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# Generalized Sasakian-Space-Forms with Quasi-Conformal Curvature Tensor

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## Abstract

In this article, we study generalized Sasakian-space-form with quasi-conformal curvature tensor. In a quasi-conformally flat generalized Sasakian-space-form we find some relations between differentiable functions $f_1$, $f_2$ and $f_3$. In a quasi-conformally flat generalized Sasakian space form we also find Ricci-tensor, Ricci-operator and scaler curvature. Ricci-tensor, Ricci-operator and scalar curvature are also found in a quasi-conformally semi-symmetric generalized Sasakian-space-form.

**Keywords and Phrases :** Generalized Sasakian-space-form, Quasi-conformal curvature tensor, Quasi-conformally semi-symmetric, Ricci-operator, Ricci-tensor, Scalar curvature
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## 1. Introduction

The notion of generalized Sasakian-space-form was introduced and studied by Alegre et al. [1]. A generalized Sasakian-space-form is an almost contact metric manifold $(M, \varphi, \xi, \eta, g)$ whose curvature tensor is given by

$$\begin{aligned}
&R(X,Y)Z \\
&= f_1\left[g(Y,Z)X - g(X,Z)Y\right] \\
&+ f_2\left[g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z\right] \\
&+ f_3[\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\
&+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi],
\end{aligned} \tag{1.1}$$

where $f_1, f_2, f_3$ are differentiable functions on $M$ and $X, Y, Z$ are vector fields on $M$. In such a case we shall write generalized Sasakian-space-form as $M(f_1, f_2, f_3)$. This type of manifold appears as a natural generalization of the well known Sasakian-space-form $M(c)$, which can be obtain as a particular case of generalized Sasakian-space-form by taking $f_1 = \frac{c+3}{4}$ and $f_2 = f_3 = \frac{c-1}{4}$, where $c$ denotes constant $\varphi$-sectional curvature. Moreover cosymplectic space forms, Kenmotsu space forms are also particular case of generalized Sasakian-space-forms $M(f_1, f_2, f_3)$. In the recent paper Alegre and Carriazo [2] ale08 studied contact metric and trans-Sasakian generalized Sasakian-space-forms. Conformally flat and locally symmetric generalized Sasakian-space-form was studied by Kim [8].

In this paper we have studied generalized Sasakian-space-forms with quasi-conformal curvature tensor. The notion of quasi-conformal curvature tensor was introduced by Yano and Sawaki [10]. A $(2n+1)$-dimensional Riemannian manifold $M$ is quasi-conformally flat if $\widetilde{C} = 0$, where $\widetilde{C}$ is quasi-conformal curvature tensor, defined as

$$\begin{aligned} &\widetilde{C}(X,Y)Z \\ &= aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y \\ &+ g(Y,Z)QX - g(X,Z)QY] \\ &- \frac{\tau}{(2n+1)}\left[\frac{a}{2n} + 2b\right][g(Y,Z)X - g(X,Z)Y], \end{aligned} \tag{1.2}$$

where $a$ and $b$ are constants and $R$, $S$, $Q$ and $\tau$ are the Riemannian curvature-tensor, the Ricci-tensor, the Ricci-operator and the scalar curvature of the manifold respectively. If $a = 1$ and $b = -\frac{1}{2n-1}$, then $\widetilde{C}$ becomes conformal curvature tensor. Thus conformal curvature tensor is a particular case of quasi-conformal curvature tensor. It is known that the quasi-conformally flat manifold is either conformally flat if $a \neq 0$ or Einstein if $a = 0$ and $b \neq 0$ [3].

In the present paper we have also studied quasi-conformally semi-symmetric generalized Sasakian-space-forms. If a Riemannian manifold satisfies $R(X,Y)\widetilde{C} = 0$ where $\widetilde{C}$ is the quasi-conformal curvature tensor, then the manifold is said to be quasi-conformally semi-symmetric manifold [5].

## 2. Preliminaries

In this section, we recall some general definitions and basic formulas which we will use later. For this, we recommend the reference [4]. A $(2n+1)$-dimensional Riemannian manifold $(M, g)$ is said to be an almost contact metric manifold if there exist a $(1,1)$ tensor field $\varphi$, a unique global non-vanishing structural vector field $\xi$ (called the vector field) and a 1-form $\eta$ such that

$$\eta(\xi) = 1, \qquad \varphi^2 X = -X + \eta(X)\xi, \qquad \varphi\xi = 0, \tag{2.1}$$

$$d\eta(X,\xi) = 0, \qquad g(X,\xi) = \eta(X), \tag{2.2}$$

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.3}$$

$$d\eta(X,Y) = g(X,\varphi Y) \qquad \eta o\varphi = 0. \tag{2.4}$$

Such a manifold is called contact manifold if $\eta\wedge(d\eta)^n \neq 0$, where $n$ is $n^{th}$ exterior power. For contact manifold we also have $d\eta = \Phi$, where $\Phi(X,Y) = g(\varphi X, Y)$ is called fundamental 2-form on $M$. If $\xi$ is killing vector field, then $M$ is said to be $K$-contact manifold. The almost contact metric structure $(\varphi, \xi, \eta, g)$ on $M$ is said to be normal if

$$[\varphi,\varphi](X,Y) + 2d\eta(X,Y)\xi = 0, \tag{2.5}$$

for any vector fields $X, Y$ on $M$, where $[\varphi,\varphi]$ denotes the Nijenhuis torsion of $\varphi$ given by

$$\begin{aligned}[\varphi,\varphi](X,Y) =& \varphi^2[X,Y] + [\varphi X, \varphi Y] \\ & - \varphi[\varphi X, Y] - \varphi[X, \varphi Y].\end{aligned} \tag{2.6}$$

An almost contact metric manifold is called trans-Sasakian manifold of type $(\alpha, \beta)$ if

$$\begin{aligned}(\nabla_X \varphi) Y =& \alpha\left(g(X,Y)\xi - \eta(X)Y\right) \\ & + \beta\left(g(\varphi X, Y)\xi - \eta(X)\varphi Y\right),\end{aligned} \tag{2.7}$$

for all vector field $X, Y$ on $M$, where $\alpha$ and $\beta$ are some smooth real valued functions. A trans-Sasakian manifold of type $(1,0)$ and $(0,1)$ is called Sasakian and Kenmotsu manifold respectively.

An almost contact metric manifold $M$ is said to be $\eta$-Einstein if its Ricci-tensor $S$ is of the form

$$S(X,Y) = cg(X,Y) + d\eta(X)\eta(Y), \tag{2.8}$$

where $c$ and $d$ are smooth functions on $M$. A $\eta$-Einstein manifold becomes Einstein if $d = 0$.

If $\{e_1, e_2, ......, e_{2n}, \xi\}$ is a local orthonormal basis of vector fields in a $(2n+1)$-dimensional almost contact manifold $M$, then $\{\varphi e_1, \varphi e_2, ............, \varphi e_{2n}, \xi\}$

is also a local orthonormal basis. It is easy to verify that

$$\sum_{i=1}^{2n} g\left(e_i, e_i\right) = \sum_{i=1}^{2n} g\left(\varphi e_i, \varphi e_i\right) = 2n, \tag{2.9}$$

$$\begin{aligned}\sum_{i=1}^{2n} g\left(e_i, Y\right) S\left(X, e_i\right) =& \sum_{i=1}^{2n} g\left(\varphi e_i, Y\right) S\left(X, \varphi e_i\right) \\ =& S\left(X, Y\right) - S\left(X, \xi\right) \eta\left(Y\right),\end{aligned} \tag{2.10}$$

for all $X, Y \in TM$. In view of (2.4) and (2.10), we get

$$\begin{aligned}\sum_{i=1}^{2n} g\left(e_i, \varphi Y\right) S\left(\varphi X, e_i\right) =& \sum_{i=1}^{2n} g\left(\varphi e_i, \varphi Y\right) S\left(\varphi X, \varphi e_i\right) \\ =& S\left(\varphi X, \varphi Y\right).\end{aligned} \tag{2.11}$$

### 2.1. Some Results on generalized Sasakian-space-form

On taking $Z = \xi$ in the equation (1.1), we have

$$R\left(X, Y\right) \xi = \left(f_1 - f_3\right)\left(\eta\left(Y\right) X - \eta\left(X\right) Y\right). \tag{2.12}$$

Again from (1.1) and taking account of $S(X,Y) = \sum_{i=1}^{(2n+1)} g(R(e_i, X)Y, e_i)$, we get

$$\begin{aligned}S\left(X, Y\right) =& \left(2nf_1 + 3f_2 - f_3\right) g\left(X, Y\right) \\ &- \left(3f_2 + \left(2n-1\right) f_3\right) \eta(X)\eta(Y).\end{aligned} \tag{2.13}$$

From (2.12), we have

$$R\left(X, \xi\right) \xi = \left(f_1 - f_3\right)\left(X - \eta\left(X\right) \xi\right), \tag{2.14}$$

$$R\left(X, \xi\right) Y = \left(f_1 - f_3\right)\left(\eta\left(Y\right) X - g\left(X, Y\right) \xi\right) \tag{2.15}$$

and from (2.13), we have

$$Q\left(X\right) = \left(2nf_1 + 3f_2 - f_3\right) X - \left(3f_2 + \left(2n-1\right) f_3\right) \eta(X).\xi, \tag{2.16}$$

$$\tau = 2n\left(2n+1\right) f_1 + 6nf_2 - 4nf_3, \tag{2.17}$$

where $Q$ is the Ricci operator and $\tau$ is scalar curvature of $M\left(f_1, f_2, f_3\right)$. Now from (2.13) and (2.16), we have

$$S\left(X, \xi\right) = 2n\left(f_1 - f_3\right) \eta(X) \tag{2.18}$$

and

$$Q\xi = 2n\left(f_1 - f_3\right) \xi. \tag{2.19}$$

From (2.18), we get

$$\sum_{i=1}^{2n} S(e_i, e_i) = \sum_{i=1}^{2n} S(\varphi e_i, \varphi e_i) = \tau - 2n(f_1 - f_3), \tag{2.20}$$

where $\tau$ is scalar curvature. In a generalized Sasakian-space-form $M(f_1, f_2, f_3)$ we also have

$$R(X, \xi, \xi, Y) = R(\xi, X, Y, \xi) = (f_1 - f_3)\, g(\varphi X, \varphi Y) \tag{2.21}$$

and

$$\begin{aligned} \sum_{i=1}^{2n} R(e_i, X, Y, e_i) &= \sum_{i=1}^{2n} R(\varphi e_i, X, Y, \varphi e_i) \\ &= S(X, Y) - (f_1 - f_3)\, g(\varphi X, \varphi Y), \end{aligned} \tag{2.22}$$

for all $X, Y \in TM$.

## 3. Quasi-conformally flat generalized Sasakian-space-form

Let $M(f_1, f_2, f_3)$ be a $(2n+1)$ -dimensional generalized Sasakian-space-form. Then the Riemannian curvature-tensor $R$, the Ricci-tensor $S$ and the Ricci-operator $Q$ of $M$ are given by equations (1.1), (2.13) and (2.16) respectively. On putting the value of $R(X, Y)Z$, $S(X, Y)$ and $QX$ in the equation (1.2), we get

$$\begin{aligned} &\widetilde{C}(X, Y) Z \\ &= a[\frac{-1}{2n+1}(3f_2 - 2f_3)\{g(Y, Z)X - g(X, Z)Y\} \\ &+ f_2\{g(X, \varphi Z)\varphi Y - g(Y, \varphi Z)\varphi X + 2g(X, \varphi Y)\varphi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X, Z)\eta(Y)\xi \\ &- g(Y, Z)\eta(X)\xi\}] \\ &+ b[\frac{(6f_2 + 2(2n-1)f_3)}{(2n+1)}\{g(Y, Z)X - g(X, Z)Y\} \\ &- (3f_2 + (2n-1)f_3)\{\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y \\ &+ g(Y, Z)\eta(X)\xi - g(X, Z)\eta(Y)\xi\}]. \end{aligned} \tag{3.1}$$

If $M(f_1, f_2, f_3)$ is quasi-conformally flat, then we have $\widetilde{C}(X, Y)Z = 0$. If we put $X = \varphi Y$ in the above equation, we get

$$\begin{aligned}
&a[\frac{-1}{2n+1}(3f_2-2f_3)\{g(Y,Z)\varphi Y-g(\varphi Y,Z)Y\}\\
&+f_2\{g(Y,Z)\varphi Y-\eta(Y)\eta(Z)\varphi Y+g(Y,\varphi Z)Y\\
&-g(Y,\varphi Z)\eta(Y)\xi+2g(Y,Y)\varphi Z-2\eta(Y)\eta(Y)\varphi Z\}\\
&+f_3\{\eta(Y)\eta(Z)\varphi Y+g(\varphi Y,Z)\eta(Y)\xi\}]\\
&+b[\frac{(6f_2+2(2n-1)f_3)}{(2n+1)}\{g(Y,Z)\varphi Y-g(\varphi Y,Z)Y\}\\
&-(3f_2+(2n-1)f_3)\{\eta(Y)\eta(Z)\varphi Y-g(\varphi Y,Z)\eta(Y)\xi\}=0.
\end{aligned} \tag{3.2}$$

If we choose a unit vector $U$ such that $\eta(U)=0$ and put $Y=U$ in the equation (3.2), then we get

$$\begin{aligned}
&a[\frac{(2f_3-3f_2)}{(2n+1)}\{g(U,Z)\varphi U-g(\varphi U,Z)U\}\\
&+f_2\{g(U,Z)\varphi U-g(\varphi U,Z)U+2\varphi Z\}]\\
&+b[\frac{(6f_2+2(2n-1)f_3)}{(2n+1)}\{g(U,Z)\varphi U-g(\varphi U,Z)U\}]=0.
\end{aligned}$$

Again taking $Z=U$ in the above equation, we get

$$[(3nf_2+f_3)a+(3f_2+(2n-1)f_3)b]\varphi U=0. \tag{3.3}$$

In view of equation (3.3) we have following theorem:

**Theorem 3.1.** In a quasi-conformally flat generalized Sasakian-space-form

$$(3nf_2+f_3)a+(3f_2+(2n-1)f_3)b=0.$$

It is known that a quasi-conformally flat manifold becomes Einstein if $a=0$ and $b\neq 0$ [3]. Thus, analogous to [3], if a quasi-conformally flat generalized Sasakian-space-form is Einstein, then theorem 3.1 implies

$$3f_2+(2n-1)f_3=0 \tag{3.4}$$

and equation (2.13) also implies the same condition for Einstein generalized Sasakian-space-form. So we have following corollary:

**Corollary 3.1.** Quasi-conformally flatness ($a=0,\ b\neq 0$) does not change the condition of Einstein generalized Sasakian-space-form.

Now under the consideration of quasi-conformally flat manifold equation (1.2) reduces to

$$
\begin{aligned}
&R(X,Y,Z,W)\\
&=\frac{b}{a}[S(X,Z)\,g(Y,W)-S(Y,Z)\,g(X,W)\\
&+S(X,W)\,g(X,Z)-S(X,W)\,g(Y,Z)]\\
&+\frac{\tau}{a(2n+1)}\left[\frac{a}{2n}+2b\right][g(Y,Z)\,g(X,W)\\
&-g(X,Z)\,g(Y,W)],
\end{aligned}
\tag{3.5}
$$

where $R(X,Y,Z,W)=g(R(X,Y)Z,W)$.

On taking $Z=\xi$ in the equation (3.5) and using equations (2.2 ), (2.12) and (2.18), we get

$$
\begin{aligned}
&(f_1-f_3)[\eta(Y)\,g(X,W)-\eta(X)\,g(Y,W)]\\
=\;&\frac{b}{a}[2n(f_1-f_3)\{\eta(X)\,g(Y,W)-\eta(Y)\,g(X,W)\}\\
&+\{\eta(X)\,S(Y,W)-\eta(Y)\,S(X,W)\}]\\
&+\frac{\tau}{a(2n+1)}\left[\frac{a}{2n}+2b\right][\eta(Y)\,g(X,W)-\eta(X)\,g(Y,W)]\,.
\end{aligned}
$$

Again putting $X=\xi$ and using equations (2.1), (2.2) and (2.18), we get

$$
\begin{aligned}
&S(Y,W)\\
&=[\frac{\tau}{b(2n+1)}\left(\frac{a}{2n}+2b\right)-2n(f_1-f_3)\\
&-(f_1-f_3)\frac{a}{b}]g(Y,W)+[(f_1-f_3)\frac{a}{b}\\
&+4n(f_1-f_3)-\frac{\tau}{b(2n+1)}\left(\frac{a}{2n}+2b\right)]\eta(Y)\,\eta(W)\,.
\end{aligned}
\tag{3.6}
$$

Putting $W=\xi$ in the above equation, we get

$$
QY=2n(f_1-f_3)\,Y. \tag{3.7}
$$

If $\{e_1,e_2,......,e_{2n},e_{2n+1}=\xi\}$ is a local orthonormal basis of vector fields in $M(f_1,f_2,f_3)$, then from equation (3.6) we get

$$\begin{aligned}\sum_{i=1}^{2n+1} S(e_i,e_i) &= [\frac{\tau}{b(2n+1)}\left(\frac{a}{2n}+2b\right)-2n(f_1-f_3)\\ &-(f_1-f_3)\frac{a}{b}]\sum_{i=1}^{2n+1} g(e_i,e_i)+[(f_1-f_3)\frac{a}{b}\\ &+4n(f_1-f_3)-\frac{\tau}{b(2n+1)}\left(\frac{a}{2n}+2b\right)]\sum_{i=1}^{2n+1}\eta(e_i)\eta(e_i).\end{aligned}$$

Using equations (2.2), (2.9) and (2.20), we get

$$\tau=\frac{2n(2n+1)(a+(2n-1)b)(f_1-f_3)}{(a+2n(2n-1)b)}. \tag{3.8}$$

**Theorem 3.2.** In a quasi-conformally flat generalized Sasakian-space-form $M(f_1,f_2,f_3)$ Ricci-tensor $S$, Ricci-operator $Q$ and scalar curvature $\tau$ are given by the equations (3.6), (3.7) and (3.8) respectively.

## 4. Quasi-conformally semi-symmetric generalized Sasakian-space-form

Let $M(f_1,f_2,f_3)$ be a $(2n+1)$-dimensional generalized Sasakian-space-form. We obtain from equation (1.2) by using equations (2.2), (2.12) and (2.18)

$$\begin{aligned}&\eta\left(\widetilde{C}(X,Y)Z\right)\\ &=\left((a+2nb)(f_1-f_3)-\frac{\tau}{(2n+1)}\left(\frac{a}{2n}+2b\right)\right)\\ &[g(Y,Z)\eta(X)-g(X,Z)\eta(Y)]\\ &+b[S(Y,Z)\eta(X)-S(X,Z)\eta(Y)].\end{aligned} \tag{4.1}$$

On taking $Z=\xi$ in the equation (4.1), we get

$$\eta\left(\widetilde{C}(X,Y)\xi\right)=0 \tag{4.2}$$

and on taking $X=\xi$ in the equation (4.1), we get

$$\begin{aligned}&\eta\left(\widetilde{C}(\xi,Y)Z\right)\\ &=\left((a+2nb)(f_1-f_3)-\frac{\tau}{(2n+1)}\left(\frac{a}{2n}+2b\right)\right)\\ &[g(Y,Z)-\eta(Y)\eta(Z)]\\ &+b[S(Y,Z)-2n(f_1-f_3)\eta(Y)\eta(Z)].\end{aligned} \tag{4.3}$$

The condition of quasi-conformally semi-symmetric manifold is

$$R(X,Y).\widetilde{C}=0. \tag{4.4}$$

In virtue of above equation, we get

$$\begin{aligned} &R(X,Y)\widetilde{C}(U,V)W-\widetilde{C}(R(X,Y)U,V)W \\ &-\widetilde{C}(U,R(X,Y)V)W-\widetilde{C}(U,V)R(X,Y)W=0, \end{aligned} \tag{4.5}$$

which implies that

$$\begin{aligned} &(f_1-f_3)[\widetilde{C}(U,V,W,Y)-\eta(Y)\eta\left(\widetilde{C}(U,V)W\right) \\ &+\eta(U)\eta\left(\widetilde{C}(Y,V)W\right)+\eta(V)\eta\left(\widetilde{C}(U,Y)W\right) \\ &+\eta(W)\eta\left(\widetilde{C}(U,V)Y\right)-g(Y,U)\eta\left(\widetilde{C}(\xi,V)W\right) \\ &-g(Y,V)\eta\left(\widetilde{C}(U,\xi)W\right)-g(Y,W)\eta\left(\widetilde{C}(U,V)\xi\right)]=0. \end{aligned} \tag{4.6}$$

The above equation states that either $f_1=f_3$ or

$$\begin{aligned} &\widetilde{C}(U,V,W,Y)-\eta(Y)\eta\left(\widetilde{C}(U,V)W\right) \\ &+\eta(U)\eta\left(\widetilde{C}(Y,V)W\right)+\eta(V)\eta\left(\widetilde{C}(U,Y)W\right) \\ &+\eta(W)\eta\left(\widetilde{C}(U,V)Y\right)-g(Y,U)\eta\left(\widetilde{C}(\xi,V)W\right) \\ &-g(Y,V)\eta\left(\widetilde{C}(U,\xi)W\right)-g(Y,W)\eta\left(\widetilde{C}(U,V)\xi\right)=0. \end{aligned} \tag{4.7}$$

If $f_1\neq f_3$ then equation (4.7) must be true. Now we proceed under the assumption that $f_1\neq f_3$. Putting $U=Y$ in (4.7) and using equations (4.1) and (4.2), we get

$$\begin{aligned} &\widetilde{C}(Y,V,W,Y)+\eta(W)\eta\left(\widetilde{C}(Y,V)Y\right) \\ &-g(Y,Y)\eta\left(\widetilde{C}(\xi,V)W\right)-g(Y,V)\eta\left(\widetilde{C}(Y,\xi)W\right)=0. \end{aligned} \tag{4.8}$$

Let $\{e_1,e_2,......,e_{2n},e_{2n+1}=\xi\}$ is a local orthonormal basis of vector fields in $M(f_1,f_2,f_3)$. Putting $Y=e_i$ in the above equation and taking the summation over $i$, $1\leq 2n+1$, we get

$$\begin{aligned} &\sum_{i=1}^{2n+1}\widetilde{C}(e_i,V,W,e_i)+\eta(W)\sum_{i=1}^{2n+1}\eta\left(\widetilde{C}(e_i,V)e_i\right) \\ &-(2n+1)\eta\left(\widetilde{C}(\xi,V)W\right)-\sum_{i=1}^{2n+1}g(e_i,V)\eta\left(\widetilde{C}(e_i,\xi)W\right)=0. \end{aligned} \tag{4.9}$$

Now using equations (2.9), (2.10), (2.20), (2.22), (4.1) and (4.3), we get

$$\begin{aligned} &S(V,W) = \\ &\frac{(2n(a+2nb)(f_1-f_3)-\tau b)}{(a-b)} g(V,W) \\ &+\frac{(2a+(6n-1)b)(\tau-2n(2n+1)(f_1-f_3))}{(2n+1)(a-b)}\eta(V)\eta(W). \end{aligned} \tag{4.10}$$

On taking $W=\xi$ in the equation (4.10), we get

$$QV = \frac{2(a+(2n-1)b)\tau - 2n(2n+1)(a+(4n-1)b)(f_1-f_3)}{(2n+1)(a-b)}V \tag{4.11}$$

and on taking $V=W=\xi$ in the equation (4.10), we get

$$\tau = \frac{4n^2(2n+1)b(f_1-f_3)}{(a+(2n-1)b)}. \tag{4.12}$$

**Theorem 4.1.** In a quasi-conformally semi-symmetric generalized Sasakian-space-form $M(f_1,f_2,f_3)$ Ricci-tensor $S$, Ricci-operator $Q$ and scalar curvature $\tau$ are given by the equations (4.10), (4.11) and (4.12) respectively.

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