Journal of The Tensor Society (J.T.S.) ISSN: 0974-5424
Vol. 16 (2022), page 81 - 92

Strange Quark Matter attached to String Cloud in General Relativity under 5D space-time

Saroj R. Kumbhare[1], Praveen Kumar[2] and Safiqul Islam[3]
[1] *Amolakchand Mahavidyalalya Yavatmal-445001, (MS) India.*
[2] *Department of Mathematics*
G H Raisoni College of Engineering, Nagpur-440016 India.
[3] *Department of Mathematics, St. Theresa International College, Thailand. E-mail[1]: moundekarsaroj@gmail.com*
E-mail[2]: pkumar6743@gmail.com
E-mail[3]: sofiqul001@yahoo.co.in

Received : August 27, 2022
Accepted : November 18, 2022
Published : December 30, 2022

In this work, we have solved Einstein's field equations in five dimensional spherically symmetric space-time with strange quark matter attached to string cloud by assuming one parameter group of conformal motions. We have also discussed the solution in five dimensional space-time.

Keywords. Five dimension, Starnge quark matter, Conformal motion.
AMS Classification 98.80.Cq, 98.80.-k, 96.36 +x, 04.50 kd.

1. Introduction.

In the recent year, it has been widely assumed that the symmetry of the universe is shattatered spontaneously during the phase change. The investigation of cosmic strings and physical processes, such strings has received wide attention **Vilenkin**[9] because they are believed to give rise to density perturbation leads to the formation of galaxies **Zel'dovich**[7].

The string theory is quantum gravity theory that, at macroscopic distances, resembles General Reativity. As the stress energy of string can be attached to the gravitation field, it may be intereting to study the gravitational effects that aries from strings. **Yilmaz** [10]**,** [11] have studied $5-D$ Kalunza-Klein cosmological models with quark matter attached to the string cloud and domain wall.

Collineation are geometrical or physical relevant quantities which concern the symmetries of the metric. The important collineation is Conformal Killing Vector (CKV) which gives a better understanding into the space time geometry connected to the astrophysical and cosmological relm (**Mak et al.** [16], **Mars et al.** [17], **Neves**

et al.[22], **Rahman et al.** [20]).

The inheritance symmetries with a CKV and special CKV in fluid space times (perfect, anisotropic, viscous and heat-conducting) has recently attracted some interest. **Yavuz and Yilmaz**[3] and **Yilmaz et al.** [4] who have considered inheriting confomal and special CKV, and also curvature inheritance symmetry in the string cosmology (Sting cloud and String fluids) respectively. **Baysal et al.**[5] have investigated conformal collineation in the string cosmology.

Strange quark matter may be attach to the string cloud. As a result strange quark matter will attach to the string cloud in this work. **Witten**[6] and **Bodmar**[1] have studied strange quark matter in different contexts.

A gravitating object when it undergoes indefinite collapse, the end product is a singularity which is marked by the divergence of physical parameters like energy density. As the singularity is approached, the density diverge and it would therefore be of relevance to consider the state of matter at ultra high density beyond the nuclear matter. One of such possible states could be the strange quark matter which consists of u, d, and s quark.

In the generalized bag model, quarks are assumed to be massless and non integrating, then we get quark pressure $P_q = \frac{\rho_q}{3}$ where ρ_q is the quark energy density. The total energy density are $\rho = \rho_q + B_c$
where B_c is the difference between the energy density of the perturbative and non perturbative quantum chromodynamics (QCD) vacuum (the bag constant) but the total pressure is $p = p_q - B_c$.

The collineation is deifined as

$$L_\xi g_{ab} = 2\psi g_{ab}, \tag{1}$$

where L_ξ is the Lie derivative along collineation vector ξ^α and $\psi = \psi(x^a)$ is the conformal factor. If $\psi_{;ab} = 0$ and $\psi_{,a} = 0$ then ξ is a special CKV where (;) is covariant derivative and (,) is ordinary derivatives. Other sub cases are homothetic vector(HV) if ξ is constant. It is supposed that the vector ξ generates the conformal symmetry and the metric g is conformally mapped onto itself along ξ. Neither ξ nor ψ need to be static, even in the case of static metric **Bohmer et al.** [13] [14].

In present work, we attach charged strange quark to the string cloud in the five dimensional spherical symmetric space time admitting one parameter group of conformal motion. This work is the extension of the work obtained early by **Yavuz et al.** [2] for four dimensional space time.

Our work is organized in the following ways: In section 2, We found Einstein field equations in five dimensional spherical symmetric space time for charged strange quark matter attach to the string cloud. In section 3, We solved the same Einstein field equations for the same matter using idea of conformal motion. In section 4, we summarize our results.

2. The Metric and Field Equations

Here we can consider the five dimensional spherically symmetric line element as

$$ds^2 = e^{\nu(r)}dt^2 - e^{\mu(r)}dr^2 - r^2 d\Omega^2, \tag{2}$$

where

$$d\Omega^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \sin^2\theta_1 \sin^2\theta_2 d\theta_3^2$$

$$x^{1,2,3,4} = r, \theta_1, \theta_2, t.$$

We assume that the total energy-momentum tensor T_{ab} is the sum of the two parts, T^Q_{ab} for string cloud and T^E_{ab} for electromagnetic contributions respectively, i.e.

$$T_{ab} = T^S_{ab} + T^E_{ab}. \tag{3}$$

The energy momentum tensor for string cloud **Letelier**[8] and **Stachel**[12] given by

$$T_{ab} = \rho u_a u_b - \rho_s x_a x_b, \tag{4}$$

where ρ is the rest energy for the string cloud and ρ_s is string tension density which are related as

$$\rho = \rho_p + \rho_s, \tag{5}$$

where, ρ_p is the particle energy density.

We know that string is vibration free. The different vibration mode of the string represents the different types of particles because of different masses or spins of different modes are seen. Therefore, here we will take quarks in the string cloud. As a result,instead of considering particle energy density in the string cloud, we examine strange quark matter energy density. Now in this case, Eq.(5) leads to

$$\rho = \rho_q + \rho_s + B_c. \tag{6}$$

From Eq.(6) and Eq.(4) we have

$$T^S_{ab} = (\rho_q + \rho_s + B_c)u_a u_b - \rho_s x_a x_b, \tag{7}$$

where, $u^a = \delta^a_5 e^{\frac{\nu}{2}}$ is the five velocity and $x^a = \delta^a_1 e^{\frac{\mu}{2}}$ is the unit space like vector in the radial direction which represent the direction of anisotropy.

$$T^E_{ab} = -\frac{1}{4\pi}(F^c_a F_{bc} - \frac{1}{4} g_{ab} F_{\alpha\beta} F^{\alpha\beta}). \tag{8}$$

where F_{ab} is the electromagnetic field tensor defined in terms of the five potential A_0 as

$$F_{ab} = A_{b;a} - A_{a;b}.$$

For the electromagnetic field we consider the gauge

$$A_a(0, 0, 0, \phi(r)).$$

Einstein-Maxwell equations can be expressed as

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi T_{ab}, \tag{9}$$

$$F_{ab;c} + F_{bc;a} + F_{ca;b} = 0, \tag{10}$$

$$F_{ab;b} = -4\pi J^a, \tag{11}$$

where J^a is the five current density that becomes $J^a = \bar{\rho_e}U^a$, $\bar{\rho_e}$ being the proper charge density.

By using Eq.(2), the field Eqs. (9)-(11) can be expressed as,

$$8\pi\rho + E^2 = -e^{-\mu}\left[\frac{3}{r^2} - \frac{3\mu'}{2r}\right] + \frac{3}{r^2}, \tag{12}$$

$$8\pi\rho - E^2 = e^{-\mu}\left[\frac{3\nu'}{2r} + \frac{3}{r^2}\right] - \frac{3}{r^2}, \tag{13}$$

$$E^2 = \frac{e^{-\mu}}{2}\left[\nu'' - \frac{\nu'\mu'}{2} + \frac{\nu'^2}{2} - \frac{(\mu' - \nu')}{r} + \frac{1}{r^2}\right] - \frac{1}{r^2}, \tag{14}$$

$$\left[r^3 E(r)\right]' = 4\pi\rho_e r^3, \tag{15}$$

where prime ($'$) denotes differentiation with respect to r and $\rho_e$is charge density which is related to the proper charge density $\bar{\rho_e}$ by

$$\rho_e = \bar{\rho_e}e^{\mu/2}. \tag{16}$$

And electric field E which is defined as

$$F_{01}F^{01} = -E^2,$$

$$E(r) = -e^{-(\nu+\mu)/2}\beta'(r),$$

$$\beta'(r) = F_{10} = -F_{01}. \tag{17}$$

3. Solutions of the Field Equations

To get the deterministic solution, we assume that the space-time admiting conformal motions i.e.

$$\pounds_{\xi} g_{ab} = 2\psi g_{ab}, \tag{18}$$

where, ψ is an arbitrary function of $'r'$. From Eqs. (1) and (18) and we get the following expressions:

$$\xi^1 \nu' = \psi, \quad \xi^1 = \frac{\psi r}{2}, \tag{19}$$

$$\lambda' \xi' + 2\xi^1_{,1} = \psi, \quad \xi^4 = C_1 = \text{constant}, \tag{20}$$

where a comma (,) denotes partial derivative with respective r.
From Eqs. (19)-(20), we obtain

$$e^{\nu} = C_2^2 r^2, \quad e^{\mu} = \left(\frac{C_3^2}{\psi}\right)^2, \tag{21}$$

$$\xi^a = \frac{\psi r}{2}\delta^a_1 + C_1 \delta^a_4, \tag{22}$$

where C_1 and C_2 are constants of integration.

Using Eq. (21) into the field Eqs. (12)-(14), we get

$$\rho + E^2 = \frac{3}{r^2}[1 - \frac{\psi^2}{C_3^2}] - \frac{3\psi\psi'}{C_3^2 r}, \tag{23}$$

$$\rho_s - E^2 = \frac{3}{r^2}\left[\frac{2\psi^2}{C_3^2} - 1\right], \tag{24}$$

$$E^2 = \frac{3\psi\psi'}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2} - \frac{1}{r^2}, \tag{25}$$

From Eqs. (23)-(25), we get

$$\rho = -\frac{6\psi\psi'}{C_3^2 r} - \frac{6\psi^2}{C_3^2 r^2} + \frac{1}{r^2}, \tag{26}$$

$$\rho_s = -\frac{3\psi\psi'}{C_3^2 r} - \frac{9\psi^2}{C_3^2 r^2} + \frac{4}{r^2}, \tag{27}$$

$$E^2 = \frac{3\psi\psi'}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2} - \frac{1}{r^2}. \tag{28}$$

From Eq.(6)

$$\rho_p = -\frac{3\psi\psi'}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2}. \tag{29}$$

Using Eq. (21), the line element Eq. (2) reduces to

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2. \tag{30}$$

From Eqs. (26)-(29), we can obtained different solutions by specifying the different choice of $\psi(r)$.

3.1. *Case I : $\psi = c_4 r$*

For this case, from Eqs. (26)-(29), we have

$$\rho = \rho_s = \frac{4}{r^2} - \frac{12C_4^2}{2C_3^2}, \tag{31}$$

$$E^2 = -\frac{1}{r^2} + \frac{6{C_4}^2}{C_3^2}, \tag{32}$$

$$\rho_p = \rho - \rho_s = 0, \tag{33}$$

where, C_4 is an integrating constant.

Now, consider that the charged sphere extends to radius r_0, then the solution of Einstein-Maxwell equation for r> r_0 which is given by the Reissner-Nordstrom metric as

$$ds^2 = \left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right] dt^2 - \left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right]^{-1} dr^2 - r^2\Omega^2 \tag{34}$$

and the radial electric field is

$$E = \frac{q}{r^3}, \tag{35}$$

where,q and M are charge and the total mass respectively.

To match the metric in Eq. (30) with the Reissner-Nordstrom metric across the boundary condition r= r_0. We required continuity of gravitational potential g_{ab} at r= r_0,

$$(C_2 r_0)^2 = (\frac{\psi}{C_3})^2 = 1 - \frac{2M}{r_0} + \frac{q^2}{r_0^2}, \tag{36}$$

And also we require the continuity of the electric field, which leads to

$$E(r_0) = \frac{q}{r_0^3}, \tag{37}$$

From Eq.(32) and (35), we get

$$\frac{q^2}{r_0^3} = 6(\frac{C_4}{C_3})^2 - \frac{1}{r_0^2}, \tag{38}$$

Putting this values back into Eq.(33) we obtain

$$\frac{M}{r_0} = \frac{1}{2} + 3r_0^3(\frac{C_3}{C_4})^2 - \frac{r_0}{2}, \tag{39}$$

Also from Eqs.(34) and (35), we get

$$M = \frac{r_0}{2} + \frac{q^2}{2r_0}. \tag{40}$$

From Eqs. (36), (38) and (39), we get

$$\psi = \pm\sqrt{3C_4^2 r_0(1 - \frac{r_0^2}{2}) + C_3^2(r_0 - \frac{1}{r_0})}. \tag{41}$$

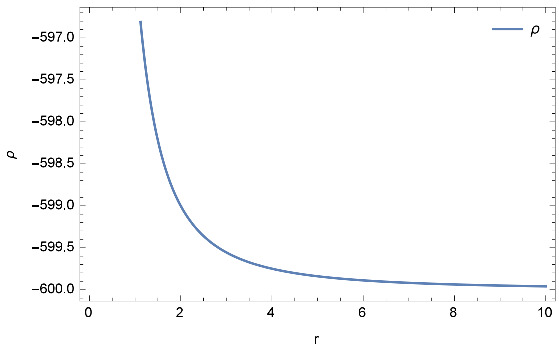

Fig. 1. $\rho(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

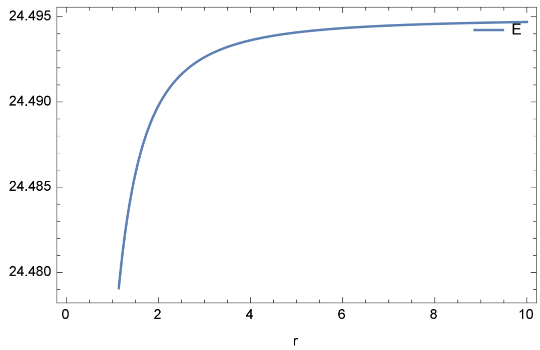

Fig. 2. $E(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_4 = 1$.

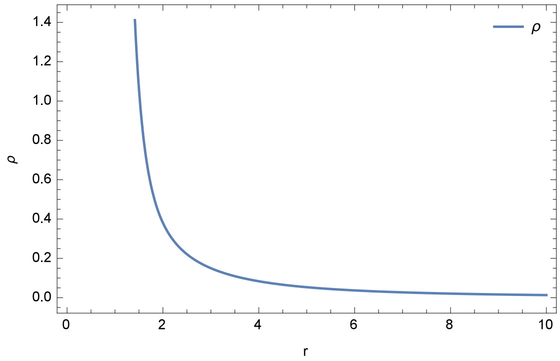

Fig. 3. $\rho(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

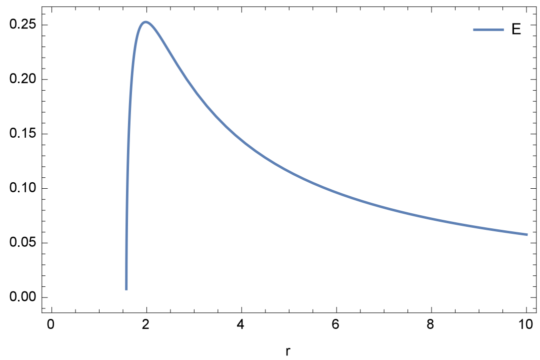

Fig. 4. $E(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

3.2. *Case II* : $\psi = \sqrt{\frac{4C_3^2}{9} + \frac{C_5}{r^6}}$

For this case, from Eqs. (26)-(29), we get

$$\rho = \rho_p = \frac{4}{3r^2} + \frac{12C_5}{C_3^2 r^8}, \rho_s = 0 \tag{42}$$

$$\rho_q = \frac{4}{3r^2} + \frac{12C_5}{C_3^2 r^8} - B_c \tag{43}$$

$$E^2 = \frac{1}{3r^2} - \frac{5C_5}{C_3^2 r^8}, \tag{44}$$

where C_5 is integrating constant.

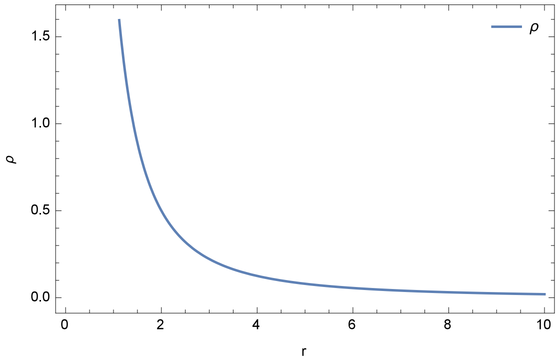

Fig. 5. $\rho(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

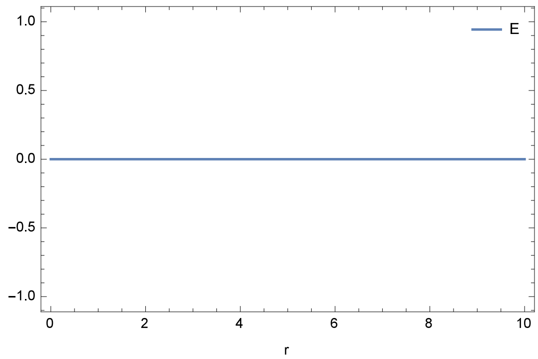

Fig. 6. $E(r)$ are shown against r. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

3.3. *Case III* : $\psi = \sqrt{\frac{1}{3}C_3^2 + \frac{C_6}{r^2}}$

For this case, from Eqs. (26)-(29), we get

$$\rho = \frac{2}{r^2}, \rho_s = \frac{1}{r^2} - \frac{6C_6}{C_3^2 r^4}, E^2 = 0 \tag{45}$$

$$\rho_p = \frac{1}{r^2} + \frac{6C_6}{C_3^2 r^4}, \tag{46}$$

$$\rho_q = \frac{1}{r^2} + \frac{6C_6}{C_3^2 r^4} - B_c, \tag{47}$$

where, C_6 is constant of integrating .

4. Conclusion

In present work, we studied charged strange quark matter in $5D$ the spherically symmetric space-time admitting one parameter group of conformal motion which is attached to string cloud. Our finding in five dimension are identically to those of Yavuz et al. (2005)[2] in four dimensions. One can deduce Yavuz et al. (2005)[2] results from ours corresponding results by assigning appropriate values to the functions concerned.

Acknowledgments

Praveen Kumar thankful to Isaac Newton Institute for Mathematical Sciences for support and hospitality during the CAT-2 & CAT-3 programme when work on this paper was undertaken. This work was supported by: EPSRC grant number EP/R014604/1.

References

1. A. R. Bodmer, Collapsed Nuclei, *Phys. Rev.* **D4**, 160 (1971).
2. I. Yavuz , I. Yilmaz and H. Baysal, Strange Quark Matter Attached to the String Cloud in the Spherical Symmetric Space-Time Admitting Conformal Motion, *Int. J. Mod. Phys.* **D14**, 1365 (2005).
3. I. Yavuz and I. Yilmaz, Inheriting Conformal and Special Conformal Killing Vectors in String Cosmology, *Gen. Rel. Grav.* **29**, 1295, (1997)
4. I. Yilmaz, I. Tharhar, I. Yavuz, H. Baysal and U. Camci, Curvature Inheritage Symmetry in Riemannian Spaces with Application to String Cloud and String fluids, *Int. J. Mod Phys.* **D8**, 659 (1999).
5. H. Baysal, U. Camci, I. Yilmaz, I. Tarhan and I. Yavuz, Conformal Collineations in Strong Cosmology, *Int. J. Mod Phys.* **D11**, 463 (2002).
6. E. Witten, Properties of O(32) Superstrings *Phys. Lett.* **B144**, 351 (1984).
7. Zel'dovich, Cosmological fluctuations produced near a singularity, *Mon. Not. R. Astron. Soc.* **192**, 663 (1980).
8. P. S. Letelier, Clouds of strings in general relativity, *Phys. Rev. D* **20**, 1249 (1979).
9. Vilenkin, Cosmic Strings, *Phys. Rev.* **121**, 163 (1980)
10. I. Yilmaz, Domain wall solutions in the nonstatic and stationary Gödel universes with a cosmological constant, *Phys. Rev. D* **71**, 103503 (2005).
11. I. Yilmaz, String cloud and domain walls with quark matter in 5-D Kaluza-Klein cosmological model, *Gen. Rel. Grav.* **38**, 1397 (2006).
12. J. S. Stachel, Thickening the string, *Phys. Rev. D* **21**, 2171 (1980).
13. C. G. Bohmer, T. Harko, F. S. N. Lobo, Conformally symmetric traversable wormholes, *Phys. Rev. D***76**, 084014 (2007).
14. C. G. Bohmer, T. Harko,F. S. N. Lobo, Wormhole Geometries with Conformal Motions, *Class Quantum Gravit.***25**, 075016 (2008).
15. R. Maartens, C. M. Melllin, Anisotropic universes with conformal motion, *Class Quantum Gravit.* **13**, 1571 (1996).
16. M. K. Mak, T. Harko, Quark Stars Admitting a One Parameter Group of conformal Motions, *Int. J. Mod Phys. D* **13**, 149 (2004).
17. M. Mars, J. M. M. Senovilla, Stationary and axisymmetric perfect fluid solutions with conformal motion, *Class Quantum Gravit.* **11**, 3049 (1994).
18. M. K. Mak, T. Harko, Vacuum solutions of the gravitational field equations in the brane world model, *Phys. Rev. D* **69**, 064020 (2004).
19. M. K. Mak, T. Harko, Can the galactic rotation curves be explained in brane world models? *Phys. Rev. D* **70**, 024010 (2004).

20. F. Rahman et al., A class of solutions for anisotropic stars admitting conformal motion, *Astrophys. Space Sci.* **330**, 249 (2010).
21. F. Rahman et al., On role of pressure anisotropy for relativistic stars admitting conformal motion, *Astrophys. Space Sci.* **325**, 137 (2010).
22. R. Neves, Braneworlds and Dark Energy, *TSPU Vestink*, **44(7)**, 94 (2006).