional P2-like Finsler space

We consider the four-dimensional Finsler space F^4 in which the Moor's frame (l_i, m_i, n_i, p_i) plays the important role. With respect to his frame, the metric tensor and (h) hv-torsion tensor are respectively given by

$$g_{ij} = l_i\,l_j + m_i\,m_j + n_i\,n_j + p_i\,p_j$$

and

$$LC_{ijk} = Hm_i\,m_j\,m_k + J\,\{n_i\,n_j\,n_k\} + H'\,p_i\,p_j\,p_k + I\,\pi_{(ijk)}\,\{m_i\,n_j\,n_k\}$$
$$+ K\,\pi_{(ijk)}\,\{m_i\,p_j\,p_k\} - (J + J')\,\pi_{(ijk)}\,\{m_i\,m_j\,n_k\} + J'\,\pi_{(ijk)}\,\{n_i\,p_j\,p_k\}$$

$$- (H' + I')\, \pi_{(ijk)} \{m_i\, m_j\, p_k\} + I'\, \pi_{(ijk)} \{n_i\, n_j\, p_k\}$$

$$+ K'\, \pi_{(ijk)} \{m_i\, (n_j\, p_k + n_k\, p_j)\}, \tag{4.1}$$

where H, I, J, K, H′, I′, J′ and K′ are eight main scalars of F^4. Taking h-covariant derivative of (4.1), using [9]

$$m_i\,|_j = n_i\, h_j + p_i\, j_j; \qquad n_i\,|_j = - m_i\, h_j + p_i\, k_j, \qquad p_i\,|_j = - m_i\, j_j - n_i\, k_j, \tag{4.2}$$

and then contracting the resulting equation with y^h, we get

$$\begin{aligned} P_{ijk} = {} & [H,_1 + 3(J + J')\, h_1 + 3(H' + I')\, J_1]\, m_i\, m_j\, m_k + [J,_1 + 3Ih_1 - 3I'k_1]\, n_i\, n_j\, n_k \\ & + [H',_1 + 3Kj_1 + 3J'k_1]\, p_i\, p_j\, p_k + [- (J + J'),_1 + (H - 2I)\, h_1 - 2K'\, j_1 + \\ & (H' + I')\, k_1]\, \pi_{(i\,j\,k)}\{m_i\, m_j\, n_k\} + [I,_1 - (3J + 2J')\, h_1 - I'J_1 - 2K'k_1] \\ & \pi_{(i\,j\,k)}\{m_i\, n_j\, n_k\} + [K,_1 - J'h_1 - (3H' + 2I')\, J_1 + 2K'\, k_1]\, \pi_{(i\,j\,k)}\{m_i\, p_j\, p_k\} \\ & + [J',_1 + Kh_1 + 2K'j_1 - (H' - 2I')\, k_1]\, \pi_{(i\,j\,k)}\{n_i\, p_j\, p_k\} + [I',_1 + 2K'h_1 + Ij_1 \\ & + (J - 2J')\, k_1]\, \pi_{(i\,j\,k)}\{n_i\, n_j\, p_k\} + [- (H' + I'),_1 - 2K'h_1 + (H - 2K)\, j_1 \\ & - (J + J')\, k_1]\;\; \pi_{(i\,j\,k)}\{m_i\, m_j\, p_k\} + [K',_1 + (H' + 2I')\, h_1 - (J' + 2J')j_1 \\ & + (I - K)\, k_1]\, \pi_{(i\,j\,k)}\{m_i\, n_j\, p_k + m_i\, p_j\, n_k\} \end{aligned} \tag{4.3}$$

where h_i, j_i, k_i are components of three h connection vectors in F^4 and

$$h_1 = h_i\, l^i, \qquad j_1 = j_i\, l^i, \qquad k_1 = k_i\, l^i, \qquad H,_1 = \left(\frac{\partial H}{\partial x^h} - \frac{\partial H}{\partial y^r}\, G^r_h\right) l^h \text{ etc.}$$

Let X_α ($\alpha = 1, 2, 3, 4$) are scalar components of X_i with respect to Moor frame (l_i, m_i, n_i, p_i), i.e.,

$$X_i = X_1\, l_i + X_2\, m_i + X_3\, n_i + X_4\, p_i\,. \tag{4.4}$$

Then $X_i\, y^i = 0$ implies that $X_1 = 0$ and $g^{ij}\, X_i\, X_j = 1$ implies that

$$X_2^2 + X_3^2 + X_4^2 = 1. \tag{4.5}$$

If F^4 is P2-like Finsler space, then from (2.1) and (4.5), we get

$$P_{ijk} = \lambda\,[X_2^3\ m_i m_j m_k - X_3^3\ n_i n_j n_k + X_4^3\ p_i p_j p_k + X_2^2\ X_3\ \pi_{(i\,j\,k)}\{m_i m_j n_k\}$$
$$+ X_2 X_3^2\, \pi_{(i\,j\,k)}\{m_i n_j n_k\} + X_2 X_4^2\, \pi_{(i\,j\,k)}\{m_i p_j p_k\}$$
$$+ X_3 X_4^2\, \pi_{(i\,j\,k)}\{n_i p_j p_k\} + X_4 X_3^2\, \pi_{(i\,j\,k)}\{p_i n_j n_k\} + X_4 X_2^2\, \pi_{(i\,j\,k)}\{p_i m_j m_k\}$$
$$+ X_2 X_3 X_4\ \pi_{(i\,j\,k)}\{m_i n_j p_k + m_i p_j n_k\}]\,. \tag{4.6}$$

Comparing (4.3) and (4.6), we get

$$\begin{aligned}
&(a) && \lambda X_2^3 = H,_1 + 3(J + J')\,h_1 + 3(H' + I')\,J_1,\\
&(b) && \lambda X_3^3 = J,_1 + 3Ih_1 - 3I'\,k_1,\\
&(c) && \lambda X_4^3 = H',_1 + 3Kj_1 + 3J'\,k_1,\\
&(d) && \lambda X_2^2 X_3 = -(J + J'),_1 + (H - 2I)\,h_1 - 2K'\,j_1 + (H' + I')\,k_1,\\
&(e) && \lambda X_2 X_3^2 = I,_1 - (3J + 2J')\,h_1 - I'\,j_1 - 2K'\,k_1,\\
&(f) && \lambda X_2 X_4^2 = K,_1 - J'\,h_1 - (3H' + 2I')\,j_1 + 2K'\,k_1,\\
&(g) && \lambda X_3 X_4^2 = J',_1 + Kh_1 + 2K'j_1 - (H' - 2I')\,k_1,\\
&(h) && \lambda X_3 X_3^2 = I',_1 + 2K'\,h_1 + Ij_1 + (J - 2J')\,k_1,\\
&(i) && \lambda X_4 X_2^2 = -(H' + I'),_1 - 2K'\,h_1 + (H - 2K)\,j_1 + (J + J')\,k_1,\\
&(j) && \lambda X_2 X_3 X_4 = K',_1 - (H' + 2I')\,h_1 - (J + 2J')\,j_1 + (I - K)\,k_1.
\end{aligned} \tag{4.7}$$

Adding (a), (e) and (f) of equation (4.7), using (4.5) and the fact that $H + I + K = LC$, we get $(LC),_1 = \lambda X_2$. Also adding (b), (d) and (g) of equation (4.7), using (4.5) and the fact that $H + I + K = LC$, we get $LC\,h_1 = \lambda\,X_3$. Similarly adding (c) (h) and (i) of equation (4.7), using (4.5) and the fact that $H + I + K = LC$, we get $LCj_1 = \lambda X_4$. Therefore

$$\lambda X_i = (LC)_{,1}\, m_i + LC\,(h_1\, n_i + j_1\, p^i) \tag{4.8}$$

and

$$\lambda^2 = [(LC)_{,1}]^2 + L^2 C^2 (h_1{}^2 + j_1{}^2). \tag{4.9}$$

Theorem (4.1). In a four dimensional P2-like Finsler space the unit vector field X_i and the scalar λ are given by (4.8) and (4.9).

Now we consider the particular case of P2-like four dimensional space for which $X_i = m_i$. Then we have $X_2 = 1$, $X_3 = X_4 = 0$. Therefore (4.7) reduces to

$$
\begin{aligned}
&(a) \quad H_{,1} + 3(J + J')\, h_1 + 3(H' + I')\, J_1 = \lambda,\\
&(b) \quad J_{,1} + 3Ih_1 - 3I'\, k_1 = 0,\\
&(c) \quad H'_{,1} + 3Kj_1 + 3J'\, k_1 = 0,\\
&(d) \quad -(J + J')_{,1} + (H - 2I)\, h_1 - 2K'\, j_1 + (H' + I')\, k_1 = 0,\\
&(e) \quad I_{,1} - (3J + 2J')\, h_1 - I'\, j_1 - 2K'\, k_1 = 0, \qquad\qquad (4.10)\\
&(f) \quad K_{,1} - J'\, h_1 - (3H' + 2I')\, j_1 + 2K'\, k_1 = 0,\\
&(g) \quad J'_{,1} + Kh_1 + 2K'j_1 - (H' - 2I')\, k_1 = 0,\\
&(h) \quad I'_{,1} + 2K'\, h_1 + Ij_1 + (J - 2J')\, k_1 = 0,\\
&(i) \quad -(H' + I')_{,1} - 2K'\, h_1 + (H - 2K)\, j_1 + (J + J')\, k_1 = 0,\\
&(j) \quad K'_{,1} - (H' + 2I')\, h_1 - (J + 2J')\, j_1 + (I - K)\, k_1 = 0.
\end{aligned}
$$

Solving these equations we get $h_1 = 0$, $j_1 = 0$, $H_{,1} = 1$ and

$$\frac{J_{,1}}{2I'} = \frac{H'_{,1}}{-3J'} = \frac{I_{,1}}{2K'} = \frac{K_{,1}}{-2K'} = \frac{J'_{,1}}{H' - 2I'} = \frac{I'_{,1}}{-(J - 2J')} = \frac{K'_{,1}}{-(I - K)} = k_1, \tag{4.11}$$

provided unified main scalar LC is non-zero.

Theorem (4.2). If the vector field X_i in four dimensional P2-like Finsler space is along one of the vector m_i of Moor's frame (l_i, m_i, n_i, p_i) then $h_1 = 0$, $j_1 = 0$, $H_{,1} = \lambda$, provided unified main scalar LC is non-zero and (4.11) holds.

Next suppose that $X_i = n_i$. Then $X_2 = X_4 = 0$, $X_3 = 1$ and hence (4.5) reduces to the same equations as given in (4.8) except the equations (a) and (b) which now become

$$\text{(a)} \quad H_{,1} + 3(J + J')\, h_1 + 3(H' + I')\, J_1 = 0, \quad \text{(b)} \quad J_{,1} + 3Ih_1 - 3I'\, k_1 = \lambda, \quad (4.12)$$

Solving these equations and using the fact that $H + I + K = LC$ we get $(LC)_{,1} = 0$, $LCh_1 = \lambda$, $h_1 = \dfrac{H_{,1}}{-3\,(J + J')}$, $j_1 = 0$, $k_1 = \dfrac{H'_{,1}}{3J'}$. Hence we have the following :

Theorem (4.3). If the vector field X_i in four dimensional P2-like Finsler space is along one of the vector n_i Moor's frame (l_i, m_i, n_i, p_i) then $(LC)_{,1} = 0$, $\lambda = LCh_1$, $h_1 = \dfrac{H_{,1}}{-3\,(J + J')}$, $j_1 = 0$ and $k_1 = \dfrac{H'_{,1}}{3J'}$.

Lastly suppose that $X_i = p_i$. Then $X_2 = X_3 = 0$, $X_4 = 1$ and hence (4.7) reduces to the same equations as given in (4.10) except the equations (a) and (c) which now becomes

$$\text{(a)} \quad H_{,1} + 3(J + J')\, h_1 + 3(H' + I')\, J_1 = 0, \quad \text{(b)} \quad H'_{,1} + 3Kj_1 + 3J'\, k_1 = \lambda. \quad (4.13)$$

Solving these equations and using the fact that $H + I + K = LC$, we get $(LC)_{,1} = 0$, $h_1 = 0$, $LCj_1 = \lambda$, $j_1 = \dfrac{H_{,1}}{-3\,(H' + I')}$ and $k_1 = \dfrac{J'_{,1}}{3I'}$. Hence, we have the following :

Theorem (4.4). If the vector field X_i in four dimensional P2-like Finsler space is along one of the vector p_i Moor's frame (l_i, m_i, n_i, p_i) then $(LC)_{,1} = 0$, $h_1 = 0$, $LCj_1 = \lambda$, $j_1 = \dfrac{H_{,1}}{-3\,(H' + I')}$ and $k_1 = \dfrac{J'_{,1}}{3I'}$.

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