Strange Quark Matter attached to String Cloud in General Relativity under 5D space-time

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In this work, we have solved Einstein’s field equations in five dimensional spherically symmetric space-time with strange quark matter attached to string cloud by assuming one parameter group of conformal motions. We have also discussed the solution in five dimensional space-time.

Keywords. Five dimension, Strange quark matter, Conformal motion.

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1. Introduction.

In the recent year, it has been widely assumed that the symmetry of the universe is shattered spontaneously during the phase change. The investigation of cosmic strings and physical processes, such strings has received wide attention Vilenkin⁹ because they are believed to give rise to density perturbation leads to the formation of galaxies Zel’dovich⁷.

The string theory is quantum gravity theory that, at macroscopic distances, resembles General Relativity. As the stress energy of string can be attached to the gravitation field, it may be intereting to study the gravitational effects that aries from strings. Yilmaz ¹⁰, ¹¹ have studied 5 – D Kaluza-Klein cosmological models with quark matter attached to the string cloud and domain wall.

Collineation are geometrical or physical relevant quantities which concern the symmetries of the metric. The important collineation is Conformal Killing Vector (CKV) which gives a better understanding into the space time geometry connected to the astrophysical and cosmological rehn (Mak et al. ¹⁶, Mars et al. ¹⁷, Neves
The inheritance symmetries with a CKV and special CKV in fluid space times (perfect, anisotropic, viscous and heat-conducting) has recently attracted some interest. Yavuz and Yilmaz and Yilmaz et al. who have considered inheriting conformal and special CKV, and also curvature inheritance symmetry in the string cosmology (String cloud and String fluids) respectively. Baysal et al. have investigated conformal collineation in the string cosmology.

Strange quark matter may be attach to the string cloud. As a result strange quark matter will attach to the string cloud in this work. Witten and Bodmar have studied strange quark matter in different contexts.

A gravitating object when it undergoes indefinite collapse, the end product is a singularity which is marked by the divergence of physical parameters like energy density. As the singularity is approached, the density diverges and it would therefore be of relevance to consider the state of matter at ultra high density beyond the nuclear matter. One of such possible states could be the strange quark matter which consists of $u$, $d$, and $s$ quark.

In the generalized bag model, quarks are assumed to be massless and non integrating, then we get quark pressure $P_q = \frac{\rho_q}{3}$ where $\rho_q$ is the quark energy density. The total energy density are $\rho = \rho_q + B_c$ where $B_c$ is the difference between the energy density of the perturbative and non perturbative quantum chromodynamics (QCD) vacuum (the bag constant) but the total pressure is $p = \rho_q - B_c$.

The collineation is defined as

$$L_\xi g_{ab} = 2\psi g_{ab},$$

where $L_\xi$ is the Lie derivative along collineation vector $\xi^a$ and $\psi = \psi(x^a)$ is the conformal factor. If $\psi_{,ab} = 0$ and $\psi_{,a} = 0$ then $\xi$ is a special CKV where (;) is covariant derivative and (,) is ordinary derivatives. Other sub cases are homothetic vector(HV) if $\xi$ is constant. It is supposed that the vector $\xi$ generates the conformal symmetry and the metric $g$ is conformally mapped onto itself along $\xi$. Neither $\xi$ nor $\psi$ need to be static, even in the case of static metric Bohmer et al. 13 14.

In present work, we attach charged strange quark to the string cloud in the five dimensional spherical symmetric space time admitting one parameter group of conformal motion. This work is the extension of the work obtained early by Yavuz et al. 2 for four dimensional space time.

Our work is organized in the following ways: In section 2, We found Einstein field
equations in five dimensional spherical symmetric space time for charged strange quark matter attach to the string cloud. In section 3, We solved the same Einstein field equations for the same matter using idea of conformal motion. In section 4, we summarize our results.

2. The Metric and Field Equations

Here we can consider the five dimensional spherically symmetric line element as

\[ ds^2 = e^{\nu(r)} dt^2 - e^{\mu(r)} dr^2 - r^2 d\Omega^2, \]  

(2)

where

\[ d\Omega^2 = d\theta_1^2 + \sin^2 \theta_1 d\theta_2^2 + \sin^2 \theta_1 \sin^2 \theta_2 d\theta_3^2 \]

\[ x^{1,2,3,4} = r, \theta_1, \theta_2, t. \]

We assume that the total energy-momentum tensor \( T_{ab} \) is the sum of the two parts, \( T^S_{ab} \) for string cloud and \( T^E_{ab} \) for electromagnetic contributions respectively, i.e.

\[ T_{ab} = T^S_{ab} + T^E_{ab}. \]  

(3)

The energy momentum tensor for string cloud Letelier\(^8\) and Stachel\(^12\) given by

\[ T_{ab} = \rho u_a u_b - \rho_s x_a x_b, \]  

(4)

where \( \rho \) is the rest energy for the string cloud and \( \rho_s \) is string tension density which are related as

\[ \rho = \rho_p + \rho_s, \]  

(5)

where, \( \rho_p \) is the particle energy density.

We know that string is vibration free. The different vibration mode of the string represents the different types of particles because of different masses or spins of different modes are seen. Therefore, here we will take quarks in the string cloud. As a result, instead of considering particle energy density in the string cloud, we examine strange quark matter energy density. Now in this case, Eq.(5) leads to

\[ \rho = \rho_q + \rho_s + B_c. \]  

(6)

From Eq.(6) and Eq.(4) we have

\[ T^S_{ab} = (\rho_q + \rho_s + B_c) u_a u_b - \rho_s x_a x_b, \]  

(7)

where, \( u^a = \delta^a_5 e^\nu \) is the five velocity and \( x^a = \delta^a_1 e^{\mu} \) is the unit space like vector in the radial direction which represent the direction of anisotropy.

\[ T^E_{ab} = -\frac{1}{4\pi}(F_a^c F_c^b - \frac{1}{4}g_{ab} F_{\alpha\beta} F^{\alpha\beta}). \]  

(8)
where $F_{ab}$ is the electromagnetic field tensor defined in terms of the five potential $A_0$ as

$$F_{ab} = A_{b,a} - A_{a,b}.$$ 

For the electromagnetic field we consider the gauge

$$A_a(0,0,0,\phi(r)).$$

Einstein-Maxwell equations can be expressed as

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}, \quad (9)$$

$$F_{abc} + F_{bca} + F_{cab} = 0, \quad (10)$$

$$F_{ab,b} = -4\pi J^a, \quad (11)$$

where $J^a$ is the five current density that becomes $J^a = \bar{\rho}_e U^a$, $\bar{\rho}_e$ being the proper charge density.

By using Eq.(2), the field Eqs. (9)-(11) can be expressed as,

$$8\pi \rho + E^2 = -e^{-\mu} \left[ \frac{3}{r^2} - \frac{3\nu'}{2r} \right] + \frac{3}{r^2}, \quad (12)$$

$$8\pi \rho - E^2 = e^{-\mu} \left[ \frac{3\nu'}{2r} + \frac{3}{r^2} \right] - \frac{3}{r^2}, \quad (13)$$

$$E^2 = \frac{e^{-\mu}}{2} \left[ \frac{\nu'' - \nu'\mu'}{2r} + \frac{\nu'^2}{2} - \frac{(\mu' - \nu')}{r} + \frac{1}{r^2} \right] - \frac{1}{r^2}, \quad (14)$$

$$[r^3 E(r)]' = 4\pi \rho_e r^3, \quad (15)$$

where prime ('') denotes differentiation with respect to $r$ and $\rho_e$ is charge density which is related to the proper charge density $\bar{\rho}_e$ by

$$\rho_e = \bar{\rho}_e e^{\mu/2}. \quad (16)$$

And electric field $E$ which is defined as

$$F_{01} F^{01} = -E^2,$$

$$E(r) = -e^{-(\nu+\mu)/2} \beta'(r),$$

$$\beta'(r) = F_{10} = -F_{01}. \quad (17)$$
3. Solutions of the Field Equations

To get the deterministic solution, we assume that the space-time admitting conformal motions i.e.

$$\mathcal{L}_\xi g_{ab} = 2\psi g_{ab}, \quad (18)$$

where, $\psi$ is an arbitrary function of $'r'$. From Eqs. (1) and (18) and we get the following expressions:

$$\xi^1 \nu' = \psi, \quad \xi^1 = \frac{\psi r}{2}, \quad (19)$$

$$\lambda' \xi' + 2\xi^1 = \psi, \quad \xi^4 = C_1 = \text{constant}, \quad (20)$$

where a comma $(,)$ denotes partial derivative with respective $r$. From Eqs. (19)-(20), we obtain

$$e^\nu = C_2^2 r^2, \quad e^\mu = \left(\frac{C_2^2}{\psi}\right)^2, \quad (21)$$

$$\xi^a = \frac{\psi r}{2} \delta^a_1 + C_1 \delta^a_4, \quad (22)$$

where $C_1$ and $C_2$ are constants of integration.

Using Eq. (21) into the field Eqs. (12)-(14), we get

$$\rho + E^2 = \frac{3}{r^2} \left[ 1 - \frac{\psi^2}{C_3^2} - \frac{3\psi\nu}{C_3^2 r} \right], \quad (23)$$

$$\rho_s - E^2 = \frac{3}{r^2} \left[ \frac{2\psi^2}{C_3^2} - 1 \right], \quad (24)$$

$$E^2 = \frac{3\psi\nu}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2} - \frac{1}{r^2}, \quad (25)$$

From Eqs. (23)-(25), we get

$$\rho = -\frac{6\psi\nu}{C_3^2 r} - \frac{6\psi^2}{C_3^2 r^2} + \frac{1}{r^2}, \quad (26)$$

$$\rho_s = \frac{3\psi\nu}{C_3^2 r} - \frac{9\psi^2}{C_3^2 r^2} + \frac{4}{r^2}, \quad (27)$$

$$E^2 = \frac{3\psi\nu}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2} - \frac{1}{r^2}, \quad (28)$$

From Eq.(6)

$$\rho_p = \frac{3\psi\nu}{C_3^2 r} + \frac{3\psi^2}{C_3^2 r^2}. \quad (29)$$
Using Eq. (21), the line element Eq. (2) reduces to

$$ds^2 = C_2^2 r^2 dt^2 - \frac{C_3^2}{\psi^2} dr^2 - r^2 d\Omega^2.$$  \hspace{1cm} (30)

From Eqs. (26)-(29), we can obtained different solutions by specifying the different choice of $\psi(r)$.

3.1. Case I : $\psi = c_4 r$

For this case, from Eqs. (26)-(29), we have

$$\rho = \rho_s = \frac{4}{r^2} - \frac{12 C_4^2}{2C_3^2},$$  \hspace{1cm} (31)

$$E^2 = -\frac{1}{r^2} + \frac{6 C_4^2}{C_3^2},$$  \hspace{1cm} (32)

$$\rho_p = \rho - \rho_s = 0,$$  \hspace{1cm} (33)

where, $C_4$ is an integrating constant.

Now, consider that the charged sphere extends to radius $r_0$, then the solution of Einstein-Maxwell equation for $r > r_0$ which is given by the Reissner-Nordstrom metric as

$$ds^2 = \left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right] dt^2 - \left[1 - \frac{2M}{r} + \frac{q^2}{r^2}\right]^{-1} dr^2 - r^2 d\Omega^2$$  \hspace{1cm} (34)

and the radial electric field is

$$E = \frac{q}{r^3},$$  \hspace{1cm} (35)

where, $q$ and $M$ are charge and the total mass respectively.

To match the metric in Eq. (30) with the Reissner-Nordstrom metric across the boundary condition $r = r_0$, We required continuity of gravitational potential $g_{ab}$ at $r = r_0$,

$$(C_2 r_0)^2 = \left(\frac{\psi}{C_3}\right)^2 = 1 - \frac{2M}{r_0} + \frac{q^2}{r_0^2},$$  \hspace{1cm} (36)

And also we require the continuity of the electric field, which leads to

$$E(r_0) = \frac{q}{r_0^3},$$  \hspace{1cm} (37)

From Eq.(32) and (35), we get

$$\frac{q^2}{r_0^2} = 6\left(\frac{C_4}{C_3}\right)^2 - \frac{1}{r_0^2},$$  \hspace{1cm} (38)
Putting this values back into Eq.(33) we obtain

\[ \frac{M}{r_0} = \frac{1}{2} + 3r_0^2 \left( \frac{C_3}{C_4} \right)^2 = \frac{r_0}{2}, \quad (39) \]

Also from Eqs.(34) and (35), we get

\[ M = \frac{r_0}{2} + \frac{q^2}{2r_0}, \quad (40) \]

From Eqs. (36), (38) and (39), we get

\[ \psi = \pm \sqrt{3C_4^2 r_0^2 \left( 1 - \frac{r_0^2}{2} \right) + C_3^2 (r_0 - \frac{1}{r_0})}. \quad (41) \]

Fig. 1. \( \rho(r) \) are shown against \( r \). We use constants \( C_3 = 0.1 \) and \( C_4 = 1 \).

Fig. 2. \( E(r) \) are shown against \( r \). We use constants \( C_3 = 0.1 \) and \( C_4 = 1 \).
Fig. 3. $\rho(r)$ are shown against $r$. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

Fig. 4. $E(r)$ are shown against $r$. We use constants $C_3 = 0.1$ and $C_5 = 0.01$.

3.2. Case II: $\psi = \sqrt{\frac{4C_3^2}{3} + \frac{C_5}{r^8}}$

For this case, from Eqs. (26)-(29), we get

$$\rho = \rho_p = \frac{4}{3r^2} + \frac{12C_5}{C_3^2 r^8}, \rho_s = 0$$  \hspace{1cm} (42)

$$\rho_q = \frac{4}{3r^2} + \frac{12C_5}{C_3^2 r^8} - B_c$$  \hspace{1cm} (43)

$$E^2 = \frac{1}{3r^2} - \frac{5C_5}{C_3^2 r^8},$$  \hspace{1cm} (44)

where $C_5$ is integrating constant.
Fig. 5. $\rho(r)$ are shown against $r$. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

Fig. 6. $E(r)$ are shown against $r$. We use constants $C_3 = 0.1$ and $C_6 = 0.1$.

3.3. **Case III**: \( \psi = \sqrt{\frac{2}{9}} C_3^2 + \frac{C_6}{r^2} \)

For this case, from Eqs. (26)-(29), we get

\[
\rho = \frac{2}{r^2}, \quad \rho_s = \frac{1}{r^2} - \frac{6C_6}{C_3^2 r^4}, \quad E^2 = 0 \quad (45)
\]

\[
\rho_p = \frac{1}{r^2} + \frac{6C_6}{C_3^2 r^4}, \quad (46)
\]

\[
\rho_q = \frac{1}{r^2} + \frac{6C_6}{C_3^2 r^4} - B_c, \quad (47)
\]

where, $C_6$ is constant of integrating.

4. **Conclusion**

In present work, we studied charged strange quark matter in 5D the spherically symmetric space-time admitting one parameter group of conformal motion which
is attached to string cloud. Our finding in five dimension are identically to those of Yavuz et al. (2005)\(^2\) in four dimensions. One can deduce Yavuz et al. (2005)\(^2\) results from ours corresponding results by assigning appropriate values to the functions concerned.

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**References**