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RESEARCH ARTICLE

Bianchi Type- III Perfect Fluid Dark Energy Cosmological Model with Dynamical Equation of State Parameter in *f(R,T)* Theory of Gravity

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Abstract
We investigate some spatially homogeneous Bianchi type-III dark energy cosmological models with equation of state parameters in the presence of perfect fluid within the framework of modified f(R, T) gravity theory with the help of a special form of deceleration parameter for FRW metric proposed by Singh and Debnath.[1] We have also assumed that the scalar expansion is proportional to the shear, and the EoS parameter is proportional to the skewness parameter in this model. Some physical and kinematical behavior of the model is also discussed in detail.

Keywords: Perfect fluid.Dark Energy. f(R,T)

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1. Introduction

The most popular issue in modern-day cosmology is the current expansion of the universe. It is now evident from observational and theoretical facts that our universe is in the phase of accelerated expansion.[2,3]. The phenomenon of dark energy and dark matter is another topic of discussion. [4 , 5] It was Einstein who first gave the concept of dark energy and introduced the small positive cosmological constant. But after some time, he remarked it was the biggest mistake in his life. However, it is now through that the cosmological constant may be a suitable candidate for dark energy. It is generally believed that some " dark energy" (DE) is pervading the whole universe. This is a hypothetical form of energy that permeates all of space and tends to increase the rate of expansion of the universe.[6] Thus DE is a prime candidate to explaining the recent cosmic observations. Most WMAP observations indicate that DE accounts for 74% of the total mass-energy of the universe.[7] The study of the anisotropic universe has a long history.[8,9]

Many authors have suggested a number of ideas to explain the current accelerating universe, partly such as scalar field model, exotic equation of state(EOS), modified gravity, and the inhomogeneous cosmology model. The dark energy EoS parameter $\omega = \frac{p}{\rho}$, where p is the dark-energy pressure and ρ is its energy density. The value $\omega < -\frac{1}{3}$ is necessary for cosmic acceleration. The simplest candidate for dark energy is the cosmological constant Ë , for which $\omega = -1$. According to Caldwell et al. [10], the matter with $\omega < -1$ give rise to Big Rip type of future singularity . Several ideas are posed to prevent the Big-Rip type of future singularity, like by introducing quantum effects terms in the action [11], Nojiri and Odintsov [12] and Astashenok et al. [13] have studied phantom cosmology without Big Rip singularity. Bama et al. [14] have reviewed different dark energy cosmologies (isotrtopic) with early deceleration late time acceleration. They have investigated $f(R)$ gravity, $\mathrm{f}(\mathrm{T})$ gravity and ΛCDM cosmological models representing the accelerating expansion with ethe quintessence/

phantom nature in detail. They have also discussed the problem of testing dark energy models to general relativity by cosmography in a Broadway. Pradhan et al. [15] have obtained a class of LRS Bianchi type-I cosmological model with a time-dependent cosmological term in the presence of a perfect fluid. Pradhan et al. [16] have also presented a new class of spatially homogeneous and anisotropic Bianchi type-I cosmological models with a time-dependent decelerate parameter and cosmological constant in the presence of a dissipative fluid.

Another proposal to justify the current expansion of universe comes from modified or alternative theories of gravity f(T) theory of gravity is one such example which has been recently developed. This theory is a generalized version of teleparallel gravity in which the Weitzenbock connection is used instead of the Levi-Civita connection. The interesting feature of the theory is that it may explain the current acceleration without involving dark energy. A considerable amount of work has been done in this theory so for [17- 21]. Another interesting modified theory is the f(R)theory of gravity, which involves a general function of Ricci scalar in standard Einstein- Hilbert Lagrangian.

Some review articles [22-23] can be helpful to understand the theory. During the last decade, there have been several modifications of general relativity to provide a natural gravitational alternative for dark energy. Among the various modifications, the f(R) theory of gravity is treated as most suitable due to cosmologically important f(R) models. It has been suggested that cosmic acceleration can be achieved by replacing the Einstein-Hilbert action of general relativity with a general function Ricci scalar,f(R). Viable f(R) gravity models have been proposed by Nojiri and Odibtsov [24]. Multamaki and Vilja [25,26], Shamir [27] which show the unification of early time inflation and late time accel- eration.

In a recent paper, Harko et al. [28] have proposed a new generalized theory known as f(R, T) gravity. According to this theory, gravitational Lagrangian involves an arbitrary function of the scalar curvature R and a trace of the energy-momentum tensor T. Recently, Several researchers[29-31] suggested that the anisotropic Bianchi universes have played important roles in observational cosmology. Some researchers [32-34] have investigated f(R, T) gravity in different contexts. Recently, Reddy et al. [35] have oobserveda Bianchi type-III cosmological mmodelin f(R,T) gravity. Shoo et al. [36] studied anisotropic cosmological models in f(R, T) gravity with variable deceleration parameters. Tiwari et al. [37] have investigated transit cosmological models with domain wwallsin f(R, T) gravity. Nagpal et al. [38] studied FLRW cosmological models with quark and strange quark matter in f(R, T) gravity. More- over, Chaubey and Shukla[39] have discussed the general class of anisotropic Bianchi cosmological models in f(R, T) modified theories of gravity with Λ. Subsequently, several authors viz. Singh and Singh [40] explored the behaviors of scalar modified f(R, T) gravity theory within the framework of a flat Friedmann-Robertson -Walker cosmological models. Chandel and Ram [41] etc. presented spatially homogeneous Bianchi type cosmological models in the presence of a perfect fluid in f(R, T) gravity theory.

The anisotropic plays a significant role in the early state of evolution of the universe, and hence, the study of anisotropic and homogeneous cosmological models becomes important.

In this paper, we have obtained some spatially homogeneous and Bianchi type -III perfect fluid dark energy cosmological model with dynamical equation of state parameter

in f(R,T) theory of gravity proposed by[35]. Our paper is organized as follows:

In Section 2, we derive the field equations. In Section 3, we deal with the solution of the field equations in the presence of perfect fluid. Section 4 includes the solution for particular cases. Section 5 is mainly concerned with the physical and kinematical properties. The last section contains a conclusion.

2. The metric and field equations

We consider a spatially homogeneous and anisotropic Bianchi type-III metric given by

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{-2mx}B^2(t)dy^2 - C^2(t)dz^2 \tag{1}$$

$A(t), B(t)$ and $C(t)$ are cosmic scale factors and m is a positive constant. The energy momentum tensor for anisotropic dark energy is given by

$$T_j^i = \text{diag}\left[1, -p_x, p_y, p_z\right]\rho = \text{diag}\left[1, -\omega_x, \omega_y, \omega_z\right]\rho \tag{2}$$

where ρ is the energy density of the fluid and p_x, p_y, p_z are the pressure along x, y and z axes respectively. Here ω is the EoS parameter of the fluid and ω_x, ω_y and ω_z are the EoS parameters in the directions of x, y and z axes respectively. The energy momentum tensor can be parameterized as

$$T_j^i = \text{diag}\left[1, -\omega, (\omega+\gamma), (\omega+\delta)\right]\rho \tag{3}$$

For the sake of simplicity we choose $\omega_x = \omega$ and the skewness parameters δ are the deviation from ω on y and z axes respectively.

The field equations in $f(R,T)$ theory of gravity for function $\mathrm{f}(\mathrm{R},\mathrm{T}) = \mathrm{R} + \mathrm{f}(\mathrm{T})$ when the matter source is a perfect fluid are given by Harko et al. [28]

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2 f'(T) T_{ij} + \left[2 p f'(T) + f(T)\right] g_{ij} \tag{4}$$

where

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} \tag{5}$$

and the prime denotes differentiation with respect to the argument.

We choose the function $f(T)$ of the trace of the tensor of the matter so that

$$f(T) = \lambda T \tag{6}$$

where λ is constant.

Now choosing comoving coordinates the field equation (4), with the help of (4) and (5) for the metric (1), can be written as

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -\rho\left[8\pi + 2\lambda + 1 - 3\omega - \gamma - \delta\right] - 2\lambda p, \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \rho\left[(8\pi + 2\lambda)\omega - (1 - 3\omega - \delta)\right] - 2\lambda p, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = \rho\left[(8\pi + 2\lambda)(\omega + \gamma) - (1 - 3\omega - \delta)\right] - 2\lambda p, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = \rho\left[(8\pi + 2\lambda)(\omega + \delta) - (1 - 3\omega - \delta)\right] - 2\lambda p, \tag{10}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{11}$$

Here the overdot denotes ordinary differentiation with respect to t

3. Solution of field equations Integrating (11), we obtain

$$B = \beta A \tag{12}$$

where β is a constant of integration which can be taken as unity without loss of generality so that we have

$$B = A \tag{13}$$

By using (13) in (8) and (9) we obtain

$$\gamma = 0 \tag{14}$$

Now using (12) and (14) in the field equations (7) - (10), we get.

$$\left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -\rho\left[8\pi + 2\lambda + 1 - 3\omega - \delta\right] - 2\lambda p, \tag{15}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} = \rho\left[(8\pi + 2\lambda)\omega - (1 - 3\omega - \delta)\right] - 2\lambda p, \tag{16}$$

$$2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2-\frac{m^2}{A^2}=\rho\left[(8\pi+2\lambda)(\omega+\delta)-(1-3\omega-\delta)\right]-2\lambda p. \tag{17}$$

Let us introduce as usual the the dynamical shear scalar σ^2, the mean anisotropy parameter A_β, the spatial volume V, the average scale factor and the Hubble parameter H for the metric (1)

$$V=a^3=A^2C \tag{18}$$

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{19}$$

where $H_1=\frac{\dot{A}}{A}=H_2, H_3=\frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x, y and z axes respectively. Using (18) and (19), we obtain

$$H=\frac{1}{3}(H_1+H_2+H_3)=\frac{\dot{a}}{a} \tag{20}$$

The expansion scalar θ, shear scalar σ^2 and the mean anisotropy parameter A_β are given by

where $\ddot{A}H_i=H_i$ - H In the field equations (15)- (17) are three independent equations with six unknowns, A, C, ρ, pδ and ω.

Hence to find a determinate solution three more conditions are necessary. We consider the following conditions:

$$\theta=2\frac{\dot{A}}{A}+\frac{\dot{C}}{C} \tag{21}$$

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)^2, \tag{22}$$

$$A_\beta=\Sigma\left(\ddot{A}\frac{H_i}{H}\right)^2 \tag{23}$$

Singh and Debnath (1) has defined a special form of deceleration parameter for FRW metric as

$$q=-\frac{\ddot{a}a}{\dot{a}^2}=-1+\frac{\alpha}{1+a^\alpha}\# \ 24 \tag{24}$$

where $\alpha>0$ is a constant and a is mean scale factor of the universe.

After solving (24) one can obtain the mean Hubble parameter H as

$$H=\frac{\dot{a}}{a}=k\left(1+a^{-\alpha}\right), \tag{25}$$

k is a constant of integration.

On integrating (25), Adhav et al. (42) obtain the mean scale factor a as

$$a=\left(e^{k\alpha t}-1\right)^{\frac{1}{\alpha}} \tag{26}$$

where k is a constant of integration, which leads to early decelerating and late time accelerating anisotropic cosmological models with dynamical EoS parameter.

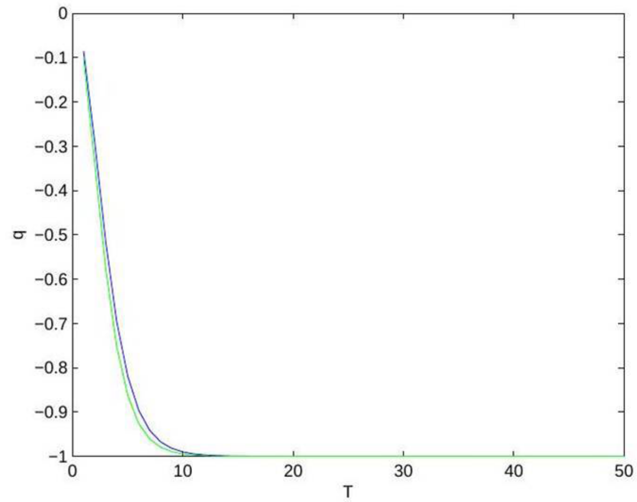

Figure 1: The plot of deceleration parameter q versus cosmic time $t, \alpha_1 = 0.3, \alpha_2 = 0.3$

We assume that the scalar expansion θ is proportional to shear scalar σ which gives us (Collins et al [9]).

$$C = A^n \tag{27}$$

$n > 1$ is a constant. The EoS parameter ω is proportional to skewness parameter δ (Reddy et al. [35]) such that

$$\omega + \delta = 0 \tag{28}$$

Now using (18), (26), (27), we obtain the solution of scale factor A, B and C as

taking $k = 1$ in (18) for convenience.

$$A = \left(e^{\alpha t} - 1\right)^{\frac{3}{\alpha(n+2)}} = B, \tag{29}$$

$$C = \left(e^{\alpha t} - 1\right)^{\frac{3n}{\alpha(n+2)}}, \tag{30}$$

4. Physical and Kinematical behaviors of the model

Physical and kinematical parameters of the model which are important for discussing the physics of the cosmological model.

For the model presented by the scale factors (29)- (30) the dynamical parameter have given values as. The generalized Hubble's parameter is given by

$$H = \left(\frac{k}{1 - e^{-\alpha t}}\right) \tag{31}$$

The spatial volume, expansion scalar, shear scalar and mean anisotropic parameter are found as

$$V = \left(e^{\alpha t} - 1\right)^{\frac{3}{\alpha}}, \tag{32}$$

$$\theta = \left(\frac{3e^{\alpha t}}{e^{\alpha t} - 1}\right), \tag{33}$$

$$\sigma^2 = 3\left(\frac{1-n}{2+n}\right)^2 \frac{e^{2\alpha t}}{\left(e^{\alpha t} - 1\right)^2} \tag{34}$$

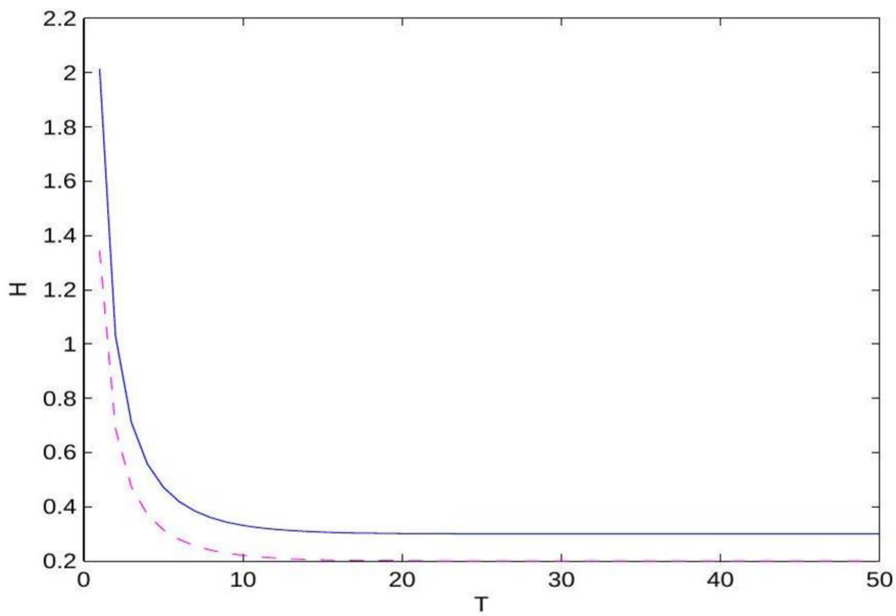

Figure 2: The plot of Hubble parameter H versus cosmic time $t, \alpha_1 = 0.3$, $\alpha_2 = 0.2$;

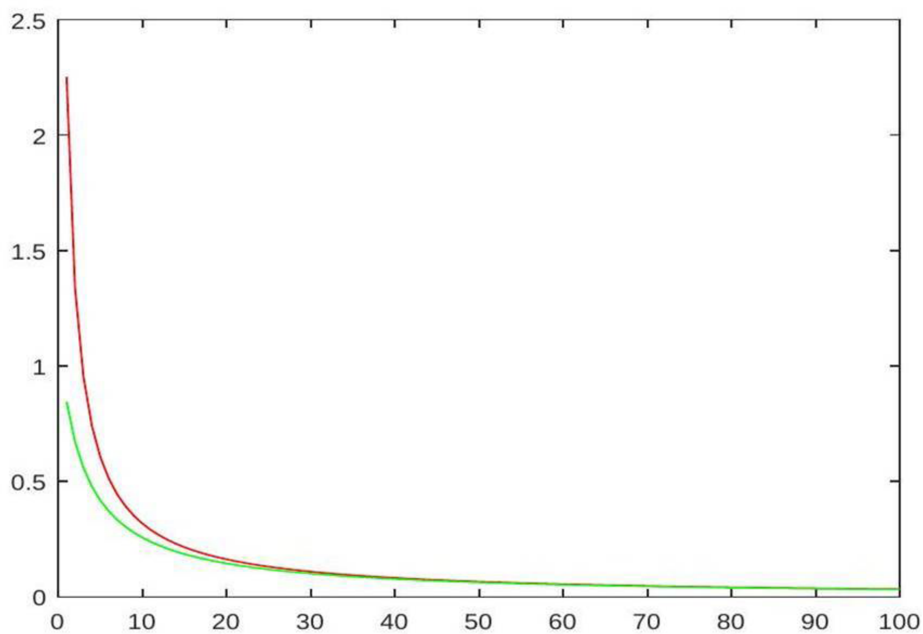

Figure 3: The plot of Expansion scalar θ versus cosmic time $t, \alpha_1 = 0.3$, $\alpha_2 = 0.2$;

$$A_\beta = \frac{9}{(n+2)^2}[n+2]-3, \tag{35}$$

model becomes isotropic for $n = 1$.

$$q = -1 + \frac{\alpha}{e^{\alpha t}}. \tag{36}$$

By using the scale factor in (15) and (17) with the help of (28), we obtain the energy density of the fluid as

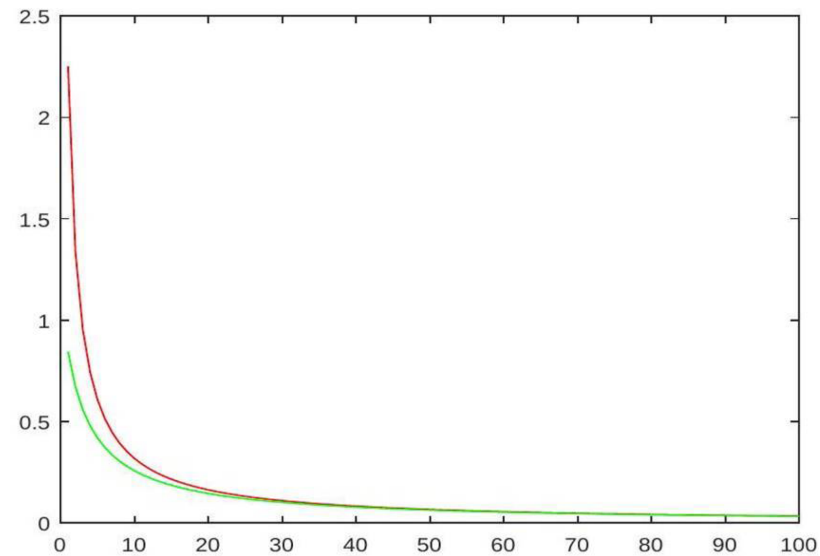

Figure 4: The plot of shear scalar σ^2 versus cosmic time $t, \alpha_1 = 0.3, \alpha_2 = 0.2$;

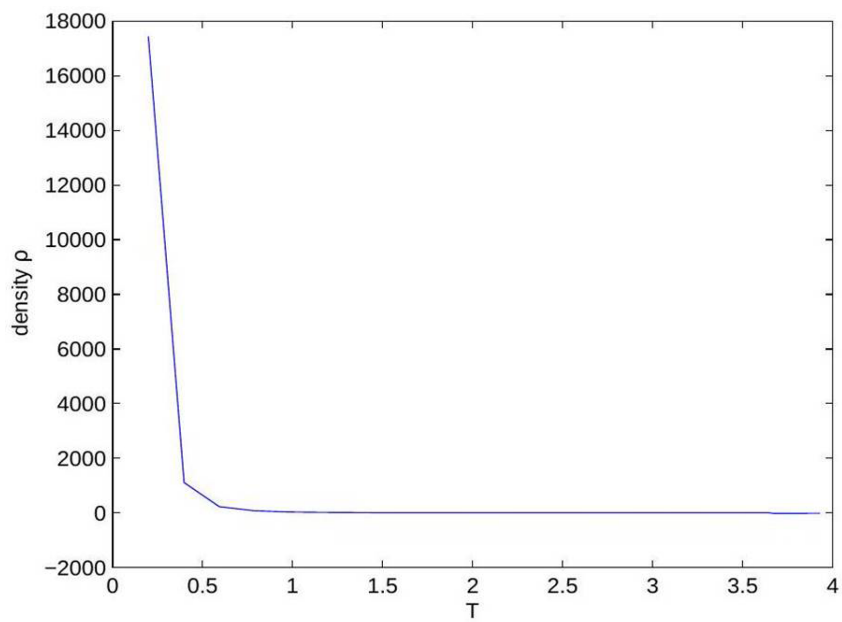

Figure 5: The plot of density ρ versus cosmic time T, $\alpha_1 = 0.2, \alpha_2 = 0.1$

$$\rho = \frac{1}{8\pi + 2\mu} \cdot \frac{18e^{2\alpha t}}{(n+2)^2 \left(e^{\alpha t} - 1\right)^2} \left[(1+n) - \frac{2}{3} e^{-\alpha t} (1+\alpha) \right] = -p \tag{37}$$

$$\omega = -1 + \frac{1}{\rho(8\pi + 2\nu)} \left[9ne^{2\alpha t} + 9n^2 e^{2\alpha t} - 6ne^{\alpha t} - \frac{6\alpha e^{\alpha t} - 3n\alpha e^{\alpha t}}{(2+n)^2 \left(e^{\alpha t} - 1\right)^2} - \frac{n^2}{\left(e^{\alpha t} - 1\right)^{\frac{6}{\alpha(2+n)}}} \right] = -\delta. \tag{38}$$

Since we have $\omega + \delta = 0$ in (28) we find that the volume V of the universe is zero at $t = 0$, and it takes infinitely

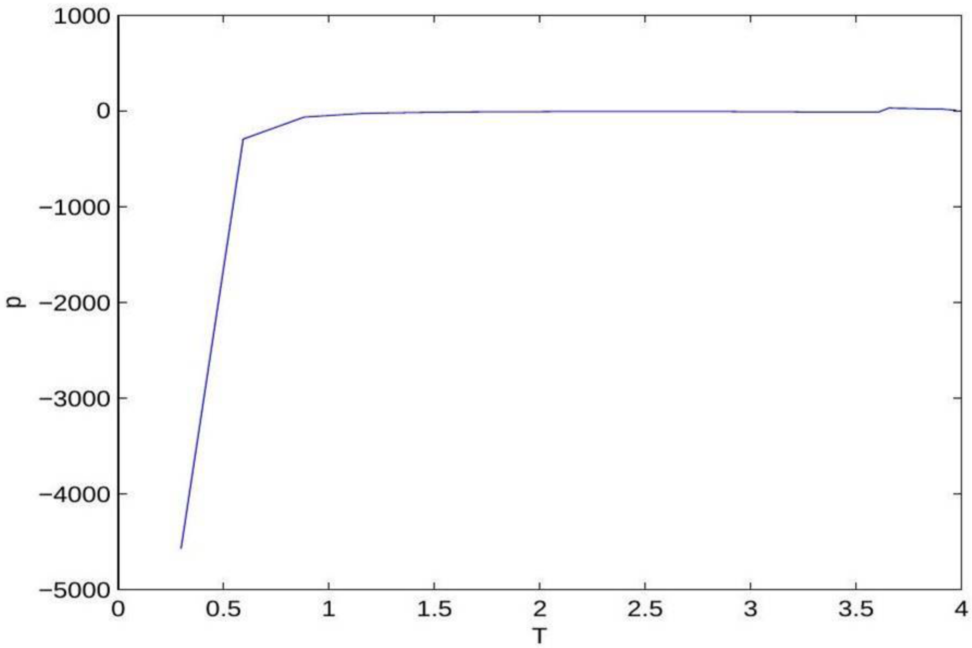

Figure 6: The plot of pressure p versus cosmic time $t, a = 0.1, k = 0.2,\ m = 1$

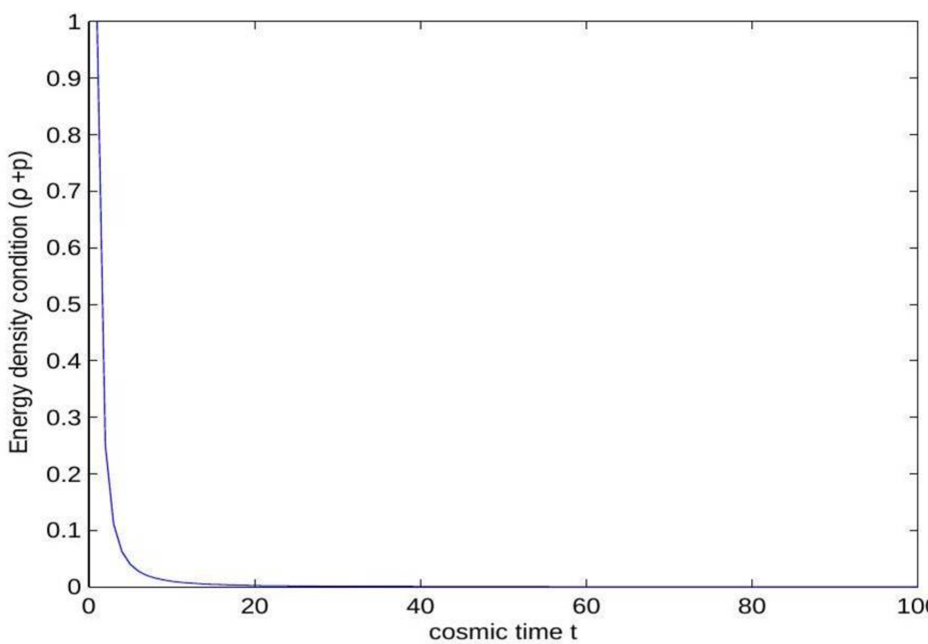

Figure 7: The plot of Energy density condition $\rho + p$ verses cosmic time t, $\alpha_1 = 0.2, \alpha_2 = 0.1, a = 0.1, k = 0.2,\ m = 1$

large value at $t \to \infty$. The expansion scalar is infinite at $t = 0$ and decreases with increase in cosmic time. Thus, the universe starts evolution with zero volume at early time with an infinite rate of expansion and the expansion rate slows down for later times of the universe. We observe that the scale factors $A(t)$ and $C(t)$ are zero at $t = 0$, which implies that the model has initial point- singularity at $t = 0$, whereas these diverge for later time of the universe. The dynamical parameter H is infinite at $t = 0$ and converge to zero at $t \to \infty$. It is also observed that energy density and pressure of the fluid is infinite at $t = 0$ and while converges to finite value at $t \to \infty$.

We also see that EoS and skewness parameters are decreasing function of t i.e at $t = 0 \omega, \delta$ take infinitely large value and converges to finite value at $t \to \infty$.

Also $\frac{\sigma^2}{\theta^2} = \frac{4}{3}\left(\frac{1-n}{2+n}\right) \neq 0$ and hence the model does not approach isotropy for large value of t. However the model becomes isotropic for $n = 1$.

It is observed that the behaviour of the physical parameters and the dark energy model in this case is quite similar to physical parameters an the Bianchi type -III anisotropic dark energy model in $f(R,T)$ gravity theory obtained by Reddy et al.[35] and Bianchi type -III anisotropic dark energy model obtained by Pradhan et al. [43].

5. Conclusion

In this paper, we have considered $[32, 35]$ spatially homogeneous and anisotropic Bianchi type -III dark energy cosmological model with dynamical equation of state parameter in $f(R,T)$ gravity theory in the presence of perfect fluid source. Reducing the anisotropy parameter of the expansion to a simple form, the exact solutions of field equations were obtained by taking special form of deceleration parameter, which was decelerating in the past and accelerating at present time, (deceleration parameter must show signature flipping (Riess et al. [34]; Amendola [4]), in order to match this observation Singh and Debnath[1] have defined for FRW metric.) The corresponding cosmological model represents an early decelerating and late time accelerating expanding universe having singularity at $t = 0$. All the physical and kinematical parameters are well defined for finite time. The $\frac{\sigma^2}{\theta^2} \neq 0$ as $t \to \infty$, and hence anisotropic is maintained throughout passage of time. However the model becomes isotropic for $n = 1$. The pressure approaches negative infinity as $t \to 0$. This strong negative pressure is an indication of dark energy. For this model, $\omega \to -1$ as $t \to \infty$ which corresponds to dark energy dominated universe. It is observed that EoS paramete, skewness parameters in the model are all functions of t. The conditions $\rho + p \geqslant 0$ are identically satisfied .It shows that the universe accelerates after an epoch of deceleration(in Fig 7).The deceleration parameter q is in the range $-1 \leqslant q \leqslant 0$ (in Fig 1)which matches with the observations made by Permutter et al. [28] and Riess et al. [33] and the present day universe is undergoing accelerated expansion.

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