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Quasi Conformally flat Sasakian Hypersurface of the Generalized Recurrent Kählerian Manifold

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(*Dedicated to Prof. K. S. Amur on his 80th birth year*)

Abstract

In the present paper we consider a Quasi-Conformally flat Sasakian hypersurface of the generalized recurrent Kählerian manifold and study the conditions under which it is η-Einstein. We also study the conditions for the scalar curvature to be constant, when its ricci tensor is cyclic parallel and η-parallel.

Keywords : Kählerian manifold, Sasakian Hypersurface, Quasi-Conformally flat, cyclic Ricci tensor, η-parallel Ricci tensor.
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1. Introduction

Let M^{2n+2} be a $2n+2$ dimensional almost Hermitian manifold, with structure tensors (J, G) and the Riemannian connection $\widetilde{\nabla}$ such that $J^2 = -I$ and $G(JX, JY) = G(X, Y)$. An almost Hermitian manifold with $\widetilde{\nabla}J = 0$ is known as Kählerian manifold. Let $\widetilde{\Omega}\left(\widetilde{X}, \widetilde{Y}\right)$ be the 2-form in Kählerian manifold such that

$$\widetilde{\Omega}\left(\widetilde{X}, \widetilde{Y}\right) = G(\tilde{X}, J\tilde{Y}) = -G(J\tilde{X}, \tilde{Y}) = -\widetilde{\Omega}\left(\widetilde{Y}, \widetilde{X}\right). \tag{1.1}$$

Let $\widetilde{K}$ and $\widetilde{S}$ denote the Curvature tensor and the Ricci tensor of the Kählerian manifold M^{2n+2}, respectively. Suppose that M^{2n+1} is a C^∞ hypersurface with unit normal N and the induced metric g. If di denotes the differential of the imbedding $i : M^{2n+1} \to M^{2n+2}$, X a vector field on M^{2n+1}, then $\tilde{X}$ is the extension of X on M^{2n+2} and is such that $\tilde{X}$ restricted to M^{2n+1} under the imbedding is diX. Also let $\Theta = \{e_i\}$, $i = 1, 2, ..., 2n+1$ be an orthonormal basis of the tangent space at any point of the manifold M^{2n+1}then $\widetilde{\Theta} = \{e_i, N\}$, $i = 1, 2, ..., 2n+1$ is an orthonormal basis for the tangent space at any point on the manifold M^{2n+2}. Hence

$$G(\tilde{X}, \tilde{Y}) = g(X, Y), \qquad G(N, N) = 1, \qquad G(\tilde{X}, N) = 0 \tag{1.2}$$

and its Riemannian connection $\widetilde{\nabla}$ is governed by Gauss-Weingarten equations

$$\widetilde{\nabla}_{\tilde{X}} \tilde{Y} = \widetilde{(\nabla_X Y)} + h(X, Y)N, \qquad \widetilde{\nabla}_{\tilde{X}} N = -\widetilde{H\,'X}, \tag{1.3}$$

where h denotes the second fundamental form and $H\,'$ the corresponding Weingarten map. Also the hypersurface M^{2n+1} inherits an almost contact metric structure (φ, ξ, η, g) given by [2] [3]

$$J\,\widetilde{X} = \widetilde{\varphi X} + \eta(X)N, \qquad JN = -\tilde{\xi}. \tag{1.4}$$

The equations (1.2) , (1.3) and (1.4) lead to the following conditions in M^{2n+1}:

$$\varphi^2 = -I + \eta \otimes \xi, \quad \eta \circ \varphi = 0, \quad \varphi\xi = 0, \quad \eta(\xi) = 1, \tag{1.5}$$

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \qquad \eta(X) = g(X, \xi). \tag{1.6}$$

An almost contact metric structure (φ, ξ, η, g) is Sasakian if and only if

$$(\nabla_X \varphi)Y = g(X, Y)\xi - \eta(Y)X. \tag{1.7}$$

If K and S denote the Curvature tensor and the Ricci tensor of the Sasakian manifold M^{2n+1} respectively, then the following conditions hold in a Sasakian manifold [2][7]:

$$S\,(X, \xi) = 2n\ \eta(X) \tag{1.8}$$

$$S\,(X, \varphi Y) = -S\,(\varphi X, Y) \tag{1.9}$$

$$S\,(\varphi Y, \varphi Z) = S\,(Y, Z) - 2n\ \eta(Y)\eta(Z) \tag{1.10}$$

$$g\,(K\,(\xi, X)\,Y, \xi) = g\,(X, Y) - \eta(X)\eta(Y) \tag{1.11}$$

$$g\,(K\,(\xi, X)\,\xi, W) = -g\,(X, W) + \eta(X)\eta(W) \tag{1.12}$$

$$g\,(K\,(X, Y)\,\xi, Z) = g\,(X, Z)\,\eta(Y) - g\,(Y, Z)\,\eta(X) \tag{1.13}$$

$$(\nabla_X \varphi)Y = K(\xi, X)Y \tag{1.14}$$

$$\nabla_X \xi = -\varphi X \tag{1.15}$$

$$(divK)\,(X, Y)\,Z = (\nabla_X S)\,(Y, Z) - (\nabla_Y S)\,(X, Z) \tag{1.16}$$

$$(\nabla_X S)\,(Y, \xi) = -2ng\,(\varphi X, Y) + S\,(Y, \varphi X)\,. \tag{1.17}$$

If Ω is the 2-form on M^{2n+1} defined by

$$\Omega(X, Y) = g(X, \varphi Y) = -g(\varphi X, Y) = -\Omega(Y, X), \tag{1.18}$$

then from (1.5), we get

$$(\nabla_X \eta)(Y) = g(X, \varphi Y). \tag{1.19}$$

The Sasakian manifold is said to be an η-Einstein manifold [1] if

$$S\,(X, Y) = pg\,(X, Y) + q\eta(X)\eta(Y), \tag{1.20}$$

where $p + q = 2n$ is a constant. On the hypersurface M^{2n+1} of the Kählerian manifold M^{2n+2}, we have the following Gauss-Codazzi equations :

$$\begin{aligned} K(X,Y,Z,W) = \quad & \widetilde{K}\ (\tilde{X}, \tilde{Y}, \tilde{Z}, \tilde{W}) \\ & + h(X,W)h(Y,Z) - h(X,Z)h(Y,W), \end{aligned} \tag{1.21}$$

and

$$\widetilde{K}\ (\tilde{X}, \tilde{Y}, \tilde{Z}, N) = (\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z), \tag{1.22}$$

where $\widetilde{K}(\tilde{X}, \tilde{Y}, \tilde{Z}, \tilde{W}) = G(\widetilde{K}(\tilde{X}, \tilde{Y})\tilde{Z}, \tilde{W})$ and $K(X,Y,Z,W)$=$g(K(X,Y)Z,W)$.

A Kählerian manifold M^{2n+2} is said to be a generalized recurrent manifold [5][8] if there exists a non-zero 1-forms $\widetilde{A}$ and $\widetilde{B}$ such that

$$(\widetilde{\nabla}_{\tilde{U}}\widetilde{K})(\tilde{X}, \tilde{Y}, \tilde{Z}, \tilde{W}) = \widetilde{A}(\tilde{U})\widetilde{K}(\tilde{X}, \tilde{Y}, \tilde{Z}, \tilde{W}) + \widetilde{B}(\tilde{U})\overline{\widetilde{F}}(\tilde{X}, \overline{\underline{\tilde{Y}}}, \underline{\tilde{Z}}, \overline{\tilde{W}}) \tag{1.23}$$

for arbitrary vector fields $\tilde{X}$,$\tilde{Y}$,$\tilde{Z}$,$\tilde{W}$ and $\tilde{U}$ on M^{2n+2}, where

$$\overline{\widetilde{F}}(\tilde{X}, \overline{\underline{\tilde{Y}}}, \underline{\tilde{Z}}, \overline{\tilde{W}}) = G(\tilde{X}, \tilde{W})G(\tilde{Y}, \tilde{Z}) - G(\tilde{X}, \tilde{Z})G(\tilde{Y}, \tilde{W}), \tag{1.24}$$

and $\widetilde{A}(\tilde{U}) = G(\tilde{U}, \tilde{\rho}_A)$ and $\widetilde{B}(\tilde{U}) = G(\tilde{U}, \tilde{\rho}_B)$ for some vector fields $\tilde{\rho}_{\widetilde{A}}$, $\tilde{\rho}_{\widetilde{B}}$. Note that in (1.24) bars above $\widetilde{Y}$ and $\widetilde{W}$ indicate that they are swaped to get the first term from $G(\tilde{X}, \tilde{Y})G(\tilde{Z}, \tilde{W})$, the bars below $\widetilde{Y}$ and $\widetilde{Z}$ indicate that they are swapped to get the second term from $G(\tilde{X}, \tilde{Y})G(\tilde{Z}, \tilde{W})$ and the bar above $\widetilde{F}$ indicates that the second term is subtracted from the first term. We follow this code through out this paper. If the second fundamental tensor $h(X,Y)$ satisfies the condition [6][9]

$$h(X,Y) = g(X,Y) + \mu\eta(X)\eta(Y) \tag{1.25}$$

then $M^{2n+1}$is called a c-umbilical hypersurface. Also the immersed manifold in Kählerian manifold is Sasakian if and only if it is c-umbilical[6][9], with

$$\mu = (2n+1)(H-1), \tag{1.26}$$

where H is the mean curvature [6]. If H is a constant then the immersed hypersurface is called **CMC** hypersurface and in this space

$$\nabla\mu = 0. \tag{1.27}$$

The Weyl conformal curvature tensor $C(X,Y,Z,W)$ and the Quasi-conformal curvature tensor $C'(X,Y,Z,W)$ on M^{2n+1} are defined as follows [4][11]:

$$\begin{aligned} C(X,Y,Z,W) = {} & K(X,Y,Z,W) \\ & -\frac{1}{(2n-1)}\{S(Y,Z)g(X,W)-S(X,Z)g(Y,W) \\ & +g(Y,Z)S(X,W)-g(X,Z)S(Y,W)\} \\ & +\frac{r}{2n(2n+1)}F(X,Y,Z,W) \end{aligned} \tag{1.28}$$

and

$$\begin{aligned} C'(X,Y,Z,W) = {} & a'K(X,Y,Z,W) \\ & +b'\{S(Y,Z)g(X,W)-S(X,Z)g(Y,W) \\ & +g(Y,Z)S(X,W)-g(X,Z)S(Y,W)\} \\ & -\frac{r}{(2n+1)}\left(\frac{a'}{2n}+2b'\right)\overline{F}\left(X,\underline{Y},\underline{Z},\overline{W}\right). \end{aligned} \tag{1.29}$$

Note that by replacing $a'=1$ and $b'=-\frac{1}{2n-1}$ in (1.29), we get (1.28).

2. Quasi-conformally flat Sasakian hypersurfaces

A Sasakian manifold M^{2n+1} is said to quasi-conformally flat [11] if

$$C'(X,Y,Z,W)=0 \tag{2.1}$$

Therefore, by using (2.1) in (1.29), we get

$$\begin{aligned} K(X,Y,Z,W) = {} & -\frac{b'}{a'}\{S(Y,Z)g(X,W)-S(X,Z)g(Y,W) \\ & +g(Y,Z)S(X,W)-g(X,Z)S(Y,W)\} \\ & +\frac{r}{(2n+1)}\left(\frac{a'}{2n}+2b'\right)\overline{F}\left(X,\underline{Y},\underline{Z},\overline{W}\right) \end{aligned} \tag{2.2}$$

From (2.2) it can be verified that M^{2n+1} is η-Einstein [11], that is ,

$$S(Y,Z)=Pg(Y,Z)+Q\eta(Y)\eta(Z) \tag{2.3}$$

with $P=\frac{r}{2n+1}\left(\frac{a'}{2nb'}+2\right)-\left(2n+\frac{a'}{b'}\right)$, $Q=-\frac{r}{2n+1}\left(\frac{a'}{2nb'}+2\right)+\left(4n+\frac{a'}{b'}\right)$ and $P+Q=2n$. On the other hand by using (2.3) in (2.2), we get

$$\begin{aligned} K(X,Y,Z,W) = {} & \lambda_1\overline{F}\left(X,\underline{Y},\underline{Z},\overline{W}\right)+(1-\lambda_1)\{\eta(X)\overline{F}\left(\xi,\underline{Y},\underline{Z},\overline{W}\right) \\ & +\eta(Y)\overline{F}\left(X,\underline{\bar{\xi}},\underline{Z},\overline{W}\right)\}, \end{aligned} \tag{2.4}$$

where $\lambda_1 = \frac{r}{2n+1}\left(\frac{1}{2n}+\frac{2b'}{a'}\right)-\frac{2b'}{a'}P = \frac{4nb'}{a'}+2-\frac{r}{2n+1}\left(\frac{1}{2n}+\frac{2b'}{a'}\right)$. Applying ∇_U on both sides in (2.4) and using (1.5),(2.4),(1.14),(1.18) and (1.17), we get

$$\begin{aligned}(\nabla_U K)(X,Y,Z,W) = (\nabla_U\lambda_1)\{&\overline{F}\left(X,\underline{\overline{Y}},\underline{Z},\overline{W}\right)-\eta(X)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\ &-\eta(Y)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\} \\ &+(1-\lambda_1)\{\Omega(U,X)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\ &+\Omega(U,Y)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{Z}\right) \\ &+\Omega(U,Z)\overline{F}\left(X,\underline{\overline{Y}},\underline{\xi},\overline{W}\right) \\ &+\Omega(U,W)\overline{F}\left(X,\underline{\overline{Y}},\underline{Z},\overline{\xi}\right)\}.\end{aligned} \tag{2.5}$$

So applying ∇_U to (1.21) and using (1.25), (1.19), (1.6) and (1.18), we get

$$\begin{aligned}(\nabla_U K)(X,Y,Z,W) = {}&(\widetilde{\nabla}_{\widetilde{U}}\widetilde{K})(\widetilde{X},\widetilde{Y},\widetilde{Z},\widetilde{W}) \\ &+U[\mu]\{\eta(X)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\ &+\eta(Y)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\} \\ &+g(U,X)\{(W[\mu]\eta(Z)-Z[\mu]\eta(W))\eta(Y)\} \\ &+g(U,Y)\{(Z[\mu]\eta(W)-W[\mu]\eta(Z))\eta(X)\} \\ &+g(U,Z)\{(Y[\mu]\eta(X)-X[\mu]\eta(Y))\eta(W)\} \\ &+g(U,W)\{(X[\mu]\eta(Y)-Y[\mu]\eta(X))\eta(Z)\} \\ &+\mu g(U,X)\{2\Omega(W,Z)\eta(Y)-\overline{F}\left(\xi,\underline{\overline{\varphi Y}},\underline{Z},\overline{W}\right)\} \\ &+\mu g(U,Y)\{2\Omega(Z,W)\eta(X)-\overline{F}\left(\varphi X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\} \\ &+\mu g(U,Z)\{2\Omega(Y,X)\eta(W)-\overline{F}\left(X,\underline{\overline{Y}},\underline{\xi},\overline{\varphi W}\right)\} \\ &+\mu g(U,W)\{2\Omega(X,Y)\eta(Z)-\overline{F}\left(X,\underline{\overline{Y}},\underline{\varphi Z},\overline{\xi}\right)\} \\ &+\mu\{\Omega(U,X)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right)+\Omega(U,Y)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\} \\ &+\mu\{\Omega(U,Z)\overline{F}\left(X,\underline{\overline{Y}},\underline{\xi},\overline{W}\right)+\Omega(U,W)\overline{F}\left(X,\underline{\overline{Y}},\underline{Z},\overline{\xi}\right)\},\end{aligned} \tag{2.6}$$

where $U[\mu] = \nabla_U\mu$. If the ambient manifold is generalized recurrent Kählerian manifold, then by using (1.23) in (2.6), we get

$$\begin{aligned}(\nabla_U K)(X,Y,Z,W) = {}&A(U)K(X,Y,Z,W) \\ &+\{B(U)-A(U)\}\overline{F}\left(X,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\ &+\{U[\mu]-\mu A(U)\}\{\eta(X)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\ &+\eta(Y)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\}\end{aligned} \tag{2.7}$$

$$
\begin{aligned}
&+ g\left(U,X\right)\left\{\left(W\left[\mu\right]\eta(Z) - Z\left[\mu\right]\eta(W)\right)\eta(Y)\right\} \\
&+ g\left(U,Y\right)\left\{\left(Z\left[\mu\right]\eta(W) - W\left[\mu\right]\eta(Z)\right)\eta(X)\right\} \\
&+ g\left(U,Z\right)\left\{\left(Y\left[\mu\right]\eta(X) - X\left[\mu\right]\eta(Y)\right)\eta(W)\right\} \\
&+ g\left(U,W\right)\left\{\left(X\left[\mu\right]\eta(Y) - Y\left[\mu\right]\eta(X)\right)\eta(Z)\right\} \\
&+ \mu g\left(U,X\right)\left\{2\Omega\left(W,Z\right)\eta\left(Y\right) - \overline{F}\left(\xi,\underline{\overline{\varphi Y}},\underline{Z},\overline{W}\right)\right\} \\
&+ \mu g\left(U,Y\right)\left\{2\Omega\left(Z,W\right)\eta\left(X\right) - \overline{F}\left(\varphi X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\right\} \\
&+ \mu g\left(U,Z\right)\left\{2\Omega\left(Y,X\right)\eta\left(W\right) - \overline{F}\left(X,\underline{\overline{Y}},\underline{\xi},\overline{\varphi W}\right)\right\} \\
&+ \mu g\left(U,W\right)\left\{2\Omega\left(X,Y\right)\eta\left(Z\right) - \overline{F}\left(X,\underline{\overline{Y}},\underline{\varphi Z},\overline{\xi}\right)\right\} \\
&+ \mu\{\Omega\left(U,X\right)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right) + \Omega\left(U,Y\right)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\} \\
&+ \mu\{\Omega\left(U,Z\right)\overline{F}\left(X,\underline{\overline{Y}},\underline{\xi},\overline{W}\right) + \Omega\left(U,W\right)\overline{F}\left(X,\underline{\overline{Y}},\underline{Z},\overline{\xi}\right)\}.
\end{aligned}
$$

Substituting (2.5) in (2.7), replacing U by ξ and simplifying using (1.5), (1.6) and (1.18), we get

$$
\begin{aligned}
&A(\xi)K(X,Y,Z,W) + \{B(\xi) - A(\xi) - \nabla_\xi\lambda_1\}\overline{F}\left(X,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\
&+ \{\xi\left[\mu\right] - \mu A(\xi) + \nabla_\xi\lambda_1\}\ \{\eta(X)\overline{F}\left(\xi,\underline{\overline{Y}},\underline{Z},\overline{W}\right) \\
&\qquad\qquad + \eta(Y)\overline{F}\left(X,\underline{\overline{\xi}},\underline{Z},\overline{W}\right)\} = 0,
\end{aligned} \tag{2.8}
$$

where

$$
\nabla_\xi\lambda_1 = -\frac{\xi\left[r\right]}{2n+1}\left(\frac{2b'}{a'} + \frac{1}{2n}\right). \tag{2.9}
$$

Substituting $X = W = e_i$ in (2.8) and summing up over i, $1 \le i \le (2n+1)$, we get

$$
S(Y,Z) = f_1 g(Y,Z) + f_2\eta(Y)\eta(Z), \tag{2.10}
$$

where

$$
f_1 = \frac{(2n-1)\left(\nabla_\xi\lambda_1\right) - 2nB(\xi) - \xi\left[\mu\right]}{A(\xi)} + (2n+\mu) \tag{2.11}
$$

and

$$
f_2 = (2n-1)\left\{\mu - \frac{\xi\left[\mu\right] + \nabla_\xi\lambda_1}{A(\xi)}\right\}. \tag{2.12}
$$

Since M^{2n+1} is η-Einstein, in view of (1.20), $f_1 + f_2 = 2n$. Therefore, from (2.11) and (2.12), we get

$$
\xi\left[\mu\right] = \mu A\left(\xi\right) - B\left(\xi\right). \tag{2.13}
$$

Now, substituting (2.13) in (2.11) and (2.12), f_1, f_2 reduce as follows:

$$f_1 = (2n-1)\left(\frac{\nabla_\xi \lambda_1 - B(\xi)}{A(\xi)}\right) + 2n \tag{2.14}$$

and

$$f_2 = (2n-1)\left\{\frac{B(\xi) - \nabla_\xi \lambda_1}{A(\xi)}\right\}. \tag{2.15}$$

Replacing $Y = Z = e_j$ in (2.10), summing up for $1 \leq j \leq (2n+1)$ and using (2.14) and (2.15), we get

$$r = 2n\left\{(2n+1) - (2n-1)\frac{B(\xi) - \nabla_\xi \lambda_1}{A(\xi)}\right\}. \tag{2.16}$$

In (2.9), $\nabla_\xi \lambda_1 = 0$ if $\xi[r] = 0$ or $\left(\frac{2b'}{a'} + \frac{1}{2n}\right) = 0$. Now, if we consider the case that $\left(\frac{2b'}{a'} + \frac{1}{2n}\right) = 0$, then by using (2.5), we state the following theorem:

Theorem 2.1 Suppose M^{2n+1} is a quasi conformally flat Sasakian hypersurface of the generalized recurrent Kählerian manifold M^{2n+2}. Then M^{2n+1} is symmetric if $a' = -4nb'$ in (1.29).

Also, if we consider the case that $\xi[r] = 0$ in (2.9), then r is a constant. Hence, using (2.16), we state the following theorem:

Theorem 2.2 Suppose M^{2n+1} is a quasi conformally flat Sasakian hypersurface with constant scalar curvature r along the characteristic vector field ξ of the generalized recurrent Kählerian manifold M^{2n+2}. Then the scalar curvature r of M^{2n+1} is given by

$$r = 2n\left\{(2n+1) - (2n-1)\frac{B(\xi)}{A(\xi)}\right\}, \tag{2.17}$$

where A and B are the 1-forms in M^{2n+1} corresponding to the 1-forms $\tilde{A}$ and $\tilde{B}$ in M^{2n+2} .

If M^{2n+1} has a constant mean curvature H, then using (1.27) in (2.13), we get

$$B(\xi) = \mu A(\xi)). \tag{2.18}$$

Using (2.18) in (2.17) we state the following corollary:

Corollary 2.3 Suppose M^{2n+1} is a quasi conformally flat Sasakian CMC hypersurface of the generalized recurrent Kählerian manifold M^{2n+2}. The vector field $\rho_B - \mu\rho_A$ is orthogonal to the characteristic vector field ξ if the scalar curvature r of M^{2n+1} is a constant along the characteristic vector field ξ and

(i) r is positive if $H > \frac{2n}{2n-1}$ and

(ii) r is negative if $H < \frac{2n}{2n-1}$,

where ρ_A,ρ_B are the vector fields associated with the 1-forms A, B and H is the mean curvature.

3. Sasakian hypersurfaces with cyclic paralled Ricci tensor

The Ricci tensor S of the Sasakian manifold is said to be cyclic parallel [7][10] if it satisfies

$$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0. \tag{3.1}$$

Now, substituting $X = W = e_i$ in (2.7), summing up over $1 \leq i \leq (2n+1)$ and using (2.10) , we get

$$\begin{aligned}(\nabla_U S)(Y,Z) = {} & ((f_1 - 2n - \mu) A(U) + 2nB(U) + U[\mu]) g(Y,Z) \\ & + ((f_2 - \mu(2n-1)) A(U) + (2n+1) U[\mu]) \eta(Y) \eta(Z) \\ & + Z[\mu] g(\varphi U, \varphi Y) + Y[\mu] g(\varphi U, \varphi Z) \\ & + 2\mu(n+1)(\Omega(U,Y)\eta(Z) + \Omega(U,Z)\eta(Y)) \\ & - \xi[\mu](g(U,Y)\eta(Z) + g(U,Z)\eta(Y)).\end{aligned} \tag{3.2}$$

Now, using (3.2) in (3.1), replacing $Y = Z = e_j$ and summing up for $1 \leq j \leq (2n+1)$, we get

$$\begin{aligned}&(2(n+1) f_1 - 2(2n(n+1) + \mu(2n+1))) A(X) \\ &+ 2n(2n+3) B(X) + 8((n+1) X[\mu] - \eta(X)\xi[\mu]) \\ &\qquad + 2(f_2 - \mu(2n-1)) A(\xi) \eta(X) = 0.\end{aligned} \tag{3.3}$$

Hence, replacing X by ξ in (3.3) , using (2.13), (2.14), (2.15) and (2.9), we get

$$\xi[r] = 0. \tag{3.4}$$

Hence we state the following:

Theorem 3.1 Suppose M^{2n+1} is a quasi conformally flat Sasakian hypersurface of the generalized recurrent Kählerian manifold M^{2n+2}. The scalar curvature r of M^{2n+1} is a constant along the characteristic vector field ξ, if the Ricci tensor of a M^{2n+1} is cyclic parallel.

4. Sasakian hypersurfaces with η -paralled Ricci tensor

The Ricci tensor S of the Sasakian manifold is called η - parallel [12] if

$$(\nabla_U S)(\varphi Y, \varphi Z) = 0. \tag{4.1}$$

Now, replacing Y, Z by φY, φZ respectively in (3.2), using (4.1) and replacing U by ξ, we get

$$((f_1 - 2n - \mu) A(\xi) + 2nB(\xi) + \xi[\mu]) g(\varphi Y, \varphi Z) = 0. \tag{4.2}$$

Since $g(\varphi Y, \varphi Z) \neq 0$, from (4.2) we get

$$((f_1 - 2n - \mu) A(\xi) + 2nB(\xi) + \xi[\mu]) = 0. \tag{4.3}$$

Using (2.13), (2.14) and (2.9) in (4.3), we get

$$\xi[r] = 0. \tag{4.4}$$

Hence we state the following theorem:

Theorem 4.1 Suppose M^{2n+1} is a quasi conformally flat Sasakian hypersurface of the generalized recurrent Kählerian manifold M^{2n+2}. The scalar r of M^{2n+1} is a constant along the characteristic vector field ξ, if the Ricci tensor of a M^{2n+1} is η-parallel.

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