Journal of The Tensor Society (J.T.S.) ISSN: 0974-5424
Vol. 16 (2022), page 20 - 39

Analysis of homogeneous anisotropic Bianchi-V cosmological model with relativistic hydrodynamic

G. S. Khadekar[1], Ather Husain[2], S. D. Tade[3]

[1] *Department of Mathematics Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur, (M.S.) 440033, India.*

[2] *Department of Mathematics, Narayanrao Kale Smruti Model College Karanja(Gh), District: Wardha, (M.S.) 442203, India.*

[3] *Department of Mathematics, Jawaharlal Nehru Arts, Commerce and Science College, Wadi, Nagpur, (M.S.) 440023, India.*

E-mail[1]: gkhadekar@yahoo.com
E-mail[2]: atherhusain1001@gmail.com
E-mail[2]: drstade25@gmail.com

Received : June 07, 2022
Accepted : September 14, 2022
Published : December 30, 2022

In the present paper, we have analyzed the general relativistic hydrodynamic source within the frame work of homogeneous and anisotropic Bianchi type-V space-time and obtained the solution of the field equations towards Hubble's law of variation which yields a constant value of deceleration parameter. In this analysis, at an initial stage both metric potentials of the derived stable model are comes out to be constant and at large time they are increases indefinitely. Also, initially the model is constant but at a specific time $t = t_s$ with vanishing metric potential the model represents singularity as well as the derived general relativistic hydrodynamic cosmological model has a phase transition from phantom field dark energy to quintessence field dark energy, which shows the model start with acceleration. Along with some other physical and kinematical parameters of general relativistic hydrodynamic model are discussing details.

Keywords. Space-Time; Relativistic Hydrodynamics; Cosmology.

AMS Classification 83C75, 76Y05, 83F05.

1. Introduction.

Einstein's hypothesis of General Theory of Relativity (GTR) is a fundamental hypothesis which assumes a significant function in astrophysical situations including neutron stars and black holes. High-energy radiation is regularly twisted in areas of strong gravitational fields near to such compact objects. Late galactic observations dependent on Type-Ia Supernovae (SNe-Ia), Large Scale Structure (LSS), and Cosmic Microwave Background Anisotropy (CMBA) definitely adjust that our universe is going over a stage change from decelerating to accelerating by Perlmutter, S. [25-26]; Riess, A.G. et al. [27] and Tonry, J. et al. [41]. This observed spectacle can basically be involve by an exotic component with negative pressure, so called mysterious energy or Dark Energy (DE). Many authors have analyzed the dark energy space-time for different purpose of used in GTR as well as scalar tensor theory and modified theories of gravity some of them are mentioned as Spergel et al.[39-40], Capoziello et al.[11], Nojiri and Odintsov [23], Sharif and Yousaf [31], Sahoo and Bhattacharjee [28], Wei et al. [42], Bhoyar et al. [7], Harko et al. [14], Sahoo et al. [29], Pawar et al. [24], Nojiri and Odintsov [22], Abbas et al. [1], Shekh et al. [35]. Along with in order to interpret the accelerating universe particular dark energy cosmological models have been proposed by a numbers of authors in GTR as well as in modified theories. Mishra et al. [18] have anticipated the anisotropic behavior of the accelerating universe in Bianchi type-V space time in presence of two non-interacting usual string fluid and dark energy fluids in the GTR, along with some others Katore and Kapse [15], Mukherjee and Banerjee [21], Sharif and Yousaf [30] examined some cosmological models in GTR. Recently Shekh and Chirde [33] have investigated the dynamical properties of plane symmetric cosmological model in regards with Einstein's general theory of relativity additionally indicated that the relativistic hydrodynamic model is completely occupied with quintessence dark energy fluid in this gravity. Accordingly to R. Mohayaee [19], when in detail the real universe is anisotropic and inhomogeneous out to distance large enough to strike on cosmological analysis, in an FLRW universe the dark energy could be an artifact of analyzing data considering that we are idealized spectators. S. Bahamonde, et al. [3], analyzed the past once two decades there has been a various experimental and theoretical trouble into perfecting our understanding of the present universe. While the cosmological equations defining the dynamics of an isotropic and homogeneous universe are system of differential equations, and important ways these can be delved is by launching them into the form of dynamical systems. Leibundgut and Sullivan [17], explained the direct observational signature of expansion of universe by instrumental in showing time dilation. Type Ia supernovae provides the Hubble constant and expansion history of the universe are key measurements. Sharma and Pradhan [32], Can Aktas [9] are discussed various dark energy model in the context of modified theories of gravitation. V. Canuto [10] formulate scale-covariant theory of gravitation and astrophysical applications, where as Brans and Dicke [8] clearly explain Mach's principle and a relativistic theory of gravitation while Shri ram

[37] construct cosmological models of Bianchi type V with perfect fluid and heat conduction in Lyra's geometry. New researches take these studies to develop more cosmological models with the help of recent investigations and theoretical observations.Also, Baumgarte and Shapiro [4] have studied under numerical development to solve Einstein's equation of general relativity in (3+1) higher dimension space-time employ the basic ADM form of the field equations. Berman [6] have studied a law of variation for Hubble's parameter that gives constant deceleration parameter cosmological models of the universe by considering a perfect fluid.

The relativistic hydrodynamics is about the physical properties of fluids in which either the bulk viscosity of the flow is comparable with the speed of light or the intensity of the gravitational field which is either the background or generated by matter itself or when the space-time curvature is large. In fact, the application of work ranges from astrophysical phenomenon to a relativistic treatment. A relativistic description is a significant topic of astrophysics in the following ways: (a) jets emanating at relativistic speed from the core of active galactic nuclei, (b) in framework which involves gravitational collapse of compact stars and flows around black holes. (c) Montero et al. [20] report on the usage of relativistic hydrodynamics, uniting with dynamical space-times, in spherical polar coordinates without symmetry assumptions.

Motivating with the above conclusions and discussions in this paper we have analyzed some aspects of homogeneous and anisotropic Bianchi-V Space-time within the framework of the general relativistic hydrodynamic. The paper is organized as follows, in section 2. We define metric, source and some kinematical parameters regarding Bianchi type-V space-time.

In Section 3, we define the solution of field equations towards the matter dominated volumetric expansion of the model based on constant deceleration parameter. In sections 4 and 5, by providing the metric potentials we discussed some physical and kinematical parameters of the general relativistic hydrodynamics. Finally, we summarize our work in Section 6.

2. Metric,Source and Kinematical Parameters

The universe having anisotropic structure that way to deal with isotropy at late times is more fitting for the description of entire expansion of the universe. Bianchi type space-times are significant in the sense that these are homogeneous and anisotropic that turns in to one isotropy. Hence, in this study we consider an anisotropic Bianchi type-V space-time of the form

$$ds^2 = dt^2 - A^2dx^2 - e^{2mx}B^2dy^2 - e^{2mx}C^2dz^2 \tag{1}$$

where A, B and C are the metric functions of cosmic time t and m is a constant. The energy momentum tensor $T_{\mu v}$ for a non-perfect (un-magnetized) fluid for general relativistic hydrodynamics is expressed as [33]

$$T_{\mu v} = \rho(1+E)u_\mu u_v + (p - \varsigma\theta)\hbar_{\mu v} - 2\eta\sigma_{\mu v} + q_\mu u_v + q_v u_\mu \tag{2}$$

where E be the specific internal energy, p be the anisotropic pressure, θ be the expansion scalar, q_μ is the energy flux vector and $\hbar_{\mu v}$ be the spatial projection tensor defined as $\hbar_{\mu v} = u_\mu u_v + g_{\mu v}$ along with η and ζ shows the shear and bulk viscosities respectively.
In this analysis, we will now neglect non-adiabatic effects, such as viscosity or heat transfer and confine ourselves as the stress-energy tensor to be that of a perfect fluid of the form

$$T_{\mu v} = \rho\hbar u_\mu u_v - p g_{\mu v} \tag{3}$$

where $\hbar$ is the relativistic specific enthalpy and it is defined by

$$\hbar = 1 + E + \frac{p}{\rho}. \tag{4}$$

Using equation (3), we have

$$T_{11} = T_{22} = T_{33} = -p, T_{44} = \rho\hbar \tag{5}$$

Next, consider the equation of state of the form

$$p = p(\rho, E). \tag{6}$$

Here, the equations of state have been sophisticated enough to consider in account to physical and chemical processes. Nevertheless, because of their simplicity, the most broadly utilized equation of state in mathematical simulation in astrophysics is of the form

$$p = (\Theta - 1)\rho E, \tag{7}$$

where Θ being the adiabatic index, and the polytropic equation of state

$$p = \Psi\rho^{\Theta} \approx \Psi\rho^{1+\frac{1}{\Sigma}}, \tag{8}$$

where Ψ is the polytropic constant and Σ being the polytropic index and the micro-physical equations of state that depict the interior of compact stars at nuclear matter densities have likewise been created.
In view of equations (7) and (8), then the equation (5) takes the form

$$\hbar = 1 + \Theta E \tag{9}$$

and

$$\hbar = 1 + E + \Psi\rho^{\frac{1}{\Sigma}}. \tag{10}$$

Considering the same source Shekh and Chirde [30] analyzed the dynamical properties of Plane symmetric cosmological model in modified theories of gravitations say GTR, $f(R)$ and $f(T)$ gravity and conclude that the unstable respectively GTR and $f(R)$ gravity models are fully occupied with Quintessence dark energy whereas $f(T)$ gravity model initially it shows standard Λ CDM (Cold Dark Matter) model and with the expansion it is fully occupied with quintessence fluid. Also, keep in mind the same source Shekh et al. [34] point towards the thermodynamical aspects of general relativistic hydrodynamics in modified $f(R,G)$ theory of gravity by governing the features the power-law inflation for the average scale factor and observed the temperature and entropy density of the proposed model both are positive and definite.

The kinematical parameters which are important to discuss the physics of the cosmological model for Bianchi type-V space-time are:
The average scale factor a and spatial volume V are given by the equations

$$a = (ABC)^{\frac{1}{3}}, \qquad V = a^3 = (ABC), \tag{11}$$

The Hubble's parameter H for the space-time is

$$H = (\ln a) = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{12}$$

The deceleration parameter in cosmology, is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{13}$$

The models of the universe can be arranged based on time dependence of the two parameters: the deceleration and Hubble's parameter. These parameters can change their sign during the evolution of the universe. At the point when the Hubble's parameter H is constant, the deceleration parameter q is constant and equal to -1 , as in the de-Sitter and steady-state universe. Accordingly, all the models of the universe can be characterized by whether they accelerate or decelerate and expand or contract. The new cosmological observations demonstrate that the present universe is expansion is presently accelerating and consequently the current dynamics belongs to the type $q < 0,\ H > 0$.
The expansion scalar is of the form

$$\theta = u^m_{;m} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{14}$$

The anisotropy parameter is expressed in terms of mean and directional Hubble parameters, defined as follows:

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{15}$$

The shear scalar is defined as

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{3}, \tag{16}$$

3. Matter Dominated Volumetric Expansion

The expansion rate of the universe appears to be accelerating at red-shift $z \approx 0.5$ is obtained by recent observational data such as Type-Ia supernovae (SN Ia). Further, the law of variation for Hubble's parameter presented by Berman and Gomide [5] yields a constant value of deceleration parameter (CDP) and is likewise approximately valid for slowly time varying deceleration parameter [38]. Besides Shekh [36] has examine of anisotropic dark energy LRS Bianchi type-I cosmological model with regards to altered gravity, the model is fully engaged with dark energy dominated era and rest existing in quintessence dominated era. By literature survey, a similar law of variation for the Hubble parameter in anisotropic space-time considered by Kumar and Singh [16], and produced solutions for Bianchi type-V space-time in GTR as well as Cunha and Lima [12] and Cunha [13].
Hence, motivating with the above investigations and observations, in the present analysis we consider the constant deceleration parameter as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = constant, \tag{17}$$

where a is the average scale factor.
Integration of above equation (17) becomes,

$$V = (mt + m_1)^{\frac{3}{1+q}}, \quad for \quad m > 0, \tag{18}$$

where $m > 0$ and m_1 are the constants of integration. From equation (18), it is observed that the condition of expansion is given by the equation $(1 + q) > 0$.

4. Field Equations and Their Solutions

The Einstein field equations are given by

$$R_{\mu v} - \frac{1}{2}Rg_{\mu v} = -kT_{\mu v}, \tag{19}$$

where $R_{\mu v}$ is the Ricci curvature tensor, R is the scalar curvature, $g_{\mu v}$ is the metric tensor and the stress energy tensor $T_{\mu v}$. Without loss of originality, we take $k = 1$. Then the field equations (19) for the fluid of general-relativistic hydrodynamics towards Bianchi-V Space-time (1) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p, \tag{20}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = -p, \tag{21}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{m^2}{A^2} = -p, \tag{22}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - 3\frac{m^2}{A^2} = \rho\hbar, \tag{23}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0, \tag{24}$$

where the overhead dot indicates that the differentiation with respect to time t.
From equations (20), (21) and (22), we obtain

$$\frac{B}{A} = \alpha_2 \exp\left\{\int \frac{\alpha_1}{a^3} dt\right\}, \tag{25}$$

$$\frac{C}{A} = \alpha_4 \exp\left\{\int \frac{\alpha_3}{a^3} dt\right\}, \tag{26}$$

$$\frac{C}{B} = \alpha_6 \exp\left\{\int \frac{\alpha_5}{a^3} dt\right\}, \tag{27}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$ are constants of integration.
In view of equation (18), the equations (25), (26), and (27) can be obtained explicitly as

$$A = \frac{k}{\alpha_4}(mt + m_1)^{\frac{1}{1+q}} \exp\left\{\lambda_1 (mt + m_1)^{\frac{q-2}{1+q}}\right\}, \tag{28}$$

$$B = \frac{\alpha_4}{k}(mt + m_1)^{\frac{1}{1+q}} \exp\left\{\lambda_2 (mt + m_1)^{\frac{q-2}{1+q}}\right\}, \tag{29}$$

$$C = k(mt + m_1)^{\frac{1}{1+q}} \exp\left\{\lambda_3 (mt + m_1)^{\frac{q-2}{1+q}}\right\}, \tag{30}$$

where

$$\lambda_1 = \frac{(\alpha_5 - 2\alpha_3)(1+q)}{3m(q-2)}, \lambda_2 = \frac{(\alpha_3 - 2\alpha_5)(1+q)}{3m(q-2)}, \lambda_3 = \frac{(\alpha_3 + \alpha_5)(1+q)}{3m(q-2)}, k = \sqrt[3]{\alpha_4\alpha_6}.$$

From equations (28) to (30) it can be seen that the metric potentials involve both term exponential and power term. Hence, the metric potentials are comes out to be constant at $t \to 0$, whereas at $t \to \infty$, all are increase indefinitely with cosmic time, which is complete agreement with the Big-bang model of the Universe.
Now, with the help of equations (28) to (30) equation (1) takes the form

$$\begin{aligned} ds^2 =& dt^2 - \frac{k^2}{\alpha_4^2}(mt + m_1)^{\frac{2}{1+q}} \exp\left\{2\lambda_1 (mt + m_1)^{\frac{q-2}{1+q}}\right\} dx^2 \\ &- e^{2mx}\begin{bmatrix} \frac{\alpha_4^2}{k^2}(mt + m_1)^{\frac{2}{1+q}} \exp\left\{2\lambda_2 (mt + m_1)^{\frac{q-2}{1+q}}\right\} dy^2 \\ +k^2(mt + m_1)^{\frac{2}{1+q}} \exp\left\{2\lambda_3 (mt + m_1)^{\frac{q-2}{1+q}}\right\} dz^2 \end{bmatrix}, \end{aligned} \tag{31}$$

From equation (31), it is observed that at $t = 0$, the model becomes constant but at a specific value for time $t = t_s = \frac{-m_1}{m}$, all the metric potentials in the model disappear hence the model shows singular model. If $\lambda_i = 0$, the exponential terms in the model will be completely vanished and metric potentials becomes identical. Hence for $\lambda_i = 0$, the model become isotropy.

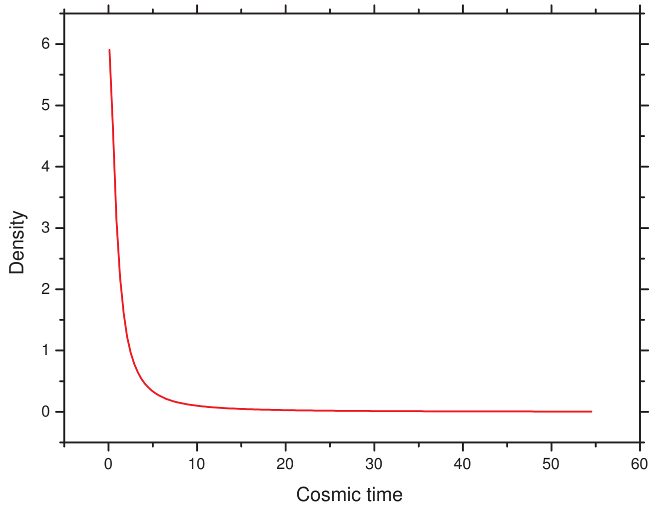

Fig. 1. The behavior of density versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.1, q = -0.5, h = 1, k = 2, \alpha_4 = 1.5$.

5. Physical Properties of the Model

The physical parameters which are important to discuss the physics of the cosmological model are:
The energy density,

$$\rho = \frac{m^2}{h}\left[\begin{array}{c}\frac{(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)(q+2)^2}{(1+q)^2}(mt+m_1)^{\frac{-6}{1+q}} \\ +\frac{2(\lambda_1+\lambda_2+\lambda_3)(q-2)}{(1+q)^2}(mt+m_1)^{\frac{-(4+q)}{1+q}} + \frac{3}{(1+q)^2}(mt+m_1)^{-2}\end{array}\right], \quad (32)$$
$$- \frac{3m^2\alpha_4^2}{hk^2}(mt+m_1)^{\frac{-2}{1+q}} exp\left\{-2\lambda_1(mt+m_1)^{\frac{q-2}{1+q}}\right\}$$

Equation (32) represents the energy density of the general relativistic hydrodynamic cosmological model. The graphical representation of energy density versus cosmic time is presented in fig 1. It is observed that the energy density of general relativistic hydrodynamic cosmological model is always positive and decreasing function of cosmic time. At initial expansion of the model it is very high but with expansion it decreases and at an infinite expansion it converges to null.

The anisotropic pressure,

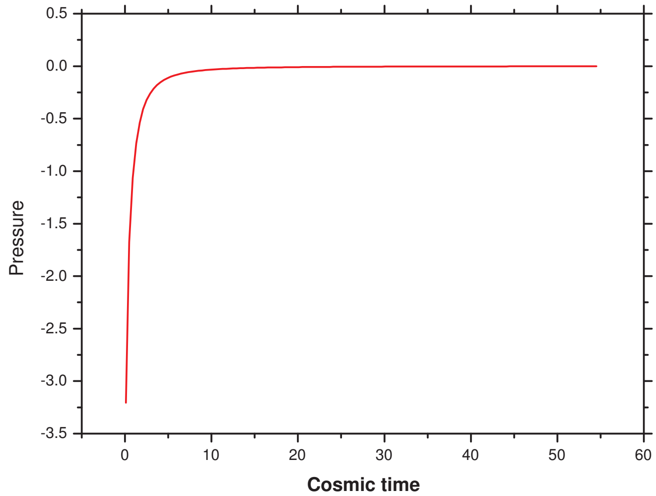

Fig. 2. The behavior of pressure versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.1, q = -0.5, k = 2, \alpha_4 = 1.5$.

$$
\begin{aligned}
p =& \frac{m^2\alpha_4^2}{k^2}(mt+m_1)^{\frac{-2}{1+q}} exp\left\{-2\lambda_1(mt+m_1)^{\frac{q-2}{1+q}}\right\} \\
&- m^2\Big[\frac{(\lambda_2^2+\lambda_3^2+\lambda_2\lambda_3)(q-2)^2}{(1+q)^2}(mt+m_1)^{\frac{-6}{1+q}} + \frac{1}{(1+q)^2}(mt+m_1)^{-2}\Big],
\end{aligned}
\tag{33}
$$

Equation (33) represents the pressure of the general relativistic hydrodynamic cosmological model. The graphical representation of pressure of the general relativistic hydrodynamic cosmological model versus cosmic time is presented in fig.2. It is observed that the pressure of general relativistic hydrodynamic cosmological model is always negative and increasing function of cosmic time.

Equation of state parameter,

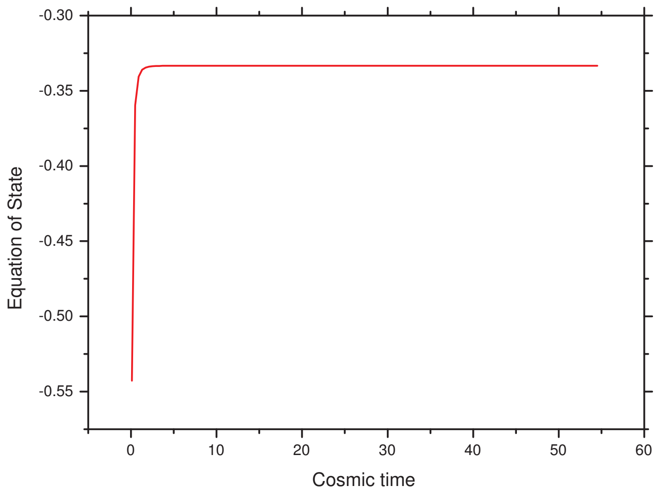

Fig. 3. The behavior of equation of state versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.1, q = -0.5, h = 1, k = 2, \alpha_4 = 1.5$.

$$\omega = \frac{\left\{ \begin{array}{l} \frac{\alpha_4^2}{k^2}(mt+m_1)^{\frac{-2}{1+q}} \exp\left\{-2\lambda_1 (mt+m_1)^{\frac{q-2}{1+q}}\right\} - \\ \left[\frac{(\lambda_2^2+\lambda_3^2+\lambda_2\lambda_3)(q-2)^2}{(1+q)^2}(mt+m_1)^{\frac{-6}{1+q}} + \frac{1}{(1+q)^2}(mt+m_1)^{-2}\right] \end{array} \right\}}{\left\{ \begin{array}{l} \frac{1}{h}\left[\frac{(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)(q+2)^2}{(1+q)^2}(mt+m_1)^{\frac{-6}{1+q}} + \right. \\ \left. \qquad \frac{2(\lambda_1+\lambda_2+\lambda_3)(q-2)}{(1+q)^2}(mt+m_1)^{\frac{-(4+q)}{1+q}} + \frac{3}{(1+q)^2}(mt+m_1)^{-2}\right] - \\ \frac{3\alpha_4^2}{hk^2}(mt+m_1)^{\frac{-2}{1+q}} \exp\left\{-2\lambda_1(mt+m_1)^{\frac{q-2}{1+q}}\right\} \end{array} \right\}}, \tag{34}$$

Equation (34) represents the equation of state parameter of the general relativistic hydrodynamic cosmological model. The graphical representation of equation of state parameter of the general relativistic hydrodynamic cosmological model versus cosmic time is presented in fig.3. It is observed that the equation of state parameter of general relativistic hydrodynamic cosmological model is always negative and increasing function of cosmic time.

The recent theoretical observations such as SNe-Ia, CMBA indicates that if the value of the equation of state parameter is -1 then the model involve dark energy, if it is equal to -1 represents a standard Λ CDM cosmology while it is less than -1 the model shows phantom field dark energy and it is a little bit upper than -1 which is a candidate for dark energy known as quintessence with. In our investigation it is observed that when the model start to expand the value of equation of state is less than -1 i.e. $\omega < -1$ which shows initially the general relativistic hydrodynamic cosmological model represents phantom dark energy with the expansion it cross the phantom divide line and by crossing phantom divide line at an infinite expansion the equation of state parameter of the general relativistic hydrodynamic cosmological model becomes $\omega > -1$ which characterizes the models involve quintessence field dark energy. Hence our general relativistic hydrodynamic cosmological model has a phase transition from phantom field dark energy to quintessence field dark energy.

The stability parameter,

$$\nu^2 = \frac{\left\{\begin{array}{l} \left\{\frac{\alpha_4^2}{k^2}\exp\left\{-2\lambda_1(mt+m_1)^{\frac{q-2}{1+q}}\right\}\times\left[\frac{-2\lambda_1(q-2)}{(1+q)}(mt+m_1)^{\frac{-5}{1+q}}\right.\right. \\ \qquad \left.\left.-\frac{2}{(1+q)}(mt+m_1)^{\frac{-(3+q)}{(1+q)}}\right]\right\}+ \\ \left[\frac{6(\lambda_2^2+\lambda_3^2+\lambda_2\lambda_3)(q-2)^2}{(1+q)^3}(mt+m_1)^{\frac{-(7+q)}{1+q}}+\frac{2}{(1+q)^2}(mt+m_1)^{-3}\right] \end{array}\right\}}{\begin{array}{l} \left\{\frac{1}{h}\left[\begin{array}{l} \quad -6\frac{(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3)(q-2)^2}{(1+q)^3}(mt+m_1)^{\frac{-(7+q)}{1+q}} \\ -\frac{2(\lambda_1+\lambda_2+\lambda_3)(q-2)(4+q)}{(1+q)^3}(mt+m_1)^{\frac{-(5+2q)}{1+q}}-\frac{6}{(1+q)^2}(mt+m_1)^{-3} \end{array}\right]\right. \\ +\frac{3\alpha_4^2}{hk^2}\exp\left\{-2\lambda_1(mt+m_1)^{\frac{q-2}{1+q}}\right\} \\ \left.\left[\frac{2\lambda_1(q-2)}{(1+q)}(mt+m_1)^{\frac{-5}{1+q}}+\frac{2}{(1+q)}(mt+m_1)^{\frac{-(3+q)}{(1+q)}}\right]\right\} \end{array}}, \tag{35}$$

Equation (35) represents the stability parameter of the general relativistic hydrodynamic cosmological model. The graphical representation of energy density versus cosmic time is presented in fig.4. It is observed that the stability parameter of general relativistic hydrodynamic cosmological model is always positive and decreasing function of cosmic time. At initial expansion of the model it is very high but with expansion it decreases and at an infinite expansion it converges to null which indicates that the general relativistic hydrodynamic cosmological model is always stable.

6. Kinematical Properties of the Model

The kinematical parameters which are important to discuss the kinematics of the model are: The spatial volume is

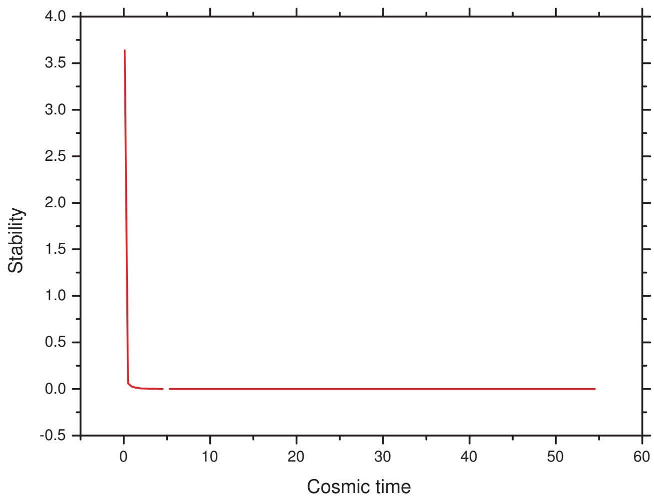

Fig. 4. The behavior of stability versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.1, q = -0.5, h = 1, k = 2, \alpha_4 = 1.5$.

$$V = ABC = a^3 = (mt + m_1)^{\frac{3}{1+q}}, \tag{36}$$

In our analysis, the spatial volume of general relativistic hydrodynamic cosmological model is obtained in equation (36) and the graphical performance is shown in fig.5. From the figure we observed that the spatial volume starts with constant value as $t \to 0$ and conquers big-bang as $t \to -\frac{m_1}{m}$, and also with the increase of cosmic time t it always expands and increase. When $t \to \infty$ then spatial volume $V \to \infty$. Thus inflation is possible in general relativistic hydrodynamic cosmological model. This shows that the universe evolves with zero volume as $t \to 0$ and expands with cosmic time t.

The mean Hubble parameter in terms of m is

$$H = \frac{m}{(1+q)(mt + m_1)}, \tag{37}$$

and the expansion scalar is as

$$\theta = 3H = \frac{3m}{(1+q)(mt + m_1)}, \tag{38}$$

The equations (37) and (38) represents the expressions of the generalized Hubble parameter and the expansion scalar of general relativistic hydrodynamic cosmologi-

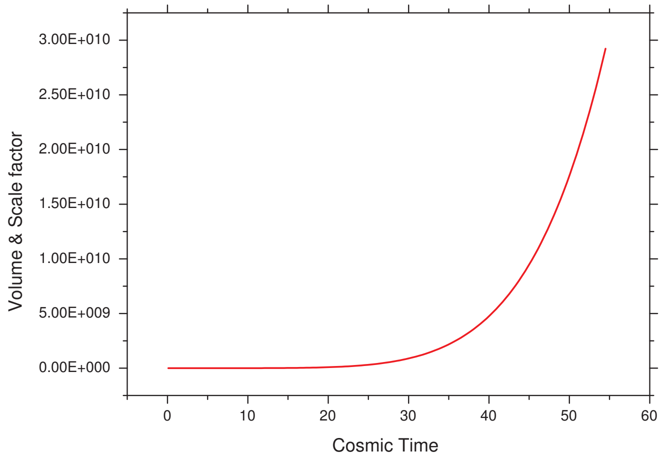

Fig. 5. The behavior of Volume and scalar factor versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, q = -0.5$.

cal model. The graphical performance is clearly seen in fig 6. The fig. 6 signifies the generalized Hubble parameter and the expansion scalar both are constant throughout the evolution of the universe are insignificant at $t \to \infty$. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion decreases to a constant value.

The anisotropy parameter,

$$A_m = \frac{(\lambda_1 + \lambda_2 + \lambda_3)(q-2)}{3}(mt + m_1)^{\frac{q-2}{1+q}}, \tag{39}$$

The shear scalar,

$$\sigma^2 = m^2\Big[\frac{(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)(q-2)^2}{(1+q)}(mt + m_1)^{\frac{-6}{1+q}} + \frac{2(\lambda_1 + \lambda_2 + \lambda_3)(q-2)}{(1+q)^2}(mt + m_1)^{\frac{-(4+q)}{1+q}}\Big], \tag{40}$$

The equations (39) and (40) represents the expressions of the anisotropy parameter and the shear scalar of general relativistic hydrodynamic cosmological model. The graphical performance is clearly seen in figs 7 and 8 respectively. The figs. 7 and 8 respectively signifies the anisotropy parameter of general relativistic hydrodynamic cosmological model is positive and decreasing function of cosmic time and approaches to small positive value of constant while the shear scalar of general relativistic hydrodynamic cosmological model is always negative and decreasing

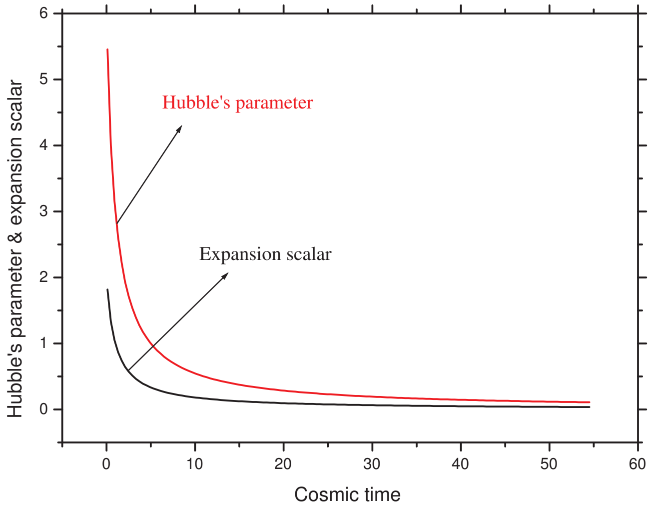

Fig. 6. The behavior of Hubble's parameter and expansion scalar versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, q = -0.5$.

function of cosmic time and remain attend negative value.

7. Discussion and the concluding Remarks

In the analysis of general relativistic hydrodynamic source within the framework of homogeneous and anisotropic Bianchi type-V space-time it is observed that the general relativistic hydrodynamic Bianchi type-V model becomes constant but at a specific value of cosmic time $t = t_s = -\frac{m_1}{m}$, all the metric potentials in the model disappear hence the model shows singular model and for $\lambda_i = 0$, the model become isotropy. The energy density of general relativistic hydrodynamic cosmological model is always positive and decreasing function of cosmic time. At initial expansion of the model it is very high but with expansion it decreases and at an infinite expansion it converges to null whereas the pressure of general relativistic hydrodynamic cosmological model is always negative and increasing function of cosmic time. Moreover, in our investigation it is observed that when the model start to expand the value of equation of state is $\omega < -1$ which shows initially the general relativistic hydrodynamic cosmological model represents phantom dark energy with the expansion it cross the phantom divide line and by crossing phantom divide line at an infinite expansion the equation of state parameter of the general relativistic hydrodynamic cosmological model becomes $\omega > -1$ which characterizes

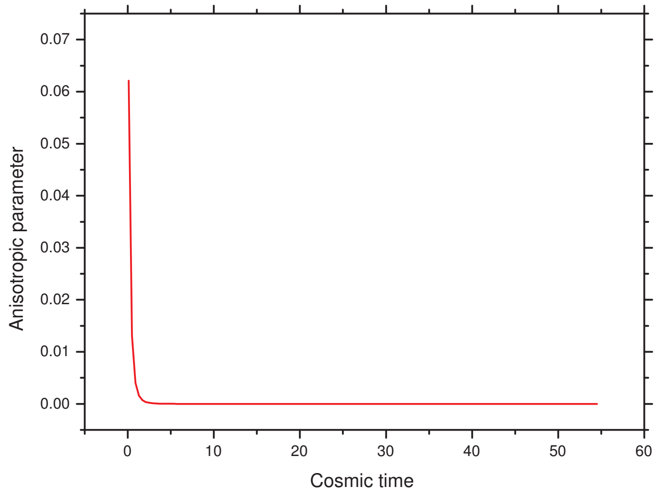

Fig. 7. The behavior of Anisotropic parameter versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.1, q = -0.5$.

the models involve quintessence field dark energy. Hence our general relativistic hydrodynamic cosmological model has a phase transition from phantom field dark energy to quintessence field dark energy along with the stability parameter of general relativistic hydrodynamic cosmological model is always positive and decreasing function of cosmic time which indicates that the general relativistic hydrodynamic cosmological model is always stable.

In our analysis, the spatial volume of general relativistic hydrodynamic cosmological model starts with constant value as $t \to 0$ and conquers big-bang as $t \to -\frac{m_1}{m}$ and also with the increase of cosmic time t it always expands and increases. Hence, inflation is possible in general relativistic hydrodynamic cosmological model. This shows that the universe evolves with zero volume as $t \to 0$ and expands with cosmic time t as well as the generalized Hubble parameter and the expansion scalar of general relativistic hydrodynamic cosmological model both are constant throughout the evolution of the universe are insignificant at $t \to \infty$. This shows that the universe is expanding with the increase of cosmic time but the rate of expansion decreases to a constant value.

Reference

1. Abbas, G., Momeni, D., Ali, M.A., Myrzakulov, R. and Qaisar, S.: Astro. Space Sci. **357**, 158 (2015).
2. Anderson, J.L. and Decanio, T.C.: Gen. Relat. Gravit. **6**, 197-237 (1975).

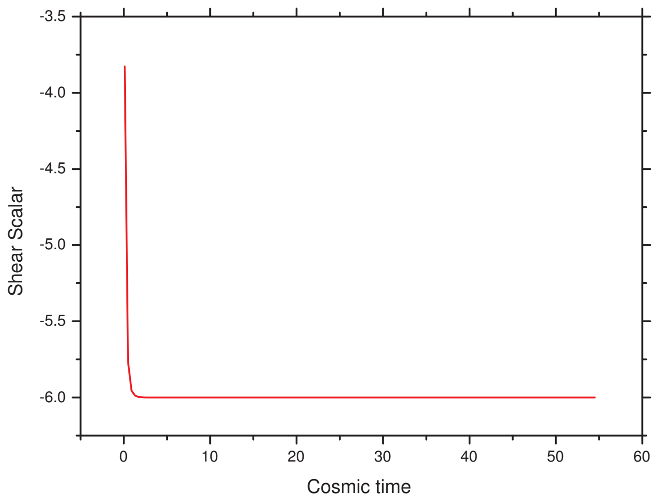

Fig. 8. The behavior of shear scalar versus cosmic time, as a representation case with appropriate choice of constants of $m = 1, m_1 = 1, \lambda_1 = 0.1, \lambda_2 = 0.1, \lambda_3 = 0.1, q = -0.5$.

3. Bahamonde, S. et al.: Physics Reports **3** (2018).
4. Baumgarte, T.W. and Shapiro, S.L.: Phys. Rev. D **59** 024007 (1998).
5. Berman, M. S.: Nuovo Cimento. **74** 182 (1983).
6. Berman, M.S. and de Mello Gomide, F.: Gen. Relat. Gravit. **20** 191-198 (1988).
7. Bhoyar, S.R., Chirde, V.R. and Shekh, S.H.: Astrophysics **60** 259 (2017).
8. Brans, C. and Dicke, R. H.: Phys. Rev. **124** 925 (1961).
9. Can Aktas.: Modern Physics Letters A. **13** 1950098 (2019).
10. Canuto, V., Adams, P. J., Hsieh, S. H. and Tsiang, E.: Phys. Rev. D **16** 16 1643 (1977).
11. Capozziello, S., Stabile, A., Troisi, A.: Classical Quantum Gravity. **24** 2153 (2007).
12. Cunha, J. V. and Lima, J. A. S.: Monthly Not. Ro.l Astro. Soci. **1** 390 (2008).
13. Cunha, J. V.: Physical Review D. **4** 79 (2009).
14. Harko, T., Lobo, F.S.N., Nojiri, S. and Odintsov, S.D.: Phys. Rev. D **1**84 (2011).
15. Katore, S. D. and Kapse, D. V.: Pramana Journal of Physics. **30** 88 (2017).
16. Kumar, S.and Singh, C.P.: Astro. Spac. Sci. **57**312 (2007).
17. Leibundgut, B. and Sullivan, M.: Space Sci. Rev. **57** 214 (2018).
18. Mishra, B., Sahoo, P.K. and Ray, P.: Int. J. Geo. Meth. Mod. Phys. **9** 14 (2017).
19. Mohayaee, R., Rameez, M. and Sarkar, S.: Eur. Phys. J. Spec. Top. **230** 2067–2076 (2021).
20. Montero, P.J., Baumgarte, T.W. and Muller, E.: Phys. Rev. D **89** 084043 (2014).
21. Mukherjee, A. and Banerjee, N.: Astrophysics and Space Sci. **2** 352 (2014).
22. Nojiri, S. and Odintsov, S.D.: J. Phys. Conf. Ser. **66** 012005 (2007).
23. Nojiri, S., Odintsov, S.: Phys. Rev. D **78** 046006 (2008).
24. Pawar, D.D., Bhuttampalle, G.G. and Agrawal, P.K.: New Astron. **1**65 (2018).
25. Perlmutter, S., Aldering, G. and Valle, M. et al.: Nature. **51**391 (1998).

26. Perlmutter, S.: The Astrophysical Journal. **2** 517 (1999).
27. Riess, A.G. et al.: The Astronomical Journal. **3** 116 (1998).
28. Sahoo, P.K. and Bhattacharjee, S.: New Astron. **77** 101351 (2020).
29. Sahoo, P.K., Mishra, B. and Chakradhar, R.: Eur. Phys. J. Plus **1** 129 (2014).
30. Sharif, M. and Yousaf, Z.: Astrophysics and Space Sci. **354** 354 (2014).
31. Sharif, M., Yousaf, Z.: Phys. Rev. D **88** 024020 (2013).
32. Sharma, U.K. and Pradhan, A.: Int. J. Geo. Meth. Mod. Phys. **1** 1850014 (2018).
33. Shekh, S. H. and Chirde, V. R.: Gen. Rel. and Grav. **51** 87 (2019).
34. Shekh, S. H., Arora, S., Chirde, V. R. and Sahoo, P. K.: Int. J. Geo. Meth. Mod. Phy. **3** 2050048 (2020).
35. Shekh, S.H., Katore, S.D., Chirde, V.R. and Raut, S.V.: New Astron. **84** 101535 (2021).
36. Shekh, S.H.: New Astronomy. **83** 101464 (2021).
37. Shri Ram, Zeyauddin, M. and Singh, C.P.: Int. j. Mod. Phys. A. **31** 4991-5005 (2008).
38. Singh, C.P., Shri Ram. and Zeyauddin, M.: Astro. Spac. Sci. **181** 315 (2008).
39. Spergel, D.N. et al.: Astrophys. J. Suppl. **10** 377 (2007).
40. Spergel, D.N. et al.: Astrophys. J. Suppl. Ser. **148** 175 (2003).
41. Tonry, J. et al.: The Astrophysical Journal. **1** 594 (2003).
42. Wei, H., Qi, H. and Ma, X.: Eur. Phys. J. C **72** 2117 (2012).