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Quasi-Conformal Curvature Tensor on a K-Contact Manifold

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(*Dedicated to Prof. K. S. Amur on his 80th birth year*)

Abstract

The object of this paper is to study K-contact manifold with quasi-conformal curvature tensor. We find some interesting results for a quasi conformally flat K-contact manifold under certain conditions. In particular it is proved that a K-contact manifold with the condition. $\tilde{C}.S = 0$ is $\eta-$ Einstein.

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1. Introduction

Let $\bar{M}$ be a $(2n+1)-$dimensional contact metric manifold. That is η is a contact form and (ϕ, ξ, η, g) is an associated almost contact metric structure. If the characteristic vector ξ is Killing vector field with respect to g. Then we call such a contact metric structure a K-contact structure. Hence for a K-contact manifold

$$\nabla_X \xi = -\phi X,$$

where ∇ is the Riemannian connection of g.

Conversely, as ϕ is a skew-symmetric operator, a contact metric structure satisfying the property $\nabla_X \xi = -\phi X$ is K-contact. Also a contact metric structure is K-contact if and only if $\pounds_\xi \phi = N^{(3)} = 0$. An interesting curvature property of K-contact manifold [5] is given by

$$R(\xi, X)\xi = -X + \eta(X)\xi,$$

and

$$g(R(\xi, X)\xi, \xi) = 0.$$

Hence we see that an a K-contact manifold of dimension$-(2n+1)$, the Ricci curvature in the direction ξ is equal to $2n$. The converse of this is also true giving a characterization of K-contact manifold [8].

A quasi- conformal curvature tensor was introduced by Yano and Sawaki [10]. According to them a quasi conformal curvature tensor is defined by

$$\begin{aligned}\tilde{C}(X,Y)Z = aR(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y + g(X,Z)QX \\ -g(X,Z)QY] - \tfrac{r}{2n+1}\left(\tfrac{a}{2n} + 2b\right)[g(Y,Z)X - g(X,Z)Y],\end{aligned} \tag{1.1}$$

where a and b are constants and R, S, Q and r are the Riemannian curvature tensor, the Ricci-tensor, the Ricci operator and the scalar curvature of the manifold respectively. If $a = 1$ and $b = -\frac{1}{2n-1}$, then $\tilde{C}$ becomes a conformal curvature tensor C, given by

$$\begin{aligned}C(X,Y)Z = R(X,Y)Z - \tfrac{1}{2n-1}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX \\ -g(X,Z)QY] + \tfrac{r}{2n(2n-1)}[g(Y,Z)X - g(X,Z)Y],\end{aligned} \tag{1.2}$$

where C is conformal curvature tensor [2]. Thus the conformal curvature tensor C is particular case of the tensor $\tilde{C}$. For this reason $\tilde{C}$ is called the quasi-conformal curvature tensor. A manifold $(\bar{M})$ is called quasi-conformally flat if the quasi-conformal curvature tensor $\tilde{C} = 0$. It is known that the quasi-conformally flat manifold is either conformally flat if $a \neq 0$ or Einstein if $a = 0$ and $b \neq 0$.

The present work is organised as follows: section 1 is introductory. Section 2 contains necessary details about K-contact manifold. In section 3, it is proved that a compact orientable $\eta-$ Einstein K-contact manifold does not admit a nonisometric conformal transformation. In section 4 we discussed a K-Contact manifold is a quasi-conformally flat if and only if it is locally isometric to the unit sphere $S^{2n+1}(1)$. In section 5 we prove that a K-contact manifold is quasi-conformally flat if and only if it is quasi-conformally semi-symmetric. A Riemannian or a semi-Riemannian manifold is to be semi-symmetric ([3],[4]) if $R(X,Y).R = 0$ where R is Riemannian curvature tensor and $R(X,Y)$ is considered as a derivation of the tensor algebra at each point of the manifold for tangent vectors X, Y. If a Riemannian manifold satisfies $R(X,Y).\tilde{C} = 0$, where $\tilde{C}$ is the quasi-conformal curvature tensor, then the manifold is said to be quasi-conformally semi-symmetric manifold. In Section 6, it is proved that a K-contact manifold with the condition $\tilde{C}.S = 0$ is $\eta-$Einstein.

2. Preliminaries

Let $\bar{M}$ be a $(2n+1)-$ dimensional contact metric manifold with contact metric structure (ϕ, ξ, η, g) [1], where ϕ is (1, 1) tensor field, ξ is a vector field, η is 1-form and g is compatible Riemannian metric defined as

$$\phi^2 X = -X + \eta(X)\xi, \qquad \eta(\xi) = 1, \qquad \phi\xi = 0, \qquad \eta\phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$g(\phi X, Y) = -g(X, \phi Y), g(X, \xi) = \eta(X), \tag{2.3}$$

for all $X, Y \in T\bar{M}$. An almost contact metric manifold is said to be contact manifold if

$$d\eta(X, Y) = \phi(X, Y) = g(\phi X, Y),$$

where $\phi(X, Y)$ is called fundamental 2-from of $\bar{M}$. In contact metric manifold it is known that

$$\nabla_\xi \phi = 0, \nabla_\xi \xi = 0. \tag{2.4}$$

If in addition ξ is Killing vector field, then $\bar{M}$ is said to K-contact metric manifold. A contact metric manifold is called a K-contact manifold if and only if

$$\nabla_X \xi = -\phi X. \tag{2.5}$$

A locally symmetric K-contact manifold is a Sasakian manifold of constant curvature one. Also if a K-contact manifold $\left(\bar{M}^{2n+1}, g\right)$ is isometrically immersed in a manifold $\left(M^{2n+2}, \bar{g}\right)$ of constant curvature one [9], then it is a Sasakian manifold. For all vector fields on $\bar{M}$.

An almost contact metric manifold is said to be $\eta-$ Einstein manifold if the Ricci tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \tag{2.6}$$

where a and b are certain scalars. A $\eta-$ Einstein manifold becomes Einstein if $b = 0$. If $\{e_1, e_2, \ldots, \ldots, e_{2n}, e_{2n+1} = \xi\}$ is a local orthonormal basis of vector fields in a $(2n+1)-$dimensional almost contact manifold $\bar{M}$. It is easy to verify that

$$\sum_{i=1}^{2n+1} g(e_i, e_i) = (2n+1). \tag{2.7}$$

and

$$\sum_{i=1}^{2n+1} g\left(e_i, Y\right) S\left(X, e_i\right) = \sum_{i=1}^{2n+1} R\left(e_i, Y, X, e_i\right)$$
$$= S(X, Y), \tag{2.8}$$

for all $X, Y \in T\bar{M}$. Also in a K-contact manifold following conditions hold

$$S\left(X, \xi\right) = 2n\eta\left(X\right), \tag{2.9}$$

$$Q\xi = 2n\xi, \tag{2.10}$$

$$\left(\nabla_X \phi\right)\left(Y\right) = R\left(\xi, X\right) Y, \tag{2.11}$$

where S is Ricci tensor.

A K-contact manifold is a contact metric manifold, while converse is true if the Lie derivative of ϕ in the characteristic direction ξ vanishes. A Sasakian manifold is always a K-contact manifold. A 3-dimensional K-contact manifold is a Sasakian manifold. A contact metric manifold is Sasakian if and only if

$$R\left(X, Y\right)\xi = \eta\left(Y\right) X - \eta\left(X\right) Y, \qquad X, Y \in T\bar{M} \tag{2.12}$$

and

$$R\left(\xi, X\right) Y = g\left(X, Y\right) - \eta\left(Y\right) X, \qquad X, Y \in T\bar{M} \tag{2.13}$$

If $\bar{M}$ is a K-contact manifold then it is known that

$$R\left(X, \xi\right)\xi = X - \eta\left(X\right)\xi, X \in T\bar{M}, \tag{2.14}$$

and

$$S\left(\xi, \xi\right) = 2n. \tag{2.15}$$

In K- Contact manifold, we also get

$$R\left(\xi, Y, Z, \xi\right) = g\left(\phi Y, \phi Z\right), Y, Z \in T\bar{M}. \tag{2.16}$$

3. $\eta-$Einstein K-contact manifold

Let l^2 be the square of the length of the Ricci tensor then

$$l^2 = \sum\nolimits_{t=1}^{2n+1} S\left(Qe_i, e_i\right), \tag{3.1}$$

where Q is the Ricci-operator and $\{e_1 e_2 \ldots\ldots e_{2n,} \xi\}$ is a local orthonormal basis of tangent space.

Now Putting $Y = Z = e_i$ in equation (2.6) and taking summation over $1 \leq i \leq 2n+1$, we get

$$r = \left(2n+1\right) a + b, \tag{3.2}$$

where r is scalar curvature.

Again from equation (2.6), we have

$$S(\xi,\xi) = a + b. \tag{3.3}$$

Now from equation (2.6) with the help of equations (3.1) (3.2) and (3.3), we get

$$l^2 = 2na^2 + (a+b)^2 \tag{3.4}$$

if the scalars a and b are constants then the scalar curvature r and the length of Ricci tensor is constant it follows that

$$L_X\, l^2 = 0, \tag{3.5}$$

where L_X is lie-differentiation with respect to X.

Now it is known [6] that if a compact Riemannian manifold $\bar{M}\,(n > 2)$ with constant scalar curvature admits an infinitesimal non isometric conformal transformation X such that $L_X l^2 = 0$ then $\bar{M}$ is isometric to a sphere. But a sphere is an Einstein manifold. Hence we can state the following;

Theorem 3.1. A compact orientable $\eta-$ Einstein $K-$contact manifold does not admit a non isometric conformal transformation.

4. Quasi- conformally flat K-contact manifold

Let $\bar{M}$ be a $(2n+1)$ $-$dimensional quasi-conformally flat K-contact manifold, then we have from equation (1.1)

$$\begin{aligned} aR(X,Y)Z = & -b\left[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right] \\ & + \tfrac{r}{(2n+1)}\left[\tfrac{a}{2n} + 2b\right]\left[g(Y,Z)X - g(X,Z)Y\right], \end{aligned}$$

$$\begin{aligned} {}'R(X,Y,Z,W) = \ & \tfrac{b}{a}\left[S(X,Z)g(Y,W) - S(Y,Z)g(X,W) + g(X,Z).\right. \\ & \left. S(Y,W) - g(Y,Z)S(X,W)\right] + \tfrac{r}{(2n+1)a}\left[\tfrac{a}{2n} + 2b\right] \\ & \left[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)\right], \end{aligned} \tag{4.1}$$

where a and b are constants and

$${}'R(X,Y,Z,W) = g(R(X,Y)Z,W).$$

On putting $Z = \xi$ in equation (4.1), we get

$$\begin{aligned} {}'R(X,Y,\xi,W) = \ & \tfrac{b}{a}\left[S(X,\xi)g(Y,W) - S(Y,\xi)g(X,W) + S(Y,W)g(X,\xi)\right. \\ & \left. -g(Y,\xi)S(X,W)\right] + \tfrac{r}{(2n+1)a}\left[\tfrac{a}{2n} + 2b\right] \\ & \left[g(Y,\xi)g(X,W) - g(X,\xi)g(Y,W)\right], \end{aligned}$$

using equation (2.1), (2.3) and (2.9), we get

$$g(R(X,Y)\xi,W)=\tfrac{b}{a}\left[2n\eta\left(X\right)g\left(Y,W\right)-2n\eta\left(Y\right)g\left(X,W\right)+S\left(Y,W\right)\right.$$
$$\left.\eta\left(X\right)-S\left(X,W\right)\eta\left(Y\right)\right]+\tfrac{r}{(2n+1)a}\left(\tfrac{a}{2n}+2b\right)$$
$$\left[\eta\left(Y\right)g\left(X,W\right)-\eta\left(X\right)g\left(Y,W\right)\right]. \tag{4.2}$$

Again putting $X=\xi$ in equation (4.2) and using (2.1), (2.3) and (2.9), we get

$$\begin{aligned} S\left(Y,W\right) &= \left[-\tfrac{a}{b}-2n+\tfrac{r}{(2n+1)b}\left(\tfrac{a}{2n}+2b\right)\right]g\left(Y,W\right) \\ &+\left[\tfrac{a}{b}+4n-\tfrac{r}{(2n+1)b}\left(\tfrac{a}{2n}+2b\right)\right]\eta\left(Y\right)\eta\left(W\right), \end{aligned} \tag{4.3}$$

equation (4.3) reduces to

$$S\left(Y,W\right)=Ag\left(Y,W\right)+B\eta\left(Y\right)\eta\left(W\right), \tag{4.4}$$

where,

$$A=\left[-\frac{a}{b}-2n+\frac{r}{\left(2n+1\right)b}\left(\frac{a}{2n}+2b\right)\right], \tag{4.5}$$

$$B=\left[\frac{a}{b}+4n-\frac{r}{\left(2n+1\right)b}\left(\frac{a}{2n}+2b\right)\right]. \tag{4.6}$$

Here $A+B=2n$. Hence the manifold is an $\eta-$ Einstein manifold. This leads to the following theorem:

Theorem 4.1. A compact orientable quasi-conformally flat K-contact manifold can not admit a nonisometric conformal transformation.

Using equation (4.4) in equation (4.1), we get

$$\begin{aligned} {}'R\left(X,Y,Z,W\right) &= \tfrac{b}{a}[Ag\left(X,Z\right)g\left(Y,W\right)+B\eta\left(X\right)\eta\left(Z\right)g\left(Y,W\right) \\ &-Ag\left(Y,Z\right)g\left(X,W\right)-B\eta\left(Y\right)\eta\left(Z\right)g\left(X,W\right)+Ag\left(Y,W\right)g\left(X,Z\right) \\ &+B\eta\left(Y\right)\eta\left(W\right)g\left(X,Z\right)-Ag\left(X,W\right)g\left(Y,Z\right)-B\eta\left(X\right)\eta\left(W\right)g\left(Y,Z\right)] \\ &+\tfrac{r}{(2n+1)a}\left(\tfrac{a}{2n}+2b\right)\left[g\left(Y,Z\right)g\left(X,W\right)-g\left(X,Z\right)g\left(Y,W\right)\right]. \end{aligned}$$

Putting the value of ${}'R\left(X,Y,Z,W\right)$ in equation (1.1) with the help of equation (4.5) and (4.6), we get

$${}'\tilde{C}\left(X,Y,Z,W\right)=0.$$

Hence $$\tilde{C}\left(X,Y\right)Z=0.$$

This leads to the following theorem:

Theorem 4.2. A K- Contact manifold is quasi-conformally flat if and only if

$$'R(X,Y,Z,W) = \left[2 + \tfrac{4nb}{a} - \tfrac{r}{(2n+1)a}\left(\tfrac{a}{2n} + 2b\right)\right][g(Y,Z)\,g(X,W) - g(X,Z)\, g(Y,W)] + \left[1 + \tfrac{4nb}{a} - \tfrac{r}{(2n+1)a}\left(\tfrac{a}{2n} + 2b\right)\right][\eta(X)\,\eta(Z)\, g(Y,W) - \eta(Y)\,\eta(Z)\,g(X,W) + \eta(Y)\,\eta(W)\,g(X,Z) - \eta(X)\,\eta(W)\,g(Y,Z)].$$

5. K-contact manifold satisfying $R(X,Y)\,\tilde{C} = 0$

We consider $(2n+1)-$dimensional K-contact manifold $\bar{M}$, satisfying the condition

$$R(X,Y)\,\tilde{C} = 0. \tag{5.1}$$

In virtue of above equation we get

$$R(X,Y)\,\tilde{C}(U,V)\,W - \tilde{C}(R(X,Y)\,U,V)\,W - \tilde{C}(U,R(X,Y)\,V)\,W - \tilde{C}(U,V)\,R(X,Y)\,W = 0. \tag{5.2}$$

Taking $X = \xi$, we have

$$R(\xi,Y)\,\tilde{C}(U,V)\,W - \tilde{C}(R(\xi,Y)\,U,V)\,W - \tilde{C}(U,R(\xi,Y)\,V)\,W - \tilde{C}(U,V)\,R(\xi,Y)\,W = 0, \tag{5.3}$$

which implies that

$$'\tilde{C}(U,V,W,Y) - \eta(Y)\,\eta\left(\tilde{C}(U,V)\,W\right) + \eta(U)\,\eta\left(\tilde{C}(Y,V)\,W\right) + \eta(V)\,\eta\left(\tilde{C}(U,Y)\,W\right) + \eta(W)\,\eta\left(\tilde{C}(U,V)\,Y\right) - g(Y,U)\,\eta\left(\tilde{C}(\xi,V)\,W\right) - \tilde{C}(U,V)\,R(\xi,Y) = 0. \tag{5.4}$$

From Equation (1.1) we get

$$\eta\left(\tilde{C}(X,Y)\,Z\right) = a\eta\,(R(X,Y)\,Z) + b\,[S(Y,Z)\,\eta(X) - S(X,Z)\,\eta(V) + g(Y,Z)\,\eta(QX) - g(X,Z)\,\eta(QY)] - \tfrac{r}{(2n+1)}\left(\tfrac{a}{2n} + 2b\right)[g(Y,Z)\,\eta(X) - g(X,Z)\,\eta(y)], \tag{5.5}$$

hence

$$\eta\left(\tilde{C}(X,Y)\,\xi\right) = 0, \tag{5.6}$$

putting $X = \xi$, we get

$$\eta(\tilde{C}(\xi,Y)Z) = \left[a + 2nb - \tfrac{r}{(2n+1)}\left(\tfrac{a}{2n} + 2b\right)\right][g(Y,Z)\eta(\xi) - g(\xi,Z)\eta(Y)] + [S(Y,Z)\,\eta(\xi) - S(\xi,Z)\,\eta(Y)],$$

$$\eta(\tilde{C}(\xi,Y)Z) = \left[a + 2nb - \tfrac{r}{2n+1}\left(\tfrac{a}{2n} + 2b\right)\right] [g(Y,Z) - \eta(Y)\eta(Z)] \\ +b\left[S(Y,Z) - 2n\eta(Z)\eta(Y)\right], \tag{5.7}$$

putting $U = Y$ in equation(5.4) with the help of (5.5) and (5.6) we get

$$'\tilde{C}(U,V,W,U) + \eta\left(\tilde{C}(U,V)U\right) - g(U,U)\eta\left(\tilde{C}(\xi,V)W\right) \\ -g(U,V)\eta\left(\tilde{C}(\xi,V)W\right) = 0. \tag{5.8}$$

Again putting $U = e_i$ and taking summation over i, $1 \leq i \leq 2n+1$ and using equation (5.5) and (5.7), we get

$$S(V,W) = \lambda g(V,W) + \mu\eta(V)\eta(W), \tag{5.9}$$

where

$$\lambda = \frac{-br + 4n^2 b + 2na}{a-b}, \tag{5.10}$$

$$\mu = \frac{b\left[r - 2n(2n+1)\right]}{a-b} \tag{5.11}$$

Hence

$$\lambda + \mu = 2n. \tag{5.12}$$

Equation (5.9) leads to the theorem;

Theorem 5.1. A quasi conformally semi-symmetric K-Contact manifold is an $\eta-$Einstein manifold.

Putting $V = W = e_i$ in equation (5.9) and taking summation over i, $1 \leq i \leq 2n+1$, we get

$$\left[a + (2n-1)b\right]\left[r - 2n(2n+1)\right] = 0, \tag{5.13}$$

therefore, either

$$b = \frac{a}{2n-1}, \tag{5.14}$$

or

$$r = 2n(2n+1), \tag{5.15}$$

hence

$$\lambda = 2n, \tag{5.16}$$

$$\mu = 0. \tag{5.17}$$

from equation (5.9) with the help of above equations

$$S(V,W) = 2ng(V,W). \tag{5.18}$$

Therefore the manifold is Einstein.

Also from equation (5.5) and (5.7) with the help of equations (5.16), (5.17) and (5.18).

$$\eta\left(\tilde{C}\left(V,U\right)W\right)=0,$$

$$\eta\left(\tilde{C}\left(\xi,U\right)W\right)=0.$$

From equation (5.4), we get

$$\tilde{C}\left(U,V,W,Y\right)=0.$$

Therefore $\bar{M}$ is quasi-conformally flat. Then it is trivially quasi-conformally semi-symmetric. So we have the following result:

Theorem 5.2. Let $\bar{M}$ be a K-contact manifold of dimension ≥ 5, Then $\bar{M}$ is quasi-conformally flat if and only if it is quasi-conformally semi-symmetric.

6. K-contact manifolds satisfying $\tilde{C}.S=0$

Let us consider a K-contact manifold $\bar{M}$ of dimension $(2n+1)$ satisfying the condition

$$\tilde{C}\left(X,Y\right)S=0, \tag{6.1}$$

which implies

$$S\left(\bar{C}\left(X,Y\right)Z,W\right)+S\left(Z,\bar{C}\left(X,Y\right)W\right)=0. \tag{6.2}$$

For $X=W=\xi$, we get

$$S\left(\bar{C}\left(\xi,Y\right)Z,\xi\right)+S\left(Z,\bar{C}\left(\xi,Y\right)\xi\right)=0. \tag{6.3}$$

Now taking $X=\xi$, in equation (1.1)

$$\begin{aligned}\tilde{C}\left(\xi,Y\right)Z=&a\left[g\left(Y,Z\right)\xi-\eta\left(Z\right)Y\right]+b\left[S\left(Y,Z\right)\xi-4n\eta\left(Z\right)Y+g\left(Y,Z\right)2n\xi\right]\\&-\tfrac{r}{2n+1}\left(\tfrac{a}{2n}+2b\right)\left[g\left(Y,Z\right)\xi-\eta\left(Z\right)Y\right].\end{aligned} \tag{6.4}$$

Putting $X=\xi$ in equation (6.4)

$$\tilde{C}(\xi,Y)\,\xi=\left[(a+4nb)-\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\right][\eta(Y)\xi-Y]. \tag{6.5}$$

In view of equation (6.4)

$$\begin{aligned}S\left(\tilde{C}\left(\xi,Y\right)Z,\xi\right)=&a\left[2ng\left(Y,Z\right)-\eta\left(Y\right)\eta\left(Z\right)\right]\\&+b\left[2nS\left(Y,Z\right)-8n^2\eta\left(Z\right)\eta\left(Y\right)+4n^2g\left(Y,Z\right)\right]\\&-\tfrac{r}{2n+1}\left(\tfrac{a}{2n}+2b\right)2n\left[g\left(Y,Z\right)-\eta\left(Y\right)\eta\left(Z\right)\right],\end{aligned} \tag{6.6}$$

$$S(\tilde{C}\,(\xi, Y)\xi, Z) = \left[(a+4nb) - \frac{r}{(2n+1)}\left(\frac{a}{2n}+2b\right)\right] 2n[\eta(Y)\eta(Z) - S(Y,Z)]. \tag{6.7}$$

From equation (6.3), we get

$$S\left(Y,Z\right) = Cg\left(Y,Z\right) + D\eta\left(Y\right)\eta\left(Z\right).$$

where

$$C = \frac{2n\left[a+2nb\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\right]}{2na-2nb+8n^2b-\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\,2n}, \tag{6.8}$$

$$D = \frac{(2n-1)a}{2na-2nb+8n^2b-\frac{r}{2n+1}\left(\frac{a}{2n}+2b\right)\,2n}. \tag{6.9}$$

This leads to the following theorem:

Theorem 6.1. A K-Contact manifold with the condition $\tilde{C}.S = 0$ is $\eta-$Einstein.

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