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Torse-forming Projective Motion in an $NP - F_n$

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(*Dedicated to Prof. K. S. Amur on his 80th birth year*)

Abstract

In the present paper, we study torse-forming projective motion in an $NP - F_n$. We obtain the necessary and sufficient condition for the torse-forming projective motion in an $NP - F_n$ to be an affine motion. We deduce a necessary condition for the torse-forming projective motion to be an N-curvature collineation. Further, we study contra affine motion, concurrent affine motion and special concircular affine motion in an $NP - F_n$. We show that every contra as well as concurrent vector field generates an affine motion but a special concircular vector field does not. We deduce that every contra vector field as well as concurrent vector field generates projective motion and N-curvature collineation in an $NP - F_n$.

Keywords : Torse-forming projective motion, N-curvature collineation, contra affine motion, concurrent affine motion and special concircular affine motion.
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1. Introduction

P. N. Pandey [1-6] discussed the infinitesimal transformation generated by contra vector field, concurrent vector field, special concircular vector field, concircular vector field, torse-forming vector field and birecurrent vector field in a Finsler space. In these papers, he established that a contra vector field as well a concurrent vector field always generates an affine motion but a special concircular vector field cannot generate an affine motion in a general Finsler space. He further obtained the necessary and sufficient condition for the concircular vector field, torse-forming vector field and birecurrent vector field to generate an affine motion. Since every affine motion is curvature collineatiion, the contra vector field and the concurrent vector field generate a curvature collineation. P. N. Pandey and V. J. Dwivedi [7] proved that a transformation is an N-curvature collineation if and only if it is a curvature collineation. Therefore, we may say that the contra vector field and the concurrent vector field generate N- curvature collineation. Also, every affine motion is a projective motion; therefore contra vector field and concurrent vector field generate curvature collineation as well as projective motion.

Let F_n be an n-dimensional Finsler manifold of class at least C^6 equipped with a metric function F satisfying the requisite conditions [8]. Let g_{ij}, G^i_{jk} and H^i_{jkh} be the components of the corresponding metric tensor, Berwald's connection parameters and components of Berwald curvature tensor respectively. The curvature tensor H^i_{jkh} is skew-symmetric in its first two lower indices and positively homogeneous of degree zero in $\dot{x}^h$. From this tensor we deduce the following tensor and vector fields:

$$\begin{aligned} &\text{(a)}\quad H^i_{jk} = H^i_{jkh}\dot{x}^h, &&\text{(b)}\quad H^i_j = H^i_{jk}\dot{x}^k,\\ &\text{(c)}\quad H_{kh} = H^i_{ikh}, &&\text{(d)}\quad H_k = H^i_{ik}. \end{aligned} \tag{1.1}$$

The Berwald covariant derivative of an arbitrary tensor T^i_j for Berwald's connection parameters is defined as

$$B_kT^i_j = \partial_kT^i_j - (\dot{\partial}_rT^i_j)G^r_{kh}\dot{x}^h + T^r_jG^i_{rk} - T^i_rG^r_{jk}, \quad \partial_k \equiv \frac{\partial}{\partial x^k}, \quad \dot{\partial}_r \equiv \frac{\partial}{\partial \dot{x}^r}. \tag{1.2}$$

The covariant derivative gives rise to the following commutation formulae

$$B_jB_kT^i_h - B_kB_jT^i_h = T^r_hH^i_{jkr} - T^i_rH^r_{jkh} - (\dot{\partial}_rT^i_h)H^r_{jk}, \tag{1.3}$$

$$\dot{\partial}_jB_kT^i_h - B_k\dot{\partial}_jT^i_h = T^r_hG^i_{jkr} - T^i_rG^r_{jkh}, \tag{1.4}$$

where $G^i_{jkh} = \dot{\partial}_hG^i_{jk}$. The partial derivatives of Berwald's connection parameters G^i_{jk} are the components of a tensor and satisfy

$$G^i_{jkh}\dot{x}^h = 0. \tag{1.5}$$

Yano [9] defined normal projective connection coefficients Π^i_{kh} by

$$\Pi^i_{kh} = G^i_{kh} - \frac{\dot{x}^i}{n+1}G^r_{khr}. \tag{1.6}$$

The partial derivative of Π^i_{kh} with respect to $\dot{x}^j$, being denoted by Π^i_{jkh}, constitutes a tensor which satisfies the following:

$$\left.\begin{array}{lll} \text{(a) } \Pi^i_{j\,kh} = \Pi^i_{j\,hk}, & \text{(b) } \Pi^i_{j\,k\,i} = G^i_{j\,k\,i}, & \text{(c) } \Pi^i_{j\,kh}\,\dot{x}^j = 0, \\ \text{(d) } \Pi^i_{i\,kh} = \frac{2}{n+1}G^i_{ikh}, & \text{(e) } \Pi^i_{j\,kh}\,\dot{x}^h = \frac{\dot{x}^i}{n+1}G^r_{jkr}. & \end{array}\right\} \tag{1.7}$$

The normal projective covariant derivative of a vector field X^i defined by

$$\nabla_k X^i = \partial_k X^i - \left(\dot{\partial}_r X^i\right)\Pi^r_{kh}\dot{x}^h + X^r \Pi^i_{kr}, \quad \partial_k \equiv \frac{\partial}{\partial x^k}, \tag{1.8}$$

gives rise to the following commutation formulae

$$\nabla_j \nabla_k X^i - \nabla_k \nabla_j X^i = X^r N^i_{jkr} - \left(\dot{\partial}_r X^i\right) N^r_{jkh}\dot{x}^h, \tag{1.9}$$

$$\dot{\partial}_j \nabla_k X^i - \nabla_k \dot{\partial}_j X^i = X^r \Pi^i_{jkr} - \left(\dot{\partial}_r X^i\right)\Pi^r_{jkh}\dot{x}^h, \tag{1.10}$$

where N^i_{jkh} are components of the normal projective curvature tensor. This tensor is skew-symmetric in its first two lower indices and positively homogeneous of degree zero in $\dot{x}^h$. The tensor N_{kh} defined by

$$N_{kh} = N^i_{ikh}, \tag{1.11}$$

satisfies

$$\left.\begin{array}{ll} \text{(a)} & N^i_{jih} = -N^i_{ijh} = -N_{jh}, \\ \text{(b)} & N^i_{jki} = N_{kj} - N_{jk}, \\ \text{(c)} & N^i_{jkh}\dot{x}^h = H^i_{jk}. \end{array}\right\} \tag{1.12}$$

P. N. Pandey [10] established the following relationship between the normal projective curvature tensor N^i_{jkh} and the Berwald curvature tensor H^i_{jkh}

$$N^i_{jkh} = H^i_{jkh} - \frac{\dot{x}^i}{n+1}\dot{\partial}_h H^r_{jkr}. \tag{1.13}$$

The normal projective covariant derivative and Berwald covariant derivative of a vector field X^i are related as

$$\nabla_k X^i = B_k X^i - X^r \frac{\dot{x}^i}{n+1} G^l_{krl}. \tag{1.14}$$

The space F_n with normal projective connection Π^i_{kh} is called normal projective Finsler space or NP-Finsler space which is denoted by $NP - F_n$.

Let us consider an infinitesimal transformation

$$\bar{x}^i = x^i + \epsilon v^i(x^j), \tag{1.15}$$

where v^i. is a contravariant vector field and ϵ is an infinitesimal constant.

The Lie-derivative of an arbitrary tensor T^i_j and the connection coefficients Π^i_{jk} for the above transformation are respectively given by

$$\pounds T^i_j = v^r \nabla_r T^i_j - T^r_j \nabla_r v^i + T^i_r \nabla_j v^r + (\dot{\partial}_r T^i_j) \nabla_s v^r \dot{x}^s \tag{1.16}$$

and

$$\pounds \Pi^i_{jk} = \nabla_j \nabla_k v^i + N^i_{hjk} v^h + \Pi^i_{hjk} \nabla_l v^h \dot{x}^l. \tag{1.17}$$

The operator $\pounds$ commutes with the operators ∇_l and $\dot{\partial}_l$ according as

$$(\pounds \nabla_l - \nabla_l \pounds)\, T^i_{jk} = \left(\pounds \Pi^i_{lh}\right) T^h_{jk} - \left(\pounds \Pi^r_{jl}\right) T^i_{rk} - \left(\pounds \Pi^r_{kl}\right) T^i_{jr}, \tag{1.18}$$

$$\dot{\partial}_h \pounds \Omega - \pounds \dot{\partial}_h \Omega = 0, \tag{1.19}$$

where Ω is any geometrical object such as vector, tensor, scalar or connection coefficients.

The Lie-derivative of the normal projective curvature tensor N^i_{jkh} expressed in the form

$$\nabla_k \left(\pounds \Pi^i_{jh}\right) - \nabla_j \left(\pounds \Pi^i_{kh}\right) = \pounds N^i_{kjh} + \left(\pounds \Pi^r_{km}\right) \dot{x}^m \Pi^i_{rjh} - \left(\pounds \Pi^r_{jm}\right) \dot{x}^m \Pi^i_{rkh}. \tag{1.20}$$

In an $NP-F_n$, the vector field v^i is called contra, concurrent, torse forming and special concircular according as it satisfies

$$\left.\begin{array}{lll} \text{(a)} & \nabla_k v^i = 0, & \\ \text{(b)} & \nabla_k v^i = c\delta^i_k, & \\ \text{(c)} & \nabla_k v^i = \mu_k v^i + \rho\delta^i_k, & \rho = \rho(x^i) \\ \text{(d)} & \nabla_k v^i = \rho\delta^i_k, & \rho = \rho(x^i) \end{array}\right\} \tag{1.21}$$

respectively, where c is a constant and μ_k are components of a non-zero covariant vector field.

In a Finsler space F_n, the vector field v^i is called contra, concurrent, torse forming and special concircular according as it satisfies

$$\left.\begin{array}{lll} \text{(a)} & B_k v^i = 0, & \\ \text{(b)} & B_k v^i = c\delta^i_k, & \\ \text{(c)} & B_k v^i = \mu_k v^i + \rho\delta^i_k, & \rho = \rho(x^i) \\ \text{(d)} & B_k v^i = \rho\delta^i_k, & \rho = \rho(x^i) \end{array}\right\} \tag{1.22}$$

respectively, where c and μ_k are the same as discussed above.

2. Projective Motion

The infinitesimal transformation (1.15) is called a projective motion if it satisfies

$$\pounds\Pi^i_{jk} \; = \; p_k\delta^i_j + p_j\delta^i_k, \tag{2.1}$$

where, $p_j = \dot{\partial}_j p,$ p being a scalar function positively homogeneous of degree one in $\dot{x}^i$'s. Due to the homogeneity of p in $\dot{x}^i$'s, we have

$$p_j\dot{x}^j = p. \tag{2.2}$$

The transformation (1.15) is called an affine motion if and only if

$$\pounds\Pi^i_{jk} \; = \; 0. \tag{2.3}$$

Thus, a projective motion is an affine motion if p vanishes.

3. Curvature Collineation

An infinitesimal transformation is said to be a curvature collineation or Ricci curvature collineation if

$$\pounds H^i_{jkh} \; = \; 0, \tag{3.1}$$

or

$$\pounds H_{kh} \; = \; 0. \tag{3.2}$$

U. P. Singh and A. K. Singh [11] defined the N-curvature collineation as an infinitesimal transformation with respect to which the normal projective curvature tensor N^i_{jkh} is Lie-invariant, i.e.

$$\pounds N^i_{jkh} \; = \; 0. \tag{3.3}$$

They [12] also defined N-Ricci curvature collineation which is characterized by

$$\pounds N_{kh} \; = \; 0. \tag{3.4}$$

P. N. Pandey and V. J. Dwivedi [7] proved that an infinitesimal transformation is an N-curvature collineation if and only if it is a curvature collineation.

4. Torse Forming Projective Motion in an $NP - F_n$

Let us consider a torse forming vector field $v^i(x^j)$ in an $NP - F_n$. Then it satisfies (1.21c). If we differentiate (1.21 c) partially with respect to $\dot{x}^h$ and use the commutation formula (1.10), we have

$$v^r\Pi^i_{hkr} = \left(\dot{\partial}_h\mu_k\right) v^i. \tag{4.1}$$

Transvecting (4.1) by $\dot{x}^k$ and using (1.7a) and (1.7e), we obtain

$$\left(-\frac{v^r\, G^m_{hrm}}{n+1}\right)\dot{x}^i + (\dot{\partial}_h\,\mu_k)\,\dot{x}^k v^i = 0.$$

P. N. Pandey [4] proved a Lemma which states that $av^i + b\dot{x}^i = 0$ implies $a = b = 0$. In view of this Lemma, the last expression provides

$$\text{(a)} \qquad \frac{v^r\, G^m_{hrm}}{n+1} = 0 \quad \text{and} \quad \text{(b)} \qquad (\dot{\partial}_h\, \mu_k)\, \dot{x}^k = 0. \tag{4.2}$$

(4.2b) leads to $\dot{\partial}_h \mu = \mu_h, \quad$ where $\mu = \mu_s \dot{x}^s$.

Using (1.14) for v^i in (1.21c), we get

$$B_k v^i - v^r \frac{\dot{x}^i}{n+1} G^l_{krl} = \mu_k v^i + \rho \delta^i_k. \tag{4.3}$$

Transvecting (4.3) by $\dot{x}^k$ and using (1.5), we have

$$B_k v^i \dot{x}^k = \mu v^i + \rho \dot{x}^i. \tag{4.4}$$

Differentiating (4.4) partially with respect to $\dot{x}^h$ and using (1.4) and (1.5), gives (1.22c). Thus (1.21c) implies (1.22c).

Conversely, if $v^i(x^j)$ is a torse forming vector field in a Finsler space F_n, then it satisfies (1.22c). Differentiating (1.22 c) partially with respect to $\dot{x}^h$ and using the commutation formula (1.10), we have

$$v^r G^i_{hkr} = \left(\dot{\partial}_h \mu_k\right) v^i. \tag{4.5}$$

Transvecting (4.5) by $\dot{x}^k$ and using the fact that $v^i \neq 0$, we obtain

$$\left(\dot{\partial}_h \mu_k\right) \dot{x}^k = 0,$$

which leads to

$$\dot{\partial}_h \mu = \mu_h, \tag{4.6}$$

where $\mu = \mu_s \dot{x}^s$.

Thus, we may write

$$\dot{\partial}_h \mu_k = \mu_{h\,k}. \tag{4.7}$$

In view of (4.7), (4.5) takes the form

$$v^r G^i_{hkr} = \mu_{hk} v^i. \tag{4.8}$$

Using (4.8) in (1.14) for v^i, we get

$$\nabla_k v^i = B_k v^i - v^r \frac{\dot{x}^i}{n+1} \mu_{rk},$$

which in view of (1.22c), gives

$$\nabla_k v^i = \mu_k v^i + \rho \delta^i_k - \frac{\dot{x}^i}{n+1} \mu_{rk} v^r. \tag{4.9}$$

Since $\mu_{rk} \neq 0$, in general, we may say that

Theorem 4.1. A torse forming vector field $v^i(x^j)$ in an $NP - F_n$ remains torse-forming in a Finsler space but converse is not true in general.

Let us consider an $NP - F_n$ admitting a projective motion generated by a torse forming vector field v^i characterized by (1.21c).

Operating the equation (1.21c) by the operator $\pounds$, and noting that the vector field v^i is Lie-invariant with respect to the infinitesimal transformation generated by it, we find

$$\pounds\, \nabla_k v^i = (\pounds\, \mu_k)\, v^i + \pounds\, \rho \delta^i_k. \tag{4.10}$$

In view of equation (1.18) and (2.1), the equation (4.10) may be written as

$$v^r \left(p_k \delta^i_r + p_r \delta^i_k \right) = (\pounds\, \mu_k)\, v^i + \pounds\, \rho \delta^i_k. \tag{4.11}$$

Transvecting (4.11) by $\dot{x}^k$, we have

$$(\pounds\, \mu - p)\, v^i + (\, \pounds\, \rho - p_r v^r)\, \dot{x}^i = 0, \tag{4.12}$$

where $\mu = \mu_s \dot{x}^s$. Since $a v^i + b \dot{x}^i = 0$ implies $a = b = 0$ (see [4]), (4.12) gives

$$(a)\;\; \pounds\, \mu = p, \quad (b)\;\; \pounds\, \rho = p_r v^r. \tag{4.13}$$

This leads to

Theorem 4.2. The condition (4.13 a) and (4.13 b) are true in an $NP - F_n$ admitting a torse forming projective motion characterized by (1.15), (2.1) and (1.21c).

We know that a projective motion is an affine motion if and only if the scalar function p vanishes identically. Therefore, in view of (4.13 a), we may conclude that $\pounds\, \mu = 0$ is necessary and sufficient for a torse forming projective motion to be an affine motion in an $NP - F_n$. Differentiating (1.21 c) partially with respect to $\dot{x}^j$ and using the commutation formula (1.18), we get

$$v^r \Pi^i_{jkr} = \left(\dot{\partial}_j \mu_k \right) v^i + \left(\dot{\partial}_j \rho \right) \delta^i_k. \tag{4.14}$$

Transvecting (4.14) by $\dot{x}^k$ and using (1.7 a) and (1.7e), we have

$$\left(\dot{x}^k \dot{\partial}_j \mu_k \right) v^i + \left(\dot{\partial}_j \rho - \frac{v^r\, G^m_{hrm}}{n+1} \right) \dot{x}^i = 0. \tag{4.15}$$

Since $av^i + b\dot{x}^i = 0$ implies [4] $a = b = 0$, (4.15) gives

$$(a)\ \dot{x}^k \dot{\partial}_j \mu_k = 0, \quad (b)\ \dot{\partial}_j \rho = \frac{v^r\, G^m_{hrm}}{n+1}. \tag{4.16}$$

Equation (4.16 a) implies $\dot{\partial}_j \mu = \mu_j$, where $\mu = \mu_s \dot{x}^s$. In view of (4.2a), (4.16b) implies that $\dot{\partial}_j \rho = 0$, i.e. the scalar ρ is at most a function of position coordinates only. Since the operator $\pounds$ and $\dot{\partial}_l$ are commutative, $\pounds\,\mu = 0$ implies $\pounds\,\mu_k = 0$. Also the transvection of $\pounds\,\mu_k = 0$ by $\dot{x}^k$ implies $\pounds\,\mu = 0$. Thus, the condition $\pounds\,\mu = 0$ and $\pounds\,\mu_k = 0$ are equivalent. Therefore $\pounds\,\mu_k = 0$ is necessary and sufficient for the torse forming projective motion to be an affine motion. This leads to

Theorem 4.3. The condition $\pounds\,\mu_k = 0$ is necessary and sufficient for the torse forming projective motion in an $NP - F_n$ to be an affine motion.

Since every affine motion is curvature collineation and every curvature collineation is N- curvature collineation [7], we have

Corollary 4.1. The condition $\pounds\,\mu_k = 0$ is necessary for the torse forming projective motion in an $NP - F_n$ to be an N- curvature collineation.

5. Some Lemmas

In this section, we shall show that the notions of Contra and Concurrent vector fields in a Finsler space are equivalent to the notions of those in an $NP - F_n$.

Lemma 5.1. The vector field $v^i(x^j)$ is a contra vector field in a Finsler space F_n if and only if it is a contra vector field in an $NP - F_n$, i.e. (1.21 a) $\Leftrightarrow$ (1.22 a).

Proof. Let $v^i(x^j)$ is a contra vector field in an $NP - F_n$, then it satisfies Equation (1.21 a). Using Equation (1.21 a) in (1.14) for $v^i(x^j)$, we get

$$B_k v^i = v^r \frac{\dot{x}^i}{n+1} G^l_{krl}.$$

Transvecting this by $\dot{x}^k$ and using (1.5), we have $\dot{x}^k B_k v^i = 0$. Differentiating $\dot{x}^k B_k v^i = 0$ partially with respect to $\dot{x}^h$ and using the commutation formula (1.4), we have (1.22 a).

Conversely, suppose that $v^i(x^j)$ is a contra vector field in a Finsler space, then it satisfies (1.22a). Differentiating partially (1.22 a) with respect to $\dot{x}^h$ and using the commutation formula (1.4), we get $v^r G^i_{khr} = 0$. Using this and (1.22 a) in (1.14) for $v^i(x^j)$, we have (1.21 a).

Lemma 5.2. The vector field $v^i(x^j)$ is a concurrent vector field in a Finsler space F_n if and only if it is a concurrent vector field in an $NP - F_n$, i.e. (1.21 b) $\Leftrightarrow$ (1.22 b).

Proof. Let $v^i(x^j)$ is a concurrent vector field in an $NP - F_n$, then it satisfies Equation (1.21b). Using Equation (1.21 b) in (1.14) for $v^i(x^j)$, we get

$$c\delta^i_k = B_k v^i - v^r \frac{\dot{x}^i}{n+1} G^l_{krl}.$$

Transvecting this by $\dot{x}^k$ and using (1.5), we have $c\dot{x}^i = \dot{x}^k B_k v^i$. Differentiating $c\dot{x}^i = \dot{x}^k B_k v^i$ partially with respect to $\dot{x}^h$ and using the commutation formula (1.4), we have (1.22 b).

Conversely, suppose that $v^i(x^j)$ is a concurrent vector field in a Finsler space, then it satisfies (1.22 b). Differentiating partially (1.22 b) with respect to $\dot{x}^h$ and using the commutation formula (1.4), we get $v^r G^i_{khr} = 0$. Using this and (1.22 b) in (1.14) for $v^i(x^j)$, we have (1.21 b).

6(a) Contra Affine Motion in an $NP - F_n$

Let us consider an infinitesimal transformation generated by contra vector field $v^i(x^j)$ characterized by (1.21 a). By Lemma 5.1, Equation (1.21 a) implies (1.22a). Differentiating (1.22a) covariantly with respect to $\dot{x}^j$, we have

$$B_j B_k v^i = 0. \tag{6.1}$$

Taking skew-symmetric part of (6.1) and using (1.3), we have

$$H^i_{jk\,r} v^r = 0. \tag{6.2}$$

P. N. Pandey [1] proved that $H^i_{jk\,r} v^r = 0$ and $H^i_{rjk} v^r = 0$ are equivalent. Therefore, Equation (6.2) implies

$$H^i_{rjk} v^r = 0. \tag{6.3}$$

Transvecting (1.13) by v^r and using (6.3), we get

$$N^i_{rjk}\, v^r = 0. \tag{6.4}$$

Now, differentiating (1.21 a) covariantly with respect to $\dot{x}^j$, we have

$$\nabla_j \nabla_k v^i = 0. \tag{6.5}$$

Using (1.21 a), (6.4) and (6.5) in (1.17), we get $\pounds\Pi^i_{jk} = 0$. Hence the infinitesimal transformation considered is an affine motion in an $NP - F_n$. Thus, we obtain

Theorem 6.1. Every contra vector generates an affine motion in an $NP - F_n$.

Every affine motion is a projective motion and by Theorem 6.1, every contra vector field generates an affine motion. Thus, we have

Corollary 6.1. Every contra vector field generates a projective motion in an $NP - F_n$.

Every contra vector field generates an affine motion i.e. $\pounds\Pi^i_{jk} = 0$. This implies $\pounds N^i_{kjh} = 0$, by (1.20). Thus, we may conclude

Corollary 6.2. Every contra vector field generates N- curvature collineation in an$NP - F_n$.

6.(b) Concurrent Affine Motion in an $NP - F_n$

Let us consider an infinitesimal transformation generated by concurrent vector field $v^i(x^j)$ characterized by (1.21 b). By Lemma 5.2, Equation (1.21 b) implies (1.22b). Differentiating (1.22 b) covariantly with respect to $\dot{x}^j$, we have (6.1). Taking skew-symmetric part of (6.1) and using (1.3), we have (6.2) and hence (6.3) as $H^i_{jk\,r}v^r = 0$ and $H^i_{rjk}v^r = 0$ are equivalent [1]. Therefore, Equation (6.2) implies (6.3). Transvecting (1.13) by v^r and using (6.3), we get (6.4). Now, differentiating (1.21 b) covariantly with respect to $\dot{x}^j$ yields (6.5). Using (1.7c), (1.21b), (6.4) and (6.5) in (1.17), we get $\pounds\Pi^i_{jk} = 0$. Hence the infinitesimal transformation considered is an affine motion in an $NP - F_n$. Thus, we have

Theorem 6.2. Every concurrent vector field generates an affine motion in an $NP - F_n$.

Every affine motion is a projective motion and by Theorem 6.2, every concurrent vector field generates an affine motion. Thus, we have

Corollary 6.3. Every concurrent vector field generates a projective motion.

Every concurrent vector field generates an affine motion i.e. $\pounds\Pi^i_{jk} = 0$. This implies $\pounds N^i_{kjh} = 0$, by (1.20). Thus, we may conclude

Corollary 6.4. Every concurrent vector field generates N-curvature collineation in an $NP - F_n$.

6.(c) Special Concircular Affine Motion in an $NP - F_n$

Let us consider an $NP - F_n$ admitting an infinitesimal transformation generated by a special concircular vector field characterized by (1.21d). If this transformation is an affine motion, we have $\pounds\Pi^i_{jk} = 0$, which by (1.17) and (1.21d) gives

$$\nabla_j\rho\delta^i_k + N^i_{hjk}v^h = 0. \tag{6.6}$$

Transvecting (6.6) by $\dot{x}^k$ and using (1.12c), we have

$$\nabla_j\rho\dot{x}^i + H^i_{hj}v^h = 0. \tag{6.7}$$

Transvecting (6.7) by y_i and using $\dot{x}^i y_i = F^2$ and $H^i_{h\,j}y_i = 0$, we have $F^2\nabla_j\rho = 0$, which implies $\nabla_j\rho = 0$. Thus we get a contradiction. Hence, we obtain

Theorem 6.3. An infinitesimal transformation generated by a special concircular vector field cannot be an affine motion in an $NP - F_n$.

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References

[1] Pandey, P. N. : Certain types of affine motion in a Finsler manifold I, Colloq. Math., 49 (2) (1985), 243-252.
[2] Pandey, P. N. : Certain types of affine motion in a Finsler manifold II, Colloq. Math., 53 (2) (1987), 219-227.
[3] Pandey, P. N. : Certain types of affine motion in a Finsler manifold III, Colloq. Math., 56 (2) (1988), 333-340.
[4] Pandey, P. N. : Certain types of projective motions in a Finsler manifold, Atti Accad. Peloritana die pericolanti cl. Sci. Fis. Math. Nat., 60 (1982/83), 287-300.
[5] Pandey, P. N. : Certain types of projective motions in a Finsler manifold II, Atti della Accad. Delle Scienze di Torino, 120 (1986), 168-178.
[6] Pandey, P. N. : On Birecurrent affine motion in a Finsler manifold, Acta Math. Hungar., 45 (34) (1985), 251-260.
[7] Pandey, P. N. and Dwivedi, V. J. : On Curvature Collineation in an NP-Finsler space, Proceedings of the 3rd Conference of International Academy of Physical Sciences, (1999), 201-213.
[8] Rund, H. : The differential geometry of Finsler spaces, Springer - Verlag: Berlin, (1959).

[9] Yano, K. : Theory of Lie derivative and its applications, North-Holland Publishing co., Amsterdam, (1957).

[10] Pandey, P. N. : On NPR-Finsler manifolds, Ann. Fac. Sci. Univ. Nat. Zaire (Kinshasa), 6 (1-2) (1980), 65-77.

[11] Singh, U. P. and Singh, A. K. : On the N-curvature collineation in Finsler space, Annales Soc. Sci. Bruxelles, 95 (1981), 69-77.

[12] Singh, U. P. and Singh, A. K. : On the N-curvature collineation in Finsler space II, Indian J. Pure Appl. Math., 12 (1981), 1208-1212.