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Mucus Flow in Human Lung Airways :A Planar Two Layer Steady State Mathematical Model

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(*Dedicated to Prof. K. S. Amur on his 80th birth year*)

Abstract

In this paper, a planar two-layer steady state mathematical model is presented to study mucus flow in human lung airways due to cilia beating and some immobile cilia forming porous matrix in serous sub layer in contact with epithelium as a highly viscous fluid. The effect of air-motion is considered by prescribing air velocity at the mucus-air interface. The effects of pressure drop and gravitational force are also considered in the model. It is shown that the mucus flow rate increases as pressure drop or gravitational force increases. It also increases as the air- velocity at the mucus air interface increases whereas it decreases as the viscosity of serous fluid or that of mucus increases.It is also shown that the increase in mucus viscosity at higher values do not have any significant effect on its transport rate.

Keywords : Mucus flow, lung airways, cilia beating, immotile cilia, air velocity.
2000 AMS Subject Classification : 92C35, 92C10, 76Z05.

1. Introduction

The muco-ciliary system is one of the most important first line of defence mechanism of the human lung and the airways for cleaning the inspired air of contaminants and for removing entrapped particles such as bacteria, viruses, carcinogens in tobacco smoke.

It consists of three layers namely : a mucus layer, a serous layer and the cilia which are small hair-like projections lining with the epithelium of the bronchial respiratory tract. The serous fluid is covered by highly viscous mucus secreted from the underlying goblet cells or sub-mucosal glands. It has been pointed out that in general, mucus flow depends upon the structure of cilia, the force imparted by cilia tips in the serous sublayer fluid, the thicknesses and the viscosities of the serous fluid and mucus and the interaction of mucus with the serous layer fluid. Mucus flow is also dependent on the pressure drop in the airways generated by the processes such as inspiration, expiration, coughing, etc. and gravitational force (Blake [4] and Sleigh et al. [9]).

In recent decades, the mucus flow in the lung has been studied by several investigators. In particular an analytical model has been presented by Barton and Raynor [2] by considering the cilium as an oscillating cylinder with a greater height during the effective stroke and a smaller height during the recovery stroke. Blake and Winet [5] suggested that if the cilia just penetrate the upper much more viscous layer, then the mucus flow rate is substantially enhanced. King et al. [6] presented a two-layer steady state mathematical model for mucus transport by introducing cilia tip velocity in their model. Agarwal and Verma [1] and Verma [11, 12] have studied the mucus transport by analysing the effect of porosity due to the formation of porous matrix bed by immotile cilia. In view of above, in this paper, a planar two-layer steady state model for mucus flow in human lung airways is presented by taking the following aspects into account:

(i) The serous layer fluid and mucus both are considered as incompressible Newtonian fluid. The mucus is considered highly viscous than the serous layer fluid.

(ii) The serous layer fluid is divided into two sublayers, one in contact with the epithelium and the other in contact with the mucus. It is assumed that cilia during beating impart a velocity at the mean level of their tips, causing the serous sublayer in contact with mucus to undergo motion. It is also assumed that some cilia are immotile and form a porous matrix in the serous sub layer in contact with with epithelium where flow may due to pressure gradient as considered by Beavers and Joseph [3]. No net flow is assumed in the serous sub-layer in contact with epithelium.

(iii) The effect of air-motion is incorporated by prescribing the air-velocity at mucus-air interface as a boundary condition.

(iv) The effects of pressure gradients and gravitational force are also taken into consideration in the model.

2. Mathematical Model

The physical situation of the flow of serous fluid and mucus in the human lung airways may be represented by a planar two-layer fluid model as shown in Fig. 1.

In the serous sublayer $0 \le y \le h_e$, no net flow of fluid due to cilia beating is assumed. However, in the serous sublayer $h_e \le y \le h_s$ and in the mucus layer $h_s \le y \le h_m$, the flow of respective fluids is governed by the interactions of cilia, air-motion in contact with the mucus, pressure gradient present in the fluids and gravitational force.

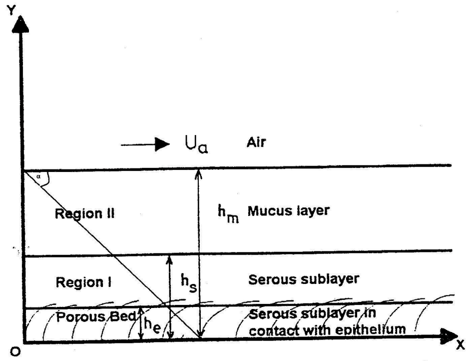

Fig. 1. Mucus flow : A planar two-layer fluid model

The equations governing the motion of the serous layer fluid and the mucus under steady state and low Reynold's number flow approximations, by taking the effect of gravitational force in the direction of the flow, can be written as follows:

Region I. Serous layer ($h_e \le y \le h_s$):

(1) $$\mu_s \frac{\partial^2 u_s}{\partial y^2} = \frac{\partial p}{\partial x} - \rho_s\, g\, \cos\, \alpha$$

Region II. Mucus layer ($h_s \le y \le h_m$):

(2) $$\mu_m \frac{\partial^2 u_m}{\partial y^2} = \frac{\partial p}{\partial x} - \rho_m\, g\, \cos\, \alpha$$

where p is the pressure that is constant across the layers; u_s and u_m are the velocity components of serous sublayer fluid and mucus in the $x-$direction respectively; ρ_s, μ_s, ρ_m and μ_m are their respective densities and viscosities; g is the acceleration due to gravity and α is the angle by which the airway in the lung is inclined with the vertical. Here, h_e is the mean thickness measured from the surface of the epithelium to the tips of cilia during its beating phase i.e. the interface between the two serous sub-layers; h_s is the thickness measured from the surface of the epithelium to the interface between the serous sublayer and mucus and h_m is the thickness measured from the surface of the epithelium to the mucus-air interface.

The following boundary and matching conditions are taken for the system of equations (1) and (2).

Boundary Conditions

$$u_s = U_0 + \beta \frac{\partial u_s}{\partial y}, \qquad y = h_e \tag{3}$$

$$u_m = U_a, \qquad y = h_m \tag{4}$$

where U_0 is the mean velocity imparted by cilia tips during beating in the serous sublayer at $y = h_e$ and β is the porosity parameter due to immotile cilia forming porous matrix in serous sub layer in contact with epithelium where flow may occur dur to pressure gradient by Beavers and Joseph [3]. The condition (4) implies that the air-velocity U_a is continuous at the mucus-air interface and incorporates the effect of air-motion similar to the analysis of Blake [4].

Matching Conditions

$$u_s = u_m = U_1, \qquad y = h_s \tag{5}$$

$$\mu_s \frac{\partial u_s}{\partial y} = \mu_m \frac{\partial u_m}{\partial y}, \qquad y = h_s \tag{6}$$

where U_1 is the mucus-serous sublayer interface velocity to be determined by using equation (6). The conditions (5) and (6) imply that the velocities and shear stresses are continuous at the mucus-serous sublayer interface.

3. Solution

Solving equations (1)- (2) and using boundary and matching conditions (3)-(6), we get

$$\begin{aligned} u_s = \frac{\phi_s}{2\mu_s}[y^2 - \{\frac{2\beta h_e + h_s^2 - h_e^2}{\beta + h_s - h_e}\}y + \{\frac{\beta h_e - (\beta - h_e)(h_s - h_e)}{\beta + h_s - h_e +}\}h_s] \\ + U_1\{\frac{\beta + y - h_e}{\beta + h_s - h_e}\} - U_0\{\frac{y - h_s}{\beta + h_s - h_e}\} \end{aligned} \tag{7}$$

$$u_m = \frac{\phi_m}{2\mu_m}(y - h_m)(y - h_s) + \frac{(y - h_s)U_a - (y - h_m)U_1}{(h_m - h_s)} \tag{8}$$

where

$$\begin{aligned} U_1 = \frac{1}{\mu_m(\beta + h_s - h_e) + \mu_s(h_m - h_s)}[\mu_m(\beta + h_s - h_e)U_a + (\frac{h_m - h_s}{2})\times \\ \{\mu_s U_0 - \phi_m(h_m - h_s)(\beta + h_s - h_e) - \phi_s(2\beta + h_s - h_e)(h_s - h_e)\}] \end{aligned} \tag{9}$$

and

$$\phi_s = \frac{\partial p}{\partial x} - \rho_s g \cos\alpha, \phi_m = \frac{\partial p}{\partial x} - \rho_m g \cos\alpha \tag{10}$$

The volumetric flow rate i.e. flux in the two layers are defined as

$$Q_s = \int_{h_e}^{h_s} u_s\, dy \qquad \text{and} \qquad Q_m = \int_{h_s}^{h_m} u_m\, dy$$

which after using (7) and (8) are found as

$$\begin{aligned} Q_s = -\frac{\phi_s}{12\mu_s}\{\frac{4\beta + h_s - h_e}{\beta + h_s - h_e}\}(h_s - h_e)^3 + (\frac{U_0}{2})\frac{(h_s - h_e)^2}{(\beta + h_s - h_e)} \\ +\{(\frac{U_1}{2})\frac{(2\beta + h_s - h_e)}{(\beta + h_s - h_e)}\}(h_s - h_e) \end{aligned} \tag{11}$$

and

$$Q_m = -\frac{\phi_m}{12\mu_m}(h_m - h_s)^3 + \frac{(U_a + U_1)(h_m - h_s)}{2} \tag{12}$$

where ϕ_s and ϕ_m are given by (10).

It can be seen by using equation of fluid continuity that Q_s and Q_m are constants, therefore, from equations (11) and (12), we note that $-\frac{\partial p}{\partial x}$ is also constant. Hence, replacing it by the pressure drop over the length L of the cilia beating zone including the cilia forming porous matrix bed zone, the expressions for the fluxes may be written as:

(13)

$$Q_s = \frac{\phi_{so}}{12\mu_s}(\frac{4\beta + h_s - h_e}{\beta + h_s - h_e})(h_s - h_e)^3 + \frac{U_0(h_s - h_e)^2}{2(\beta + h_s - h_e)} + \frac{U_1(2\beta + h_s - h_e)}{2(\beta + h_s - h_e)}(h_s - h_e)$$

and

$$Q_m = \frac{\phi_{mo}}{12\mu_m}(h_m - h_s)^3 + \frac{(U_a + U_1)(h_m - h_s)}{2} \tag{14}$$

where

$$\begin{aligned} U_1 = \frac{1}{\mu_m(\beta + h_s - h_e)}\{\mu_m(\beta + h_s - h_e) - \mu_s(h_m - h_s)\}[U_a + (\frac{h_m - h_s}{2}) \\ \{2\mu_s U_0 + \phi_{mo}(h_m - h_s)(\beta + h_s - h_e) - \phi_{so}(2\beta + h_s - h_e)(h_s - h_e)\}] \end{aligned} \tag{15}$$

and

$$\phi_{so} = \frac{\Delta p}{L} + \rho_s g \cos\alpha, \qquad \phi_{mo} = \frac{\Delta p}{L} + \rho_m g \cos\alpha \tag{16}$$

and $\triangle p = p_0 - p_L$; $p = p_0$ at $x = 0$, $p = p_L$ at $x = L$. It is noted that the effect of acceleration due to gravity is similar to that of the pressure drop.

4. Results and Discussion

To study the effect of various parameters on mucus flow rate quantitatively, the expression for $\overline{Q_m}$ given by (14)on using the value of U_1 from (9)can be written in non-dimensional form as:

$$
\begin{aligned}
(17)\quad \overline{Q_m} = {} & \frac{\overline{\phi_{m0}}}{12\overline{\mu_m}}[4 - \frac{3\overline{\mu_s}(1-\overline{h_s})}{\overline{\mu_m}(\overline{\beta}+\overline{h_s}-\overline{h_e})}](1-\overline{h_s})^3 \\
& + \frac{\overline{\phi_{s0}}}{4\overline{\mu_m}}[1 - \frac{\overline{\mu_s}(1-\overline{h_s})}{\overline{\mu_m}(\overline{\beta}+\overline{h_s}-\overline{h_e})}](\frac{2\overline{\beta}+\overline{h_s}-\overline{h_e}}{\overline{\beta}+\overline{h_s}-\overline{h_e}})(\overline{h_s}-\overline{h_e})(1-\overline{h_s})^2 \\
& + \frac{\overline{\mu_s}}{2\overline{\mu_m}}[1 - \frac{\overline{\mu_s}(1-\overline{h_s})}{\overline{\mu_m}(\overline{\beta}+\overline{h_s}-\overline{h_e})}](1-\overline{h_s}) + \frac{\overline{U_a}}{2}[2 - \frac{\overline{\mu_s}(1-\overline{h_s})}{\overline{\mu_m}(\overline{\beta}+\overline{h_s}-\overline{h_e})}](1-\overline{h_s})
\end{aligned}
$$

by using the following non-dimensional parameters:

$$
\begin{aligned}
& \overline{h}_e = \frac{h_e}{h_m}, \quad \overline{h}_s = \frac{h_s}{h_m}, \quad \overline{\beta} = \frac{\beta}{h_m}, \quad \overline{\mu}_s = \frac{\mu_s}{\mu_0}, \quad \overline{\mu}_m = \frac{\mu_m}{\mu_0}, \\
(18)\quad & \overline{U_a} = \frac{U_a}{U_0}, \quad \overline{\phi}_{so} = \frac{\phi_{so}h_m^2}{\mu_0 U_0}, \quad \overline{\phi}_{mo} = \frac{\phi_{mo}h_m^2}{\mu_0 U_0}, \quad \overline{Q}_m = \frac{Q_m}{h_m U_0}
\end{aligned}
$$

where μ_0 is the viscosity of the serous sublayer fluid in contact with epithelium.

Expression given by (17) is plotted in Figures 2 to 6, using the following set of parameters which have been calculated by using typical values of various characteristics related to airways [King et al. [6], Agarwal and Verma [1]]:

$\overline{h}_e = 0.10$, $\overline{h}_s = 0.20 - 0.60$, $\overline{\beta} = 0.02 - 0.10$ $\overline{\mu}_s = 1.0 - 3.0$, $\overline{\mu}_m = 100 - 1000$, $\overline{U_a} = 0.2 - 0.4$, $\overline{\phi}_{so} = 1$, $\overline{\phi}_{mo} = 2 - 25$.

In Fig. 2, we have shown the variation of $\overline{Q}_m$ with $\overline{\mu}_m$ for different values of $\overline{\mu}_s$ and for fixed values of $\overline{\beta} = 0.02$, $\overline{h}_e = 0.10$, $\overline{h}_s = 0.20$, $\overline{U}_a = 0.4$, $\overline{\phi}_{so} = 1$ and $\overline{\phi}_{mo} = 20$. The figure illustrates that the mucus flow rate decreases as the viscosity of the serous layer fluid or mucus increases; however increase in mucus viscosity at higher values do not have any significant effect on its transport rate. These observations are in line with the experimental observations of King et al. [7, 8] and analytical results of King et al. [6], Agarwal and Verma [1] and Verma [11, 12].

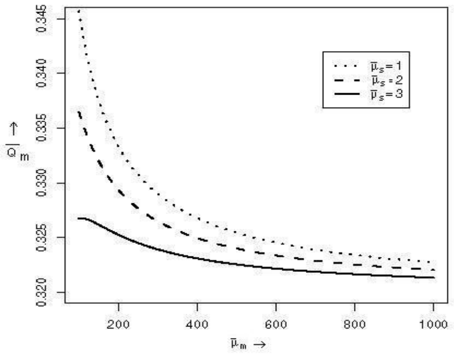

Fig. 2. Variation of $\overline{Q}_m$ with $\overline{\mu}_m$ for different values of $\overline{\mu}_s$

In Fig. 3, we have shown the variation of $\overline{Q}_m$ with $\overline{\phi}_{mo}$ for different values of $\overline{\mu}_m$ and fixed values of $\overline{\beta} = 0.02$, $\overline{h}_e = 0.10$, $\overline{h}_s = 0.20$, $\overline{\mu}_s = 1.0$, $\overline{U}_a = 0.4$ and $\overline{\phi}_{so} = 1$. The figure illustrates that the mucus flow rate increases as the pressure drop or gravitational force increases, but it decreases with increase in mucus viscosity, the relative decrease being larger at higher values of the pressure drop or gravitational force. These results are in line with the conclusions drawn by King et al. [6] and Agarwal and Verma [1] in their mathematical models.

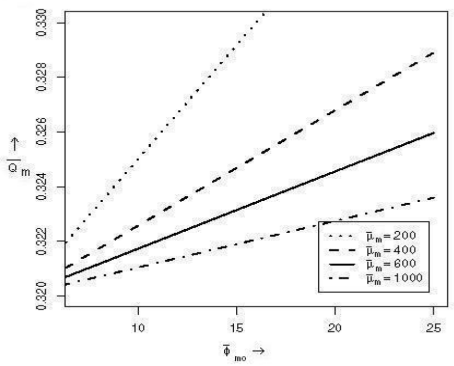

Fig. 3. Variation of $\overline{Q}_m$ with $\overline{\phi}_{mo}$ for different values of $\overline{\mu}_m$

In Fig. 4, we have shown the variation of $\overline{Q}_m$ with $\overline{h}_s$ for different values of $\overline{\mu}_m$ and fixed values of $\overline{\beta} = 0.02$, $\overline{h}_e = 0.10$, $\overline{\mu}_s = 1.0$, $\overline{U}_a = 0.4$, $\overline{\phi}_{so} = 1$ and $\overline{\phi}_{mo} = 20$. The figure illustrates that the mucus flow rate decreases as $\overline{h}_s$ increases. We further note that the mucus transport rate decreases with increase in its viscosity, the relative decrease being smaller for higher values of $\overline{h}_s$.

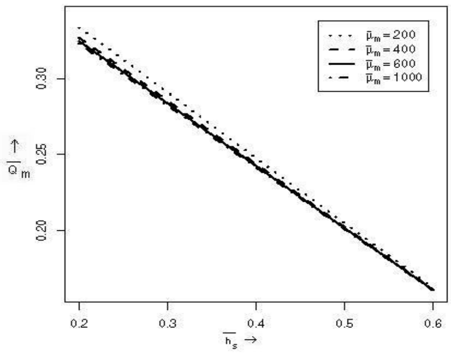

Fig. 4. Variation of $\overline{Q}_m$ with $\overline{h}_s$ for different values of $\overline{\mu}_m$

In Fig. 5, we have shown the variation of $\overline{Q_m}$ with $\overline{U_a}$ for different values of $\overline{\mu_m}$ and fixed values of $\overline{\beta} = 0.02$, $\overline{h_e} = 0.10$, $\overline{h_s} = 0.20$, $\overline{\mu_s} = 1.0$, $\overline{\phi_{so}} = 1$ and $\overline{\phi_{mo}} = 20$. From this figure, it is seen that mucus flow rate increases linearly as the air velocity increases and it decreases as its viscosity increases.

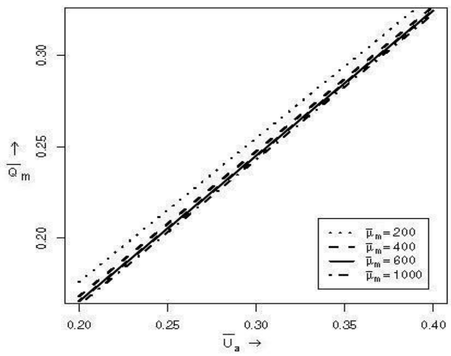

Fig. 5. Variation of $\overline{Q}_m$ with $\overline{U}_a$ for different values of $\overline{\mu}_m$

In Fig. 6, we have shown the variation of $\overline{Q_m}$ with $\overline{\beta}$ for different values of $\overline{\mu_m}$ and fixed values of $\overline{h_e} = 0.10$, $\overline{h_s} = 0.20$, $\overline{\mu_s} = 1.0$, $\overline{U_a} = 0.4$, $\overline{\phi_{so}} = 1$ and $\overline{\phi_{mo}} = 20$. From this figure, it is seen that mucus flow rate increases as the porosity parameter increases and it decreases as its viscosity increases.

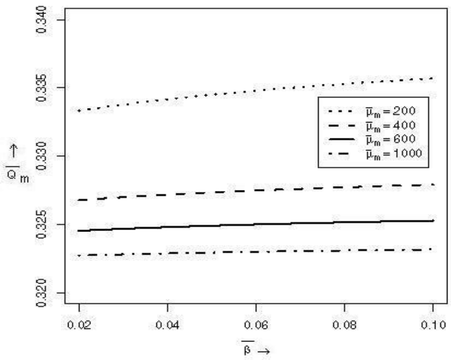

Fig. 6. Variation of $\overline{Q}_m$ with $\overline{\beta}$ for different values of $\overline{\mu}_m$

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