J. T. S.
Vol. 4 (2010), pp.57-68
https://doi.org/10.56424/jts.v4i01.10423

# Pseudo-Slant Submanifolds of a Generalised Almost Contact Metric Structure Manifold

**Jaya Upreti and S. K. Chanyal**

Department of Mathematics
Kumaun University, S. S. J. Campus, Almora, Uttrakhand, India
(Received: July 21, 2009)

**Abstract**

In this paper we have studied pseudo-slant submanifolds of a Generalised almost contact metric structure manifold and established integrability conditions of distributions and some interesting results on this submanifold.

**Keywords and Phrases :** Generalised Almost Contact Metric Structure Manifold, Slant Submanifold Pseudo-Slant Submanifold.

## 1. Introduction

The geometry of slant submanifolds was initiated by B. Y. Chen. He defined slant immersions in the complex geometry as a natural generalization of both holomorphic and totally real immersions [4]. A. Lotta introduced the notion of slant immersions of a Riemannian manifold into an almost contact metric manifold [5]. In [2], J. L. Cabererizo et. all studied and characterised slant submanifolds of K-contact and Sasakian manifolds with several examples. Recently Khan and Khan studied Pseudo-slant submanifolds of a Sasakian manifold [5].

The purpose of this paper is to study pseudo-slant submanifolds of Generalised almost contact metric structure manifold. In section 3 we defined slant immersions and slant distributions on Generalised almost contact metric structure manifold and Hyperbolic Hermite manifold and proved some characterisation theorem. In section 4 we defined pseudo-slant submanifolds of these manifolds and established a relation between them. We also worked out integrability conditions of distributions on pseudo-slant submanifolds of Generalised almost contact metric structure manifold.

## 2. Preliminaries

First we define a Generalised almost contact metric structure manifold.

**Definition (2.1) [8].** An odd dimensional Riemannian manifold $(\overline{M}, g)$ is said to be a Generalised almost contact metric structure manifold if, there exits a tensor $\phi$ of the type (1, 1) and a global vector field $\xi$ and a 1-form $\eta$ satisfying the following equations:

$$\phi^2 X = a^2 X + \eta(X)\xi \tag{1}$$

$$\eta(\phi X) = 0 \tag{2}$$

$$\eta(\xi) = -a^2 \tag{3}$$

$$\phi(\xi) = 0 \tag{4}$$

$$\eta(X) = g(X, \xi) \tag{5}$$

$$g(\phi X, \phi Y) = -a^2 g(X, Y) - \eta(X)\eta(Y), \tag{6}$$

where $X, Y \in T\,\overline{M}$, $a$ be a complex number and $g$ be the metric of $\overline{M}$.

From above definition it is clear that almost contact metric manifold is a particular case of a Generalised almost contact metric structure manifold for $a^2 = -1$.

If $'\Phi$ is a 2-form defined on $\overline{M}$ as

$$'\Phi(X, Y) = g(\phi X, Y),$$

then $'\Phi$ is alternating i.e.

$$'\Phi(Y, X) = -{}'\Phi(X, Y)$$

or

$$g(\phi X, Y) = -g(\phi Y, X). \tag{7}$$

Now let M be a submanifold immersed in $\overline{M}$ and we denote by the same symbol $g$ the induced metric on $M$. let $TM$ be the Lie algebra of the vector fields in $M$ and $T^{\perp}M$ denote the set of all vector fields normal to $M$. Then, the Gauss and Weingarten equations are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{8}$$

$$\overline{\nabla}_X V = -A_V X + \nabla^{\perp}{}_X V, \tag{9}$$

for all $X, Y \in TM$, $V \in T^{\perp}M$.

Where $\overline{\nabla}$, $\nabla$ are respectively the Levi-Civita connexions on $\overline{M}$ and $M$ and $\nabla^{\perp}$ is induced connexion in normal bundle of $M$ i.e. $T^{\perp}M$, $h$ is symmetric bilinear vector valued function called second fundamental form and $A_V$ is the shape operator associated with $V$. The second fundamental form $h$ and the shape operator $A$ are related by

$$g(A_V X, Y) = g(h(X, Y), V). \tag{10}$$

For any$X \in TM$, we write,

$$\phi X = TX + NX, \tag{11}$$

where $TX$ is the tangential component of $\phi X$ and $NX$ is the normal component of $\phi X$. Similarly for any $V$ in $T^{\perp}M$, we write

$$\phi V = tV + nV, \tag{12}$$

where $tV$ (resp. $nV$) denotes the tangential (resp. normal) component of $\phi V$.

The submanifold $M$ is said to be an invariant submanifold if $N$ is identically zero i.e. $\phi X = TX$ for any $X \in TM$. On the other hand the submanifold $M$ is called anti-invariant submanifold in $T$ is identically zero i.e. $\phi X = NX$.

The covariant derivatives of $T$ and $N$ are defined as

$$(\overline{\nabla}_X T)Y = \nabla_X(TY) - T(\nabla_X Y) \tag{13}$$

and

$$(\overline{\nabla}_X N)Y = \nabla^{\perp}{}_X(NY) - N(\nabla_X Y). \tag{14}$$

The distribution spanned by the structure vector ? is denoted by $< \xi >$.

## 3. Slant distributions and slant immersions

Let M be a Riemannian manifold, isometrically immersed in a Generalised almost contact metric structure manifold $(\overline{M}, \phi, g, a, \eta, \xi)$. Suppose that the structure vector $\xi$ is tangent to $M$. if we denote by $D$ the orthogonal distribution to $\xi$ in $TM$. Then

$$TM = D \oplus < \xi > .$$

For each nonzero vector $X$ tangent to $M$ at $x$, such that $X$ is not proportional to $\xi_x$, we denote by $\theta(X)$ the angle between $\phi X$ and $T_x M$. Since $\phi(\xi) = 0$, thus $\theta(X)$ is the angle between $\phi X$ and $D_x$.

**Definition (3.1) :** $M$ is said to be slant if the angle $\theta(X)$ is constant, i.e. which is independent of the choice of $x \in M$ and $X \in TM - < \xi_x >$. The angle $\theta$ of a slant immersion is called the slant angle of the immersion.

From this definition, it is evident that invariant and anti-invariant immersions slant immersions with slant angle $\theta = 0$ and $\theta = \pi/2$ respectively. A slant immersion, which is neither invariant nor anti-invariant, is called proper slant immersion.

A useful characterization of slant submanifolds in Generalised almost contact metric structure manifold is given by the following theorem.

**Theorem (3.1) :** Let $M$ be a submanifold isometrically immersed in a Generalised almost contact metric structure manifold $(\overline{M}, \phi, g, a, \eta, \xi)$ such that $\xi \in TM$, then $M$ is slant if and only if there exists a constant $\lambda \in [0, 1]$ such that

$$T^2 = a^2\lambda I + \lambda\eta \otimes \xi. \tag{15}$$

Furthermore, in this case, if $\theta$ is slant angle of $M$, then $\lambda = \cos^2\theta$.

**Proof :** Let $X, Y \in TM$, then for any slant submanifold, we have

$$\begin{aligned}
&g(TX, TY) = \cos^2\theta.g(\phi X, \phi Y)\\
&\Leftrightarrow g(TX, TY) = \cos^2\theta.[-a^2g(X, Y) - \eta(X)\eta(Y)] \text{ from (6)}\\
&\Leftrightarrow -g(T^2X, Y) = -\cos^2\theta.[a^2g(X, Y) + \eta(X)\eta(Y)] \;\Theta\; g(TX, Y) = -g(X, TY)\\
&\Leftrightarrow g(T^2X, Y) = \cos^2\theta.[a^2g(X, Y) + \eta(X)\eta(Y)] \;\forall\; Y \in TM\\
&\Leftrightarrow T^2X = \cos^2\theta.[a^2X + \eta(X)\xi] \;\forall\; X \in TM\\
&\Leftrightarrow T^2 = \cos^2\theta.[a^2I + \eta \otimes \xi]\\
&\Leftrightarrow T^2 = a^2\lambda I + \lambda\eta \otimes \xi
\end{aligned}$$

where $\lambda = \cos^2\theta$, $\theta$ is the slant angle.

Hence the theorem.

Now we define slant distributions.

**Definition (3.2) :** A differentiable distribution $\nu$ on M is said to be a slant distribution if for each $x \in M$ and each nonzero vector $X \in \nu_x$, the angle $\theta\nu(X)$ between $\phi X$ and the vector space $\nu_x$ is constant, i.e. which is independent of the choice of$x \in M$ and $X \in \nu_x$. In this case the constant angle $\theta\nu$ is called the slant angle of the distribution $\nu$.

Thus we see that if a submanifold is slant, then there exists a slant distribution on $M$.

The following theorem provides a useful characterization for the existence of a slant distribution on a Generalised almost contact metric structure manifold.

**Theorem (3.2) :** Let $\nu$ be a distribution on $M$, orthogonal to $\xi$. Then $\nu$ is slant if and only if there exists a constant $\lambda \in [0,1]$ such that $(PT)^2X = a^2\lambda X$, for any $X \in \nu$.

Furthermore, in this case, if $\theta$ is slant angle of $M$, then $\lambda = \cos^2\theta$.

**Proof :** The proof is straightforward and may be obtained from theorem (3.1).

Now we define slant distributions on a submanifold of Hyperbolic Hermite manifold.

**Definition (3.2) :** Given a submanifold $S$, isometrically immersed in a Hyperbolic Hermite manifold $(\overline{S}, J, g_1)$, a differentiable distribution $D$ on $S$ is said to be a slant distribution if for any nonzero vector $X \in D_x$, $x \in S$, the angle between $JX$ and the vector space $Dx$ is constant, i.e. which is independent of the choice of $x \in S$ and $X \in D_x$. In this case the constant angle is called the slant angle of the distribution $D$ (compare with the definition (3.2)).

## 4. Pseudo-slant submanifolds of Generalised almost contact metric structure manifold

We first define pseudo-slant submanifolds of Hyperbolic Hermite manifold.

**Definition (4.1) :** A submanifold S of a Hyperbolic Hermite manifold $(\overline{S}, J, g_1)$ is called a pseudo-slant submanifold, if there exists on $S$, two differentiable orthogonal distributions $D_1$ and $D_2$ such that $TM = D_1 \oplus D_2$, where $D_1$ is totally real distribution i.e. $JD_1 \subset T^{\perp}S$ and $D_2$ is slant distribution with slant angle $\theta \neq \pi/2$, in particular if dim $D_1 = 0$ and $\theta \in (0, \pi/2)$, then $S$ is proper slant submanifold of $(\overline{S}, J, g_1)$.

In the following paragraph we show that there is a relationship between slant submanifold of Generalised almost contact metric structure manifold and pseudo-slant submanifolds of Hyperbolic Hermite manifold.

Let $(\overline{M}, \phi, g, a, \eta, \xi)$ be a Generalised almost contact metric structure manifold. Then we consider the manifold $\overline{M} \times R$. We denote by $\left(X, f\frac{d}{dt}\right)$ a vector field on $\overline{M} \times R$, where $X$ is tangent to $\overline{M}$, $t$ is the coordinate of $R$ and $f$ is a differentiable function on $\overline{M} \times R$.

If we define a tensor $J$ of type (1, 1) on $\overline{M} \times R$ defined by

$$J\left(X, f\frac{d}{dt}\right) = \frac{1}{a}\left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right) \tag{16}$$

Then we have, $J^2\left(X, f\frac{d}{dt}\right) = \frac{1}{a}J\left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right)$ from (16)

$$\begin{aligned}
&= \tfrac{1}{a}\cdot\tfrac{1}{a}\left(\phi(\phi X - f\xi) - \eta(X)\xi, \eta(\phi X - f\xi)\tfrac{d}{dt}\right) \\
&= \tfrac{1}{a^2}\left(\phi^2 X - f\phi\xi - \eta(X)\xi, (\eta(\phi X) - f\eta(\xi))\tfrac{d}{dt}\right) \\
&= \tfrac{1}{a^2}\left(a^2 X, (a^2 f)\tfrac{d}{dt}\right), \text{ from (1), (2), (3) and (4)} \\
&= \left(X, f\tfrac{d}{dt}\right)
\end{aligned}$$

i.e.

$$J^2\left(X, f\frac{d}{dt}\right) = \left(X, f\frac{d}{dt}\right). \tag{17}$$

Now we define the metric $g_1$ on $\overline{M} \times R$ as

$$g_1\left[\left(X, f\frac{d}{dt}\right), \left(Y, h\frac{d}{dt}\right)\right] = g(X, Y) + fh. \tag{18}$$

Then we obtain

$$g_1\left[J\left(X, f\frac{d}{dt}\right), J\left(Y, h\frac{d}{dt}\right)\right] = g_1\left[\frac{1}{a}\left(\phi X - f\xi, \eta(X)\frac{d}{dt}\right), \frac{1}{a}\left(\phi Y - h\xi, \eta(Y)\frac{d}{dt}\right)\right], \text{ by (16)}$$

$$\begin{aligned}
&= \tfrac{1}{a^2}g_1\left[\left(\phi X - f\xi, \eta(X)\tfrac{d}{dt}\right), \left(\phi Y - h\xi, \eta(Y)\tfrac{d}{dt}\right)\right] \\
&= \tfrac{1}{a^2}\left[g\left(\phi X - f\xi, \phi Y - h\xi\right) + \eta(X)\eta(Y)\right] \text{ by (18)} \\
&= \tfrac{1}{a^2}\left[g(\phi X, \phi Y) - g(\phi X, h\xi) - g(f\xi, \phi Y) + g(f\xi, h\xi) + \eta(X)\eta(Y)\right] \\
&= \tfrac{1}{a^2}\left[-a^2 g(X, Y) - \eta(X)\eta(Y) - a^2 fh + \eta(X)\eta(Y)\right], \\
&\qquad \text{by (3), (4), (5), (6) and (7)} \\
&= -\left[g(X, Y) + fh\right] \\
&= -g_1\left[\left(X, f\tfrac{d}{dt}\right), \left(Y, h\tfrac{d}{dt}\right)\right], \text{ by (18)}
\end{aligned}$$

Therefore we have

$$g_1\left[J\left(X, f\frac{d}{dt}\right), J\left(Y, h\frac{d}{dt}\right)\right] = -g_1\left[\left(X, f\frac{d}{dt}\right), \left(Y, h\frac{d}{dt}\right)\right], \tag{19}$$

from (17) and (19), we see that $(\overline{M} \times R, J, g_1)$ is a Hyperbolic Hermite structure manifold.

Now we state the following theorem, which provides a method to obtain a pseudo- slant submanifold of $\overline{M} \times R$ from slant submanifold of $\overline{M}$.

**Theorem (4.1) :** Let $M$ be a non anti-invariant slant submanifold of a Generalised almost contact metric structure manifold $\overline{M}$ with slant distribution $D$ and $\xi$ is orthogonal to $M$. then $M \times R$ is a pseudo-slant submanifold of the Hyperbolic Hermite manifold $\overline{M} \times R$ with totally real distribution $D_1 = \left\{(0.\frac{d}{dt})\right\}$ and slant distribution $D_2 = \{(X, 0)|X \in D\}$.

**Proof :** Since we have,

$$g_1 \left[(X, 0), \left(0, \frac{d}{dt}\right)\right] = g(X, 0) + 0 = 0.$$

and $\left(X, f\frac{d}{dt}\right) = (X, 0) + f\left(0, \frac{d}{dt}\right), \forall \left(X, f\frac{d}{dt}\right) \in T(M \times R)$,

therefore $T(M \times R) = D_1 \oplus D_2$ is an orthogonal direct decomposition.

Also $\quad J\left(0, \frac{d}{dt}\right) = \frac{1}{a}(-\xi, 0) \subset T^{\perp}(M \times R)$ from (16)

$\therefore D_1$ is totally real distribution. It is easy to see that $D_2$ is slant distribution with slant angle $\theta$ (which is slant angle of $D$) in the sense of Papaghuic [9].

To introduce pseudo-slant submanifold of a Generalised almost contact metric structure manifold; first we define bislant submanifolds of a Generalised almost contact metric structure manifold.

**Definition (4.2) :** $M$ is said to be a bislant submanifold of a Generalised almost contact metric structure manifold $\overline{M}$ if there exists two orthogonal distributions $D_1$ and $D_2$ such that

(i) $TM$ admits the orthogonal direct decomposition $TM = D_1 \oplus D_2 \oplus <\xi>$

(ii) The distribution $D_1$ is slant with angle $\theta_1$

(iii) The distribution $D_2$ is slant with angle $\theta_2$.

Now we define pseudo-slant submanifold of a Generalised almost contact metric structure manifold as a particular case of bislant submanifold.

**Definition (4.3) :** $M$ is said to be a pseudo-slant submanifold of a Generalised almost contact metric structure manifold $\overline{M}$ if there exists two orthogonal distributions $D_1$ and $D_2$, such that

(i) $TM$ admits the orthogonal direct decomposition $TM = D_1 \oplus D_2 \oplus <\xi>$

(ii) The distribution $D_1$ is anti-invariant i.e. $\phi D_1 \subset T^{\perp}M$

(iii) The distribution D2 is slant with angle $\theta \neq \pi/2$.

If we denote by $d_i$, the dimension of $D_i$, for $i = 1, 2$, then we find the following cases

(a) If $d_2 = 0$, then $M$ is an anti-invariant submanifold.

(b) If $d_1 = 0$ and $\theta = 0$, then $M$ is an invariant submanifold.

(c) If $d_1 = 0$ and $\theta \neq 0$, then $M$ is a proper slant submanifold with slant angle $\theta$.

(d) If $d_1 \neq 0$ and $\theta = 0$, then $M$ is a semi invariant submanifold.

Let $M$ be a pseudo-slant submanifold of a Generalised almost contact metric structure manifold $\overline{M}$. Then, for any $X \in TM$, we write

$$X = P_1X + P_2X + \eta(X)\xi \tag{20}$$

where $P_i$ denotes the projection map on the distribution $D_i$, $i = 1, 2$.

Now operating on both sides of the equ. (20), we obtain

$$\phi X = NP_1X + TP_2X + NP_2X, \tag{21}$$

because

$$\phi P_1X = NP_1X, \qquad TP_1X = 0. \tag{22}$$

It is easy to see that

$$TX = TP_2X \, NX = NP_1X + NP_2X \tag{23}$$

and

$$TP_2X \in D_2. \tag{24}$$

Since $D_2$ is slant distribution, by theorem (3.2)

$$T^2X = a^2\cos^2\theta X, \qquad \forall X \in D_2. \tag{25}$$

Now we have the following theorem.

**Theorem (4.2) :** Let $M$ be a submanifold of a Generalised almost contact metric structure manifold $\overline{M}$, such that $\xi \in TM$. Then $M$ is a pseudo-slant submanifold is and only if there exists a constant $\lambda \in (0, 1]$, such that

(i) $D = \{X \in TM | T^2X = a^2\lambda X\}$ is a distribution on $M$.

(ii) For any $X \in TM$, orthogonal to $D$, $TX = 0$.

Furthermore, in this case, $\lambda = \cos^2\theta$ where $\theta$ denotes the slant angle of $D$.

**Proof :** Putting $\lambda = \cos^2\theta$, it is obvious that for any $X \in D$, $T^2X = a^2\cos^2\theta X$ therefore $D = D_2$ from equ. (25).

Thus $D$ is a distribution on $M$.

Also for any $X \in TM$, orthogonal to $D$, we have

$$\phi X \in T^\perp M \text{ and } \phi\xi = 0, \text{ i.e. } TX = 0.$$

Hence the condition is necessary.

Conversely, consider the orthogonal direct decomposition $TM = D \oplus D^\perp \oplus < \xi >$, then by (i) and theorem (3.2), we find D is a slant distribution. From (ii) it is evident that $D^\perp$ is an anti-invariant distribution.

Therefore $M$ is a pseudo-slant submanifold, hence the theorem.

In the following paragraph, we discuss on the integrability conditions of the distributions involved in a pseudo-slant submanifolds of $\overline{M}$.

If $\mu$ be the invariant subspace of $T^\perp M$, then in case of pseudo-slant submanifold, consider the direct decomposition of $T^\perp M$ as

$$T^\perp M = \mu \oplus ND_1 \oplus ND_2 \tag{26}$$

Since $D_1$ and $D_2$ are orthogonal, therefore $g(Z, X) = 0$. $\forall X \in D_1, Z \in D_2$

This implies that $g(NZ, NX) = g(\phi Z, \phi X) = 0$ $Q$ $g(TZ, NX) = 0$.

Therefore (26) gives orthogonal direct decomposition of $T^\perp M$.

First, we prove some important lemmas.

**Lemma (4.1) :** $A_{\phi X}Y = A_{\phi Y}X$, if and only if

$$g((\overline{\nabla}_z\phi)X, Y) = 0, \qquad \forall X, Y \in D_1, Z \in TM.$$

**Proof :** Let $X, Y \in D_1$ and $Z \in TM$, then

$$g(A_{\phi Y}X, Z) = g(h(X, Z), \phi Y)$$
$$= g(h(Z, X), \phi Y) = g(\overline{\nabla}_Z X - \nabla_Z X, \phi Y) = g(\overline{\nabla}_Z X, \phi Y) = -g(\phi(\overline{\nabla}_Z X), Y)$$
$$= -g(\overline{\nabla}_Z(\phi X) - (\overline{\nabla}_Z \phi)X, Y) = -g(-A_{\phi X}Z + \nabla^{\perp}_Z \phi X, Y) + g((\overline{\nabla}_Z \phi)X, Y)$$
$$= g(A_{\phi X}Z, Y) + g((\overline{\nabla}_Z \phi)X, Y) = g(A_{\phi X}Y, Z) + g((\overline{\nabla}_Z \phi)X, Y) \tag{27}$$

By (27), we have the lemma.

**Lemma (4.2) :** $[X, \xi] \in D_1$ if and only if

$$g((\nabla_X \phi)\xi, Z) = g((\nabla_\xi \phi)X, Z, \quad \forall X \in D_1, Z \in D_2.$$

**Proof :** For any $X \in D_1$ and $Z \in D_2$, we have

$$g([X, \xi], TZ) = g(\overline{\nabla}_X \xi - \overline{\nabla}_\xi X, TZ)$$
$$= g(\nabla_X \xi - \nabla_\xi X, \phi Z) = -g(\phi(\nabla_X \xi - \nabla_\xi X), Z) \text{ using equ. (8)}$$
$$= g((\nabla_X \phi)\xi + \nabla_\xi(\phi X) - (\nabla_\xi \phi)X, Z) = g((\nabla_X \phi)\xi - (\nabla_\xi \phi)X, Z).$$

Hence the lemma is followed by last equation.

**Lemma (4.3) :** For any $X, Y \in D_1 \oplus D_2$, $[X, Y] \in D_1 \oplus D_2$, if and only if

$$g(\phi Y, (\overline{\nabla}_X \phi)\xi) = g(\phi X, (\overline{\nabla}_Y \phi)\xi).$$

**Proof :** We have for any $X, Y \in D_1 \oplus D_2$,

$$g([X, Y], \xi) = g(\overline{\nabla}_X Y - \overline{\nabla}_Y X, \xi). \tag{28}$$

Now

$$g(Y, \xi) = 0 \Rightarrow g(\overline{\nabla}_X Y, \xi) = -g(Y, \overline{\nabla}_X \xi) \tag{29}$$

and $\qquad g(\phi Y, \phi Z) = -a^2 g(Y, Z)\ \forall Z \in \overline{TM}.$

Replacing $Z$ by $\overline{\nabla}_X \xi$ in the last equ., we obtain

$$g(Y, \overline{\nabla}_X \xi) = -\frac{1}{a^2} g(\phi Y, \phi(\overline{\nabla}_X \xi))$$
$$= \frac{1}{a^2} g(\phi Y, (\overline{\nabla}_X \phi)\xi), \tag{30}$$

making the use of (29) and (30) in (28), we obtain

$$g([X, Y], \xi) = \frac{1}{a^2}[g(\phi X, (\overline{\nabla}_Y \phi)\xi) - g(\phi Y, (\overline{\nabla}_X \phi)\xi)],$$

but $[X,Y] \in D_1 \oplus D_2$, if and only if $g([X,Y],\xi) = 0$.

Hence the lemma follows from last equation.

For any $X,Y \in D_1$ and $Z \in TM$, we have

$$g([X,Y],TP_2Z) = -g(\phi[X,Y],P_2Z) = -g(\phi(\overline{\nabla}_X Y - \overline{\nabla}_Y X),P_2Z)$$
$$= -g(\overline{\nabla}_X(\phi Y) - (\overline{\nabla}_X\phi)Y - \overline{\nabla}_Y(\phi X) + (\overline{\nabla}_Y\phi)X, P_2Z)$$
$$= -g(-A_{\phi Y}X + \nabla^{\perp}_X(\phi Y) + A_{\phi X}Y - \nabla^{\perp}_Y(\phi X) - (\overline{\nabla}_X\phi)Y + (\overline{\nabla}_Y\phi)X, P_2Z),$$
using (27)
$$= g((\overline{\nabla}_X\phi)Y - (\overline{\nabla}_Y\phi)X, P_2Z) + g(\overline{\nabla}_{P_2Z}\phi)X,Y), \tag{31}$$

Since, $[X,Y] \in D_1$ if and only if $g([X,Y],TP_2Z) = 0$.

Thus, the required integrability conditions are obtained from (31) and lemma (4.1).

Similarly, for the distribution $D_1\oplus < \xi >$, the integrability conditions are obtained from (31) and lemma (4.2).

Now, for any $X,Y \in D_2$ and $Z \in D_1$, we have

$$g(\phi[X,Y],\phi Z) = -\,a^2 g([X,Y],Z)$$
$$\Rightarrow a^2 g([X,Y],Z) = -g(\phi[X,Y],NZ) = -g(\overline{\nabla}_X(\phi Y) - \overline{\nabla}_Y(\phi X) - (\overline{\nabla}_X\phi)Y$$
$$+(\overline{\nabla}_Y\phi)X, NZ)$$
$$= g(h(Y,TX) - h(X,TY) + \nabla^{\perp}_Y NX - \nabla^{\perp}_X NY + (\overline{\nabla}_X\phi)Y - (\overline{\nabla}_Y\phi)X, NZ). \tag{32}$$

Therefore, the integrability of the slant distribution $D_2$ is obtained from lemma (4.3), and the fact that $ND_1$ and $ND_2$ are orthogonal in the equ. (32).

In similar manner we easily find the integrability conditions for the distribution $D_2\oplus < \xi >$.

## References

1. Blair, D. E. : Contact Manifolds in Riemannian Geometry, Lecture Notes in Math. 509, Springer-Verlag, New York, 1976.
2. Cabrerizo, J. L., Carriazo, A., Fernandez, L. M. and Fernandez, M. : Slant submanifolds in Sasakian manifold, Glasgow Math. Jour., 42 (2001).
3. Cabrerizo, J. L., Carriazo, A., Fernandez, L. M. and Fernandez, M. : Semi-Slant submanifolds of a Sasakian manifold, Geomeatriae Dedicata., 78 (1999), 183-199.

4. Chen, B. Y. : Geometry of Slant Submanifolds, Katholieke Universiteit Leuven, Leuven, (1990).
5. Khan, V. A. and Khan, M. A. : Pseudo-Slant submanifolds of a Sasakian Manifold, Indian Jour. of Pure and Applied Math., 38 (1), (2007), 31-42.
6. Lotta, A. : Slant Submanifold in Contact Geometry, Bull. Math. Soc. Romanie, 39 (1996), 183-198.
7. Mishra, R. S. : Structures on a Differentiable manifold and their applications, Chandrama Prakashan, Allahabad (India), 1984.
8. Pant, J. : Hypersucfaces immersed in a GF-structure manifold. Demonstratio Mathematica, vol.XIX, no. 3 (1986), 693-697.
9. Papaghuic, N. : Semi-Slant Submanifolds of Kaehlerian manifold. An. Stiint. Al. I. Cuza. Univ. Lasi., 40 (1994), 55-61.