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Implications of One-Loop Quantum Correction in the Background Geometry of 5-Dimensional Kaluza-Klein Cosmology

M. Ansari

Department of Mathematics
North-Eastern Hill University, NEHU Campus,
Shillong-793022, India
E-mail: mansari@nehu.ac.in ; mansari12_2006 @yahoo.co.in
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Abstract

Through dimensional reduction and one-loop quantum correction of scalar and spinor fields, time-dependent cosmological constant Λ_{eff}, effective gravitational constant G_{eff} and fine structure constant are derived in 5-dimensional Kaluza-Klein model for cosmology. If the internal manifold contracts with time and stabilizes itself at some later time, one possibility gets fine-structure constant equal to

$$\frac{1}{137},\ \ G_{eff} \simeq G_N \ \ and \ \ \Lambda_{eff} \simeq 0\ .$$

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1. Introduction

In the context of unification of gravity with other fundamental forces, Kaluza-Klein theory is important. Basically, in this theory 5-dimensional manifold is considered as $M^4 \times S^1$ where M^4 is the 4-dimensional manifold and S^1 is a circle. Our observable universe is 4-dimensional, so it is expected that radius of S^1 is extremely small (undetectable). Hence, it is very natural to think that if extra manifold was a reality at very high energy scale and is undetectable now because of nonavailability of energy of required order, it should manifest itself in some way or the other. Employing the method of heat - Kernel method, Toms [3] calculated one-loop effective action in 5-dimensional background geometry and obtained induced cosmological constant, gravity and Maxwell's term as manifestation of fifth dimension of the space. But the cosmological constant obtained by him is very large. The model, considered by him (Toms) contains static component of metric tensor corresponding to extra space which completely ignores its dynamical contribution.

This note offers calculation of time-dependent cosmological constant, effective gravitational constant(time dependent) as well as Maxwell's terms using the heat-Kernal method (adapted by Toms) to evaluate one-loop effective action for scalar fields as well as Dirac spinors. The 5-dimensional cosmological model proposed here is given by the line-element

$$ds^2 = dt^2 - a^2(t)[(dx^1)^2 + (dx^2)^2 + (dx^3)^2] - b^2(t)(dy - kA_\mu(x)dx^\mu)^2 \qquad (1)$$

where t is the cosmic time, $a(t)$ is the expanding scale factor for spatially flat subspace of M^4, $b(t)$ is the contracting scale factor for S^1, $A_\mu(\mu = 0,1,2,3)$ is the four-dimensional electromagnetic field and k is a constant of $(mass)^{-1}$ dimension to make $kA_\mu(x)$ dimensionless.

Using horizontal lift basis [4,5] the action in the background geometry given by (1) is written as

$$\begin{aligned} S = &- \frac{1}{16\pi G_5}\int d^4xdy\sqrt{-g_5}R_5 + \int d^4xdy\sqrt{-g_5}\frac{1}{2}[g^{m'n'}(D_{m'}\Phi)^* \\ &(D_{n'}\Phi) - \xi R_5\Phi^*\Phi - M_0^2\Phi^*\Phi] + \frac{1}{2}\int d^4xdy\sqrt{-g_5}\bar{\Psi}(ir^{m'}D_{m'} - M_{\frac{1}{2}})\Psi \end{aligned} \qquad (2)$$

where $G_5 = G_N L$ (G_N is the Newtonian gravitational constant equal to M_p^{-2} where M_p is Planck mass, $0 \le y \le L$). 5-dim. Ricci scalar $R_5 = R_4 - \frac{1}{4}k^2F_{\mu\nu}F^{\mu\nu}$ (R_4 is 4-dim. Ricci scalar, $F_{\mu\nu} = D_\nu A_\mu - D_\mu A_\nu$, $D_\mu = \bigtriangledown_\mu + kA_\mu$, $D_5 = \bigtriangledown_5$ ($\bigtriangledown_\mu$ and $\bigtriangledown_5$ are convariant derivatives in curved space). 5-dim. Dirac matrices $\gamma^{m'}(m' = 0,1,2,3,5)$ in curved space are given as $\gamma^{m'} = h_a^{m'}\tilde{\gamma}^a(\tilde{\gamma}^0,\tilde{\gamma}^1,\tilde{\gamma}^2,\tilde{\gamma}^3)$ are Dirac matrices in 4-dimensional flat space and $\tilde{\gamma}^5 = \tilde{\gamma}^0\tilde{\gamma}^1\tilde{\gamma}^2\tilde{\gamma}^3, h_a^{m'}$ are defined as $h_a^{m'}h_b'\eta^{ab} = g^{m'n'}$ with $\eta^{ab} = diag(1,-1,-1,-1,-1)$, ξ is a coupling constant, Φ is a scalar field with mass M_o, Ψ is the Dirac spinor with mass $M_{\frac{1}{2}}$ and g_5 is the determinant of the metric tensor $g_{m'n'}$ given as

$$g_{m'n'} = diag(1,-a^2,-a^2,-a^2,-b^2)$$

in horizontal lift basis. $\hbar = c = 1$ is used as fundamental unit where $\hbar$ and c have their usual meaning.

2. Gravity

5-dimensional action for gravity given by (2) can be reduced to 4-dimensional action employing the method of Pollock[6]. In this method, $g_{m'n'}$ can be conformally transformed to $g'_{m'n'}$ as

$$g_{m'n'} = b^2(t) g'_{m'n'} = b^2(t) \begin{pmatrix} \tilde{\tilde{g}}_{\mu\nu} & 0 \\ 0 & -1 \end{pmatrix} \tag{3}$$

where $\tilde{\tilde{g}}$ is the resulting metric tensor on M^4. So, on ignoring term of total divergence,

$$S_g = -\frac{1}{16\pi G_5} \int d^4x dy \sqrt{-\tilde{\tilde{g}}_4} b^3 \left[\tilde{\tilde{R}}_4 - 12 b^{-2} (\tilde{\tilde{\nabla}} b)^2 - \frac{1}{4} b^{-2} k^2 \tilde{\tilde{F}}_{\mu\nu} \tilde{\tilde{F}}^{\mu\nu} \right] \tag{4}$$

where $\tilde{\tilde{\nabla}}$ is the covariant derivative, $\tilde{\tilde{R}}_4$ is Ricci scalar and $\tilde{\tilde{F}}_{\mu\nu}$ is electromagnetic field strength corresponding to $\tilde{\tilde{g}}_{\mu\nu}$.

Further conformal transformation is done over $\tilde{\tilde{g}}_{\mu\nu}$ only as

$$\tilde{\tilde{g}}_{\mu\nu} = e^{2v} g_{\mu\nu} \tag{5}$$

where v is function of $b(t)$. Now using this conformal transformation and integrating over y,

$$\begin{aligned} S_g^{(4)} = & -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g_4(x)} b^3 e^{2v} \\ & \left[R_4 - \frac{1}{4} b^{-2} k^2 e^{-2v} F^{\mu\nu} F_{\mu\nu} - 12(\dot{v})^2 - 12(\frac{\dot{b}}{b})^2 - 18\dot{v}(\frac{\dot{b}}{b}) \right] \end{aligned} \tag{6}$$

where dot (.) denotes derivative with respect to t(time). Choosing $v = -\frac{3}{2} ln\ b(t)$, one gets 4-dimensional action for gravity as

$$S_g^{(4)} = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g_4(x)} \left[R_4 - \frac{1}{4} k^2 F_{\mu\nu} F^{\mu\nu} - 12(\frac{\dot{b}}{b})^2 \right] \tag{7}$$

constant k was introduced with intention to keep the theory dimensionally correct. So, without any harm to physics, k may be identified with $(16\pi G_N)^{\frac{1}{2}}$.

3. Scalar fields

The extra manifold is a circle which is not simply-connected, hence any field on it can be either untwisted(periodic in y) or twisted(anti-periodic in y)[7]. Hence, in either case, one may write

$$\Phi(x^\mu, y) = [Lb(t)]^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} \Phi_n(x^\mu) exp[i(n+\alpha)My] \tag{8}$$

where $M = 2\pi L^{-1}$ (L is circumference of S^1) and $\alpha = 0 \left(\frac{1}{2}\right)$ for untwisted (twisted) field.

Substituting $\Phi(x^\mu, y)$ given by (8) in the action for scalar field given by (2) and integrating over Y

$$S_\Phi^{(4)} = \frac{1}{2} \sum_{n=-\infty}^{\infty} \int d^4x \sqrt{-g_4}(x)[g^{\mu\nu}(D_\mu^{(n)}\Phi_n)^*(D_\nu^{(n)}\Phi_n) - M_n^2 \Phi_n^* \Phi_n - \xi(R_4 - \frac{1}{4}k^2 F_{\mu\nu}F^{\mu\nu})\Phi_n^*\Phi_n] \tag{9}$$

where

$$D_\mu^{(n)}\Phi_n = \bigtriangledown_\mu \Phi_n + iq_n A_n \Phi_n, \tag{10a}$$

$$M_n^2 = M_0^2 + \frac{(n+\alpha)^2}{b^2}M^2 - \frac{3}{2}\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{1}{4}\left(\frac{\dot{b}}{b}\right)^2 - \frac{1}{2}\frac{d}{dt}\left(\frac{\dot{b}}{b}\right) \tag{10b}$$

and

$$q_n = (n+\alpha)e = (n+\alpha)kM \tag{10c}$$

Here q_n is the charge of the scalar particle in nth mode which is integral (half-integral) multiple of e ($= kM$) for untwisted (twisted) field.

Now one loop effective action for Φ_n is calculated for nth mode and summed up over all modes to get [3]

$$\Gamma_\Phi^{(1)} = \frac{i}{2} \sum_{n=-\infty}^{\infty} ln\, det \Delta_n \tag{11}$$

where Δ_n is the operator defined as

$$\Delta_n = g^{\mu\nu} D_\mu^{(n)} D_\nu^{(n)} + M_n^2 + \xi\left(R_4 - \frac{1}{4}k^2 F_{\mu\nu}F^{\mu\nu}\right) \tag{12}$$

Using the kernal $k_n(s, x, x)$ for Δ_n, (11) can be re-written as

$$\Gamma_\Phi^{(1)} = \frac{i}{2} \sum_{n=-\infty}^{\infty} \int d^4x \sqrt{-g_4} \int_0^\infty \frac{ds}{s} tr\, k_n(s, x, x) \tag{13}$$

where

$$k_n(s,x,x) = i\mu^{4-N}(4\pi is)^{-\frac{N}{2}} exp(-iM_n^2 s)\sum_{k=0}^{\infty}(is)^k a_k(x)$$

(N is the space-time dimension used as dimensional regulator with $N \to 4$ and μ is a constant of mass dimension to get dimensionless action). For Δ_n given by (12) [8,9]

$$a_0(x) = 1 \tag{14a}$$

$$a_1(x) = (\frac{1}{6} - \xi)R_4 + \frac{1}{4}\xi k^2 F_{\mu\nu}F^{\mu\nu} \tag{14b}$$

$$a_2(x) = -\frac{1}{12}k^2 M^2 (n+\alpha)^2 + \ldots \tag{14c}$$

Only relevant terms are mentioned here.

Integrating over s in (13) and using (14)

$$\begin{aligned}\Gamma_{\Phi}^{(1)} = &-\frac{1}{2(4\pi)^2}\int d^4x\sqrt{-g_4}\,[\lim_{N\to 4}\lceil(-\frac{N}{2})\sum_{n=-\infty}^{\infty}\Big\{\frac{(n+\alpha)^2M^2}{b^2}+\bar{M}^2(t)\Big\}^{\frac{N}{2}}+\\ &\lim_{N\to 4}\sqrt{(1-\frac{N}{2})}\sum_{n=-\infty}^{\infty}\Big\{\frac{(n+\alpha)^2M^2}{b^2}+\bar{M}^2(t)\Big\}^{\frac{N}{2}-1}(\frac{1}{6}-\xi)R_4+\\ &\lim_{N\to 4}\{\frac{1}{4}\xi k^2\sqrt{(1-\frac{N}{2})}\sum_{n=-\infty}^{\infty}[\frac{(n+\alpha)^2M^2}{b^2}+\bar{M}^2(t)]^{\frac{N}{2}-1}-\\ &\frac{1}{12}\sqrt{2-\frac{N}{2}}\sum_{n=-\infty}^{\infty}k^2M^2(n+\alpha)^2[\frac{(n+\alpha)^2M^2}{b^2}+\bar{M}^2(t)]^{\frac{N}{2}-2}\}+\ldots]\end{aligned} \tag{15}$$

where

$$M^{-2}(t) = M_0^2 - \frac{3}{2}\frac{\dot{a}}{a}\frac{\dot{b}}{b} - \frac{1}{4}\left(\frac{\dot{b}}{b}\right)^2 - \frac{1}{2}\frac{d}{dt}\left(\frac{\dot{b}}{b}\right)$$

Using the formulae (B6) of ref.[10],

$$\sum_{n=-\infty}^{\infty}[(n+c)^2+d^2]^{-\lambda} = \pi^{\frac{1}{2}}d^{1-2\lambda}\frac{\sqrt{(\lambda-\frac{1}{2})}}{\sqrt{\lambda}} + 4\sin\pi\lambda f_{\lambda}(c,d)$$

(where Re $\lambda > \frac{1}{2}$ and c and d are real), series in (15), for $\bar{M}^2(t) > 0$ is summed to yield when $\alpha = 0$,

$$\begin{aligned}\Gamma_{\Phi}^{(1)} = &-\frac{1}{2(4\pi)^2}\int d^4x\sqrt{-g_4}[-\frac{8\pi}{15}\{\bar{M}(t)\}^5\frac{b}{M} + \frac{4\pi b}{3M}\{\bar{M}(t)\}^3 \times \\ &(\frac{1}{6}-\xi)R_4 + \frac{k^2}{4}\left(\frac{4\pi\xi b}{3M}\{\bar{M}(t)\}^3 + \frac{M^2\varsigma(3)}{24\pi^2}\right)F_{\mu\nu}F^{\mu\nu} + \ldots]\end{aligned} \quad (16)$$

where $\varsigma(p)$ is the Riemann-zeta function.

When $\alpha = \frac{1}{2}$

$$\begin{aligned}\Gamma_{\Phi}^{(1)} = &-\frac{1}{2(4\pi)^2}\int d^4x\sqrt{-g_4}[-\frac{8\pi}{15}\{\bar{M}(t)\}^5 + \frac{4\pi b}{3M}\{\bar{M}(t)\}^3 \times \\ &(\frac{1}{6}-\xi)R_4 + \frac{k^2}{4}\left(\frac{4\pi\xi b}{3M}\{\bar{M}(t)\}^3 - \frac{M^2\varsigma(3)}{4\pi^2}\right)F_{\mu\nu}F^{\mu\nu} + \ldots]\end{aligned} \quad (17)$$

If $N_0^+(N_0^-)$ is the number of untwisted (twisted) scalar fields in the theory,

$$\begin{aligned}\Gamma_{\Phi}^{(1)} = &-\frac{1}{2(4\pi)^2}\int d^4x\sqrt{-g_4}[-\frac{8\pi}{15}\frac{b}{M}\{\bar{M}(t)\}^5(N_0^+ + N_0^-) + \\ &\frac{4\pi b}{3M}(N_0^+ + N_0^-)\{\bar{M}(t)\}^3(\frac{1}{6}-\xi)R_4 + \\ &\frac{k^2}{4}\left(\frac{4\pi\xi b}{3M}\{\bar{M}(t)\}^3(N_0^+ + N_0^-) + \frac{M^2\varsigma(3)}{24\pi^2}(N_0^+ - \frac{3}{2}N_0^-)\right)F_{\mu\nu}F^{\mu\nu} + \ldots]\end{aligned} \quad (18)$$

4. Dirac spinors

Like scalar fields, $\Psi(x^\mu, y)$ may also be written as

$$\Psi(x^\mu, y) = [Lb(t)]^{-\frac{1}{2}}\sum_{n=-\infty}^{\infty}\Psi_n(x^\mu)exp[i(n+\alpha)My] \quad (19)$$

Using this anstaz for $\Psi(x^\mu, y)$ in the action for $\Psi(x^\mu, y)$ given by (2) and integrating over y,

$$S_{\Psi}^{(4)} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int d^4x\sqrt{-g_4}\bar{\Psi}_n\left[i\gamma^\mu D_\mu^{(n)} - \frac{\tilde{\gamma}^5(n+\alpha)M}{b} - M_{\frac{1}{2}}\right]\Psi_n \quad (20)$$

Under chiral rotations [11,12], the mass term for Ψ_n gets the canonical form

$$\bar{\Psi}_n \left[\frac{(n+\alpha)^2}{b^2} + M^2_{\frac{1}{2}} \right] \Psi_n \tag{21}$$

Now one-loop correction terms for Ψ_n can be calculated by repeating the procedure adopted for scalar fields with

$$t_r a_0(x) = p \tag{22a}$$

$$t_r a_1(x) = -\frac{1}{12} p R_4 + \frac{pk^2}{16} F_{\mu\nu} F^{\mu\nu} \tag{22b}$$

$$t_r a_2(x) = -\frac{p}{12} k^2 M^2 + (n+\alpha)^2 F_{\mu\nu} F^{\mu\nu} + \dots \tag{22c}$$

Here also only relevant terms are mentioned, p in (22) is the number of spinor components which is 4 for Ψ_n. If number of untwisted (twisted) spinors are $N^+_{\frac{1}{2}}(N^-_{\frac{1}{2}})$

$$\begin{aligned} \Gamma^{(1)}_{\Psi} = & -\frac{1}{2(4\pi)^2} \int d^4x \sqrt{-g_4}\, [-\frac{32\pi b}{15M} M^5_{\frac{1}{2}} (N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}}) + \\ & \frac{4\pi b}{9M} M^3_{\frac{1}{2}} (N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}}) R_4 + \frac{k^2}{4} \{ \frac{-4\pi b}{3M} M^3_{\frac{1}{2}} (N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}}) - \\ & \frac{2M^2}{3\pi^2} \varsigma(3) (N^+_{\frac{1}{2}} - \frac{3}{2} N^-_{\frac{1}{2}}) \} F_{\mu\nu} F^{\mu\nu} + \dots \end{aligned} \tag{23}$$

5. Effective action for gravity

From (7), (18) and (23), effective action for 4-dimensional gravity is written as

$$\begin{aligned} S^{(4)eff}_g = & \int d^4x \sqrt{-g_4}\, [-\frac{1}{16\pi G_n} + \frac{b}{24\pi M} \{\bar{M}(t)\}^3 (N^+_0 + N^-_0)(\frac{1}{6} - \xi) + \\ & \frac{b}{72\pi M} M^3_{\frac{1}{2}} (N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}}) R_4 + \frac{3}{4\pi G_N} \left(\frac{\dot{b}}{b} \right)^2 + \frac{1}{60\pi} \frac{b}{M} \{\bar{M}(t)\}^5 \times \\ & (N^+_0 + N^-_0) - \frac{b}{15\pi M} M^5_{\frac{1}{2}} (N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}})] \end{aligned} \tag{24}$$

which yields the effective 4-dimensional gravitational constant as

$$\frac{1}{16\pi G_{eff}} = \frac{1}{16\pi G_N} + \frac{b}{72\pi M} [3\{\bar{M}(t)\}^3 (N^+_0 + N^-_0)(\frac{1}{6} - \xi) + M^3_{\frac{1}{2}} (N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}})] \tag{25a}$$

and effective cosmological constant $\wedge_{eff}$ as

$$\frac{\wedge_{eff}}{8\pi G_{eff}} = \frac{3}{4\pi G_N}(\frac{\dot{b}}{b})^2 + \frac{b}{60\pi M}[\{\bar{M}(t)\}^5(N_0^+ + N_0^-) - 4M_{\frac{1}{2}}^5(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-)] \quad (25b)$$

Thus, one finds that G_{eff} and $\wedge_{eff}$ are time dependent. Also it is interesting to see that if $\xi > \frac{1}{6}$ and at a particular time t'

$$\frac{1}{16\pi G_N} < \frac{b(t')}{72\pi M}[3\{\bar{M}(t)\}^3(N_0^+ + N_0^-)(\xi - \frac{1}{6}) - M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-)] \qquad (25c)$$

$G_{eff} < 0$. It means that under above circumstances, gravity becomes repulsive contrary to its usually believed nature. Possibility of anti-gravity has also been discussed by Yoshimura[13] in the context of his finite temperature theory of higher-dimensional Kaluza-Klein type cosmology. But if $\xi \leq \frac{1}{6}$, $G_{eff} > 0$. Even if $\xi > \frac{1}{6}$, $G_{eff} > 0$ is possible provided that a particular time t''

$$M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-) > 3\{\bar{M}(t'')\}^3(N_0^+ + N_0^-)(\xi - \frac{1}{6})$$

6. Induced Maxwell's terms

From (7),(18) and (23), induced Maxwell's term in the action is given as

$$S_{F^2}^{(4)} = \frac{1}{4}\int d^4x\sqrt{-g_4}\frac{e^2}{M^2}[\frac{b}{16\pi G_N} + \frac{4\pi\xi b}{3M}\{\bar{M}(t)\}^3(N_0^+ + N_0^-) + \frac{M^2\varsigma(3)}{6\pi^2}\times$$
$$(N_0^+ - \frac{3}{2}N_0^-) - \frac{4\pi b}{M}M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-) - \frac{2M^2}{3\pi^2}\varsigma(3)(N_{\frac{1}{2}}^+ - \frac{3}{2}N_{\frac{1}{2}}^-)F_{\mu\nu}F^{\mu\nu}] \quad (26)$$

The normalization condition for A_μ yields [14,15,16]

$$b(t)\left[\frac{M_p^2}{16\pi} + \frac{4\pi\xi}{3M}\{\bar{M}(t)\}^3(N_0^+ + N_0^-) - \frac{4\pi}{M}M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-)\right] + \frac{M^2\varsigma(3)}{6\pi^2}(N_0^+ - \frac{3}{2}N_0^-) - \frac{2M^2}{3\pi^2}\varsigma(3)(N_{\frac{1}{2}}^+ - \frac{3}{2}N_{\frac{1}{2}}^-) = \frac{M^2}{e^2} \quad (27)$$

If $N_0^+ = 4N_{\frac{1}{2}}^+$ and $N_0^- = 4N_{\frac{1}{2}}^-$, (27) gets a more convenient from as

$$b(t)\left[\frac{M_p^2}{16\pi} + \frac{16\pi\xi}{3M}\{\bar{M}(t)\}^3 - \frac{4\pi}{M}M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-)\right] = \frac{M^2}{e^2} \quad (28)$$

It is interesting to see from (27) and (28) that e (gauge coupling constant for electromagnetic field) is time-dependent. As a result fine structure constant (for $N_0^+ = 4N_{\frac{1}{2}}^+$ and $N_0^- = 4N_{\frac{1}{2}}^-$) is given as

$$\frac{e^2}{4\pi} = \frac{M^2}{4\pi}[b(t)]^{-1}\left[\frac{M_p^2}{16\pi} + \frac{16\pi\xi}{3M}\{\bar{M}(t)\}^3 - \frac{4\pi}{M}M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-)\right]^{-1} \quad (29)$$

is time-dependent which shows that when $b \to \infty$, $\frac{e^2}{4\pi} \to 0$ and as $b \to 0$, $\frac{e^2}{4\pi} \to \infty$. But we know that at low mass scale (large t), $\frac{e^2}{4\pi} \simeq \frac{1}{137}$. This well-known result puts a constraint on $b(t)$ that $b(t)$ should stabilize itself at some time t_1, during the course of evolution of the universe around the value $b_1 = b(t_1)$ given by

$$\frac{1}{137} = \frac{M^2}{4\pi}b_1^{-1}\left[\frac{M_p^2}{16\pi} + \frac{16\pi\xi M_0^3}{3M} - \frac{4\pi}{M}M_{\frac{1}{2}}^3(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-)\right]^{-1} \quad (30)$$

In (30), if M_0 and $M_{\frac{1}{2}}$ are sufficiently small,

$$b_1 \simeq \frac{548M^2}{M_p^2} \quad (31)$$

The effective radius of the extra manifold (circle) is Lb(t). If extra manifold is hidden, at the compactification time t_c

$$Lb(t_c) \lesssim L_p \quad (32)$$

Constraint obtained above and the fact that $b(t)$ is a contracting scale factor, imply that

$$b(t_c) \geq b_1 \quad (33)$$

Thus, one gets

$$Lb_1 \leq Lb(t_c) \lesssim L_p \quad (34)$$

Now (31) and (34) imply compactification mass $M \lesssim \frac{M_p}{548}$ and $b_1 \lesssim 1.8 \times \bar{10}^3$.

From (25a) and (34), one gets at $t = t_1$

$$\frac{1}{16\pi G_{eff}} \lesssim \frac{1}{16\pi G_N} + \frac{M_p^{-1}}{72\pi}\left\{12M_0^3(\frac{1}{6} - \xi) + M_{\frac{1}{2}}^3\right\}(N_{\frac{1}{2}}^+ + N_{\frac{1}{2}}^-) \approx \frac{1}{16\pi G_N} \quad (35)$$

(25b) and (34) imply that at $t = t_1$

$$\frac{\wedge_{eff}}{8\pi G_{eff}} \lesssim \frac{M_p^{-1}}{15\pi}(N^+_{\frac{1}{2}} + N^-_{\frac{1}{2}})(M_0^5 - N^5_{\frac{1}{2}}) \tag{36}$$

which shows that if $M_0 \simeq M_{\frac{1}{2}}, \wedge_{eff} = 0$, otherwise also $\wedge_{eff} \approx 0$.

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