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Tilted LRS Bianchi Type-VIo Dark Energy Cosmological Model

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Abstract

Tilted LRS Bianchi-VIo dark energy cosmological model is investigated in Einstein's theory of gravitation. We obtained solutions by considering shear scalar proportional to expansion scalar and deceleration parameter. Further we obtained an accelerating universe subject to the small value of the anisotropic parameter. Some physical and geometrical properties of the solutions are also discussed.

Keywords: Dark fluid; Tilted cosmological model; Heat conduction.

1. Introduction

Locally Rotationally Symmetric (LRS) Bianchi type-VIo cosmological model is said to be tilted if the fluid velocity vector is not orthogonal to the group of orbits. In the tilted LRS Bianchi type-VIo cosmological model, the tilt can become extreme in a finite time as measured along the fluid congruence, which implies that the group of orbits becomes time-like. The general dynamics of tilted cosmological models have been studied by Kings and Ellis,[1] Ellis and King,[2] Collins and Ellis,[3] Bali and Sharma,[4,5] Pawar et al,.[6] Rao et al.,[7] and Sahu et al.[8,9]

The interesting problem in astrophysics and cosmology is to know about the nature of dark energy. Observations from distant type Ia supernovae (SNIa) predict that universe is accelerating.[10-13] The presence of dark energy with a negative pressure are observed from the experiments such as X-ray clusters [14] and Baryon Acoustic Oscillations (BAO).[15] The dark energy yield negative pressure which helps to accelerate the universe. Dark energy constitute 68.3 % dark energy, 26.8% dark matter and 4.9% baryonic matter.[16,17]. Cosmic microwave background (CMB) radiation form from Wilkinson microwave anisotropy probe (WMAP) predict the presence of Λ dominated cold dark matter model (ΛCDM).[18,19]

We derived the Einstein's field equations for the tilted LRS Bianchi type VIo metric in sect. 2. We solved the Einstein's field equations in sect. 3. We mentioned the physical and geometrical properties of the solution in sect. 4 and given concluding remarks in sect.5.

2. Field Equations

We consider here the LRS Bianchi type VI_o metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(e^{-2hx}dy^2 + e^{2hx}dz^2) \tag{1}$$

where A and B are function of cosmic time t only and h is a constant.

Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. Recent studies helps to extract the properties of dark energy component of the Universe from observational data in order to determine the equation of state, EoS (ϖ (t)), where

$$\omega(t) = \frac{p}{\rho} \quad (2)$$

is the ratio of the pressure to its energy density

The directional Hubble parameter H_1 and H_2 are given by

$$H_1 = \frac{A_4}{A} \quad (3)$$

and

$$H_2 = \frac{B_4}{B} \quad (4)$$

The mean Hubble parameter is written as

$$H = \frac{1}{3}(H_1 + 2H_2) \quad (5)$$

The scalar expansion is proportional to shear tensor.[8] This condition yields a relation between the Hubble parameters as

$$H_1 = nH_2 \quad (6)$$

The deceleration parameter q is defined by

$$q = -1 - \frac{\dot{H}}{H^2} \quad (7)$$

Here the dot over the variable represents the ordinary differentiation with respect to time

The Einstein's field equations in the presence of dark energy as well as heat conduction vector are given by

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \quad (8)$$

where

$$T_i^j = (1+\omega)\rho u_i u^j + \omega\rho g_i^j + q_i u^j + u_i q^j \quad (9)$$

together with

$$g_{ij} u^i u^j = -1 \quad (10)$$

$$q_i q^i > 0 \quad (11)$$

$$q_i u^i = 0 \quad (12)$$

where ρ is the energy density of dark energy, q_i is the heat conduction vector orthogonal to u^i. The fluid vector u^i has the components

$\left(\frac{\sinh\lambda}{A}, 0, 0, \cosh\lambda\right)$ satisfying equation (10) and λ is the tilt angle.

The dimensionless Planckian units play an important rolein cosmology. All the fundamental physical constants, such as the gravitational constant G, the speed of light c and Boltzmann's constant k_B, describe the nature the universe. The units used in the work are G(=c=k_B) =1.

With the help of equations (2-6), the explicit form of the field equation (8) for the metric (1) can be written as

$$6(n+2)\dot{H} + 27H^2 + (n+2)^2\frac{h^2}{A^2} = -(n+2)^2\left[(1+w)\rho\sinh^2\lambda + w\rho + 2q_1\frac{\sinh\lambda}{A}\right] \tag{13}$$

$$3(n^2+3n+2)\dot{H} + 9(n^2+n+1)H^2 - (n+2)^2\frac{h^2}{A^2} = -(n+2)^2 w\rho \tag{14}$$

$$9(2n+1)H^2 - (n+2)^2\frac{h^2}{A^2} = -(n+2)^2\left[-(1+w)\rho\cosh^2\lambda + w\rho - 2q_1\frac{\sinh\lambda}{A}\right] \tag{15}$$

$$(1+w)\rho A\sinh\lambda\cosh\lambda + q_1\cosh\lambda + q_1\frac{\sinh^2\lambda}{\cosh\lambda} = 0 \tag{16}$$

3. Solutions

Eq. (7) yields

$$H = \frac{1}{(1+q)t} \tag{17}$$

where

$$q = \frac{-2(1-n)}{n+2} \tag{18}$$

Using eqs. (3), (6) and (17) in eq. (5) , we get

$$A = t^k \tag{19}$$

where

$$k = \frac{3n}{(n+2)(1+q)}$$

and

$$B = t^{\frac{k}{n}} \tag{20}$$

Thus the corresponding metric of our solution can be written as

$$ds^2 = -dt^2 + t^{2k}dx^2 + t^{\frac{2k}{n}}\left(e^{-2hx}dy^2 + e^{2hx}dz^2\right) \tag{21}$$

4. Physical and Geometrical Properties of the Solutions

Equating eqs. (13 - 15) ,we obtain

$$\rho = \frac{3}{n^2 t^2} + \frac{h^2}{t^{2k}} \tag{22}$$

The methods of restoration of the quantity, ω(t) from experimental data have been developed by.[20] The simplest candidate for dark energy is the vacuum energy (ω = -1), which is mathematically equivalent to the cosmological constant. Observational results coming from SNeIa data with cosmic microwave background radiation anisotropy and galaxy clustering statistics provide, -1.33 $< \omega <$ -0.79 [21-23] considered EoS parameter as constant with phase wise value −1, 0, +1/3, and +1 which represent vacuum fluid, dust fluid, radiating fluid and stiff fluid universe.

Substituting eqs. (17), (19) and (22) in eq. (14), we get

$$\omega = -\left(\frac{1 - h^2 n^2 t^{-2(1-k)}}{3 + h^2 n^2 t^{-2(1-k)}}\right) \tag{23}$$

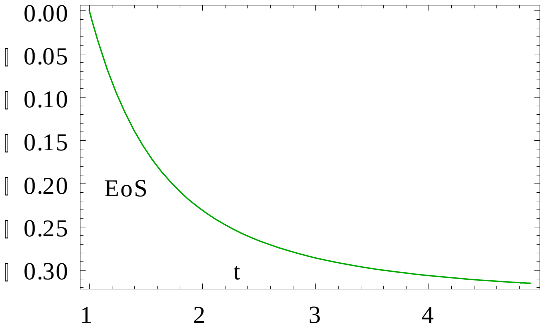

Figure 1: Graphical representation of time verses EoS parameter

From Figure 1, we obtained constant EoS parameter when t $\rightarrow \infty$.

Using eqs. (22-23) in eq. (2), we get

$$p = \frac{\frac{3h^2}{t^{2(2-k)}} + \left(n^2 h^2 - \frac{3}{n^2}\right)\frac{1}{t^2} - \frac{h^2}{t^{2k}}}{3 + \frac{n^2 h^2}{t^{2(1-k)}}} \tag{24}$$

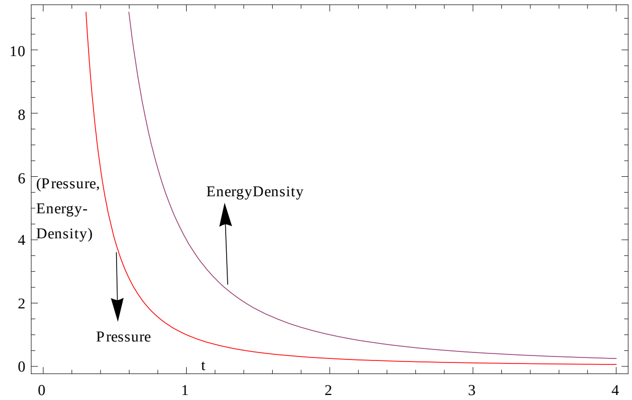

Figure 2: Graphical representation of time verses pressure and energy density

From Figure 2, we find that p → ∞ and ρ → ∞ as t → 0 and pressure and density decreases as t →∞. This implies that in this case the model (21) starts expanding with a big-bang at t = 0.
Substituting eqs. (17), (19) and (22-23) in eqs. (13) and (16), we get

$$\lambda = Co\sec h^{-2}\left[\frac{\left(\frac{2+2n^2h^2t^{-2(1-k)}}{3+n^2h^2t^{-2(1-k)}}\right)\left(\frac{3}{n^2h^2}+\frac{h^2}{t^{2k}}\right)(n+2)^2}{\frac{(n+2)^2}{t^2}\left(\frac{3}{n^2}-2\right)+\frac{(n+2)^2h^2}{t^{2k}}}-2\right] \tag{25}$$

and

$$q_1 = \left(\frac{2+h^2n^2t^{-2(1-k)}}{3+h^2n^2t^{-2(1-k)}}\right)\left(\frac{3}{n^2t^{2-k}}+\frac{h^2}{t^k}\right)\frac{\sinh\lambda}{1+\tanh\lambda} \tag{26}$$

The scalar expansion for the model (21) is obtained as

$$\theta = \frac{(n+2)}{nt}\cosh\lambda + \sinh\lambda \tag{27}$$

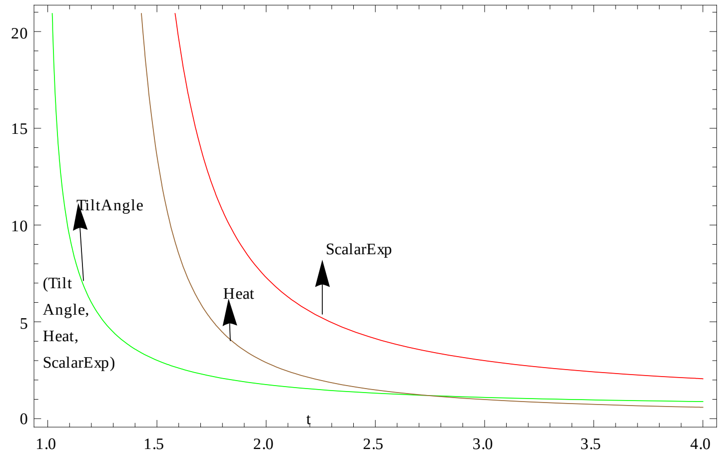

Figure 3: Graphical representation of time verses tilt angle, heat conduction vector and scalar expansion

From Figure 3 we obtain that λ→ ∞, q_1→ ∞ and θ→ ∞ at t =0 . Further we find that λ→ ∞, q_1→ ∞ and θ→ ∞ as t → ∞.

The components of shear for the model (21) are given by

$$\sigma_{11} = \frac{2}{3}\cosh\lambda\left(1-\frac{kt^{2k-1}}{n}\right)\left(1-\sinh^2\lambda\right)-t^{2k}\sinh\lambda\left[\left(\frac{1}{3}+\cosh\lambda\right)+\frac{1}{3}\sinh^2\lambda\right] \tag{28}$$

$$\sigma_{22} = \left[-t^{\frac{2k}{n}}\sinh\lambda\left(\frac{h}{t^k}+\frac{1}{3}\right)+\frac{k}{3}t^{\frac{2k-1}{n}}\cosh\lambda\left(\frac{1-n}{n}\right)\right]e^{-2hx} \tag{29}$$

$$\sigma_{33} = \left[-t^{\frac{2k}{n}}\sinh\lambda\left(\frac{h}{t^k}+\frac{1}{3}\right)+\frac{k}{3}t^{\frac{2k-1}{n}}\cosh\lambda\left(\frac{1-n}{n}\right)\right]e^{2hx} \tag{30}$$

$$\sigma_{44} = \frac{-1}{3}\left[2\sinh\lambda - \left(\frac{n+2}{n}\right)\frac{k}{t}\cosh\lambda\right]\left(1+\cosh^2\lambda\right) - \frac{k}{2t}\sinh 2\lambda \sinh\lambda \tag{31}$$

$$\sigma_{14} = t^k\left[\cosh\lambda - \frac{1}{3}\left(\frac{n+2}{nt}\cosh^2\lambda\sinh\lambda + \sinh^2\lambda\cosh\lambda\right) + \left(\frac{k}{2t}\sinh\lambda + \frac{1}{2}\cosh\lambda\right)\left(1+2\sinh^2\lambda\right)\right] \tag{32}$$

The acceleration for the model (21) is obtained as

$$a_i = \cosh\lambda\sinh\lambda\left(kt^{k-1}-1\right) + t^k\cosh^2\lambda - kt^{-1}\sinh^2\lambda \tag{33}$$

Here the acceleration

$$a_i \neq 0 \text{ as } t \to \infty$$

confirms that the matter particles does not follow geodesic in this theory
It is observed

$$\frac{\sigma^2}{\theta^2} \neq 0 \text{ as } t \to \infty .$$

implies that the model does not approach isotropy for the large value of time.

The jerk parameter for the model (21) is given as

$$j = \frac{R_{444}}{RH^3} = \frac{10n^2 - 14n + 4}{(n+2)^2} \tag{34}$$

Here we find that jerk parameter is constant quantity. It depend upon the anisotropic parameter.[24]

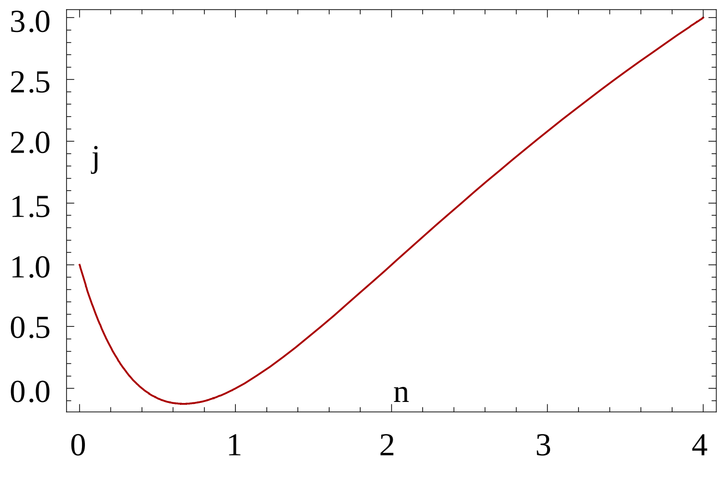

Figure 4: Graphical representation of anisotropic parameter verses jerk parameter

Observations from type Ia Supernovae,[25] SNIa data obtained from the SNLS project [14] and the X-ray galaxy cluster distance measurements,[26] provide the value of the jerk parameter as $j = 2.16^{+0.81}_{-0.75}$

In our case we observe that, Figure 4 support the present accepted range of j.

Further form Figure 4, we obtained that $j=1$ for n=2, which conforms ΛCDM model.[26]

From eq.(18), we find that deceleration parameter is independent of cosmic time [24]. The positive deceleration parameter yields decelerating universe where as the negative deceleration parameter provides accelerating universe. Observations from type-Ia Supernovae predict an accelerating universe with deceleration parameter q=-0.81±0.14.[26] Further Type Ia Supernovae with Baryon Acoustic Oscillation and CMB observation provide the deceleration parameter $q=-0.53^{+0.17}_{-0.13}$.[27,28]

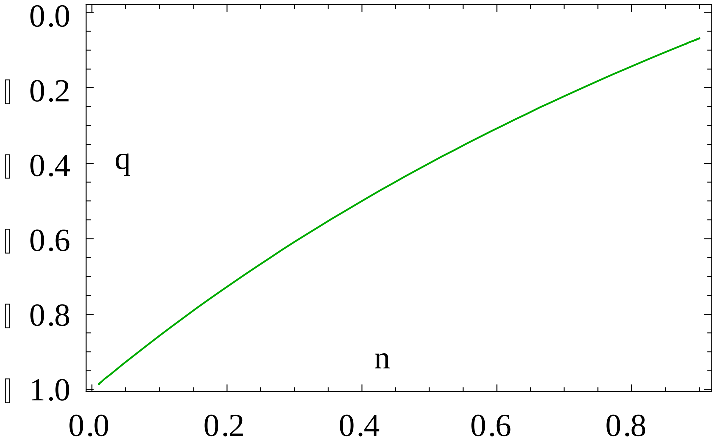

Figure 5: Graphical representation of anisotropic parameter verses deceleration parameter

Figure 5 is in agreement with the present accepted range of q for an accelerating universe subject to the small value of the anisotropic parameter.

5. Conclusion

In this paper, we have investigated Tilted LRS Bianchi- VI_0 dark fluid cosmological model in the frame work of Einstein's theory. Here the EoS parameter of the dark energy turned out to be a constant in the tilted universe. We observed that universe starts with a big-bang at the initial stage. We also observed that universe is accelerating for the small value of the anisotropic parameter.

Authors' Contributions

Subrata Kumar Sahu derived Einstein's field equations with the presence of dark energy . Mihiret Tulamo Tupala obtained the solutions of tilted LRS Bianchi VI_0 cosmological model. Neha written the abstract, introduction and has given concluding remarks. Sultan Ahmad analyzed the geometrical interpretations of the solutions. All authors have read and approved the final manuscript.

Data Availability

The data used to support the findings of the study are included within the article.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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