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Particle Creation with Generalised Gravitational and Cosmological Constants

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Abstract

In this paper we study the effect of particle creation on the evolution of FRW cosmological model. The universe has been considered as an open thermodynamic system when particle creation leads to supplementary negative creation pressure in addition to the thermodynamic pressure. We also discuss dynamical behaviors of the cosmological solutions of radiation dominated models and stiff models.

Keywords and Phrases : Particle creation, open thermodynamic system, negative pressure.

1. Introduction

Many problems in standard cosmology have been explained by the inflationary models of (Guth [1], Linde [2]). Different alternative theories have been proposed to explain the cosmological problems of the early universe (Brans and Dicke [8], Bergmann [9], Nortvedt [10], Wagoner [11]). Now we are facing a new problem in cosmology. Earlier cosmologists hold the view that our expanding universe will slow down come to a halt and start contracting under the influence of gravity. On the contrary high red-shift supernovae Ia (SNe Ia) observations (Riess et al. 1998, Perlmutter et al. 1999) reveals the accelerated expansion of the universe which is big blow to all the cosmologist. But what causes such accelerated expansion of the universe is yet to be confirmed. Dark energy models have suggested by many cosmologist to explain such observation. The idea of particle production in cosmology has been dealt by many authors (Zeldovich

[12], Parker [13], Brout et al. [14]). Prigogine et al. [15] have studied thermodynamics of open systems in the reference of cosmology and suggested a quantitative expression for the particle production out of gravitational energy. They have presented a new concept of adiabatic transformation from closed to open systems. The negative creation pressure associated with particle creation may explain the observed accelerated expansion of the universe. If we consider the universe as an adiabatic open thermodynamic system, allowing for irreversible matter production from the gravitational field, then the thermodynamic energy conservation equation becomes

$$d(\rho V) + pdV - \frac{(\rho + p)}{n} dN = 0, \tag{1}$$

where V is the volume of the system, $n = N/V$ is the particle number density, and N is the number of particles in V. This conservation equation may be written as

$$d(\rho V) + (p + p_c)dV = 0. \tag{2}$$

Here p_c is the supplementary pressure corresponding to creation of matter and expressed as

$$p_c = -\frac{(\rho + p)}{N} \frac{dN}{dt} \frac{1}{3H}, \tag{3}$$

the creation pressure p_c is negative or zero depending on the presence or absence of particle production. Thus, the effect of production of new particles is equivalent to adding a supplementary pressure term p_c to the thermodynamic pressure p so that the conservation equation for a closed system

$$d(\rho V) + pdV = 0 \tag{4}$$

is the modified to eq.(2) for an open system. For an adiabatic open system, the increase in entropy is only due to the creation of matter and since entropy S is an extensive property of the system (i.e., S is proportional to the number of particle included in the system), we have the relation

$$\frac{dS}{S} = \frac{dN}{N}. \tag{5}$$

The second law of thermodynamics requires that $dS \geq 0$, which imposes the condition that the only particle number variations admitted are such that $dN \geq 0$. Several authors (Calvao et al. [16], Triginer and Pavon [17]) have studied the thermodynamics of particle production in different contexts. A theory of gravitation, using G and Λ as no constant coupling scalars, was studied by Abdel-Rahman [18], Beesham ([19], [20]). Its motivation was to include a G-varying 'constant' of gravity as pioneered by Dirac [21]. We have the Einstein's field equations

$$R^{ij} - \frac{1}{2}Rg^{ij} = -8\pi G T^{ij} - \Lambda g^{ij}, \tag{6}$$

where T^{ij} is the matter energy-momentum tensor, g^{ij} the metric tensor, R the scalar curvature, R^{ij} the Ricci tensor, G and Λ are coupling scalars. As in Einstein's theory we assume the principle of equivalence i.e. the equality of gravitational and inertial mass and the gravitational time dilation, then we require that the equation of motion of particles and photons must not contain G and Λ but only g^{ij}. So, the interchange of energy between matter and gravitation is given by the conservation laws

$$T^{ij}_{;j} = 0. \tag{7}$$

And of course the Bianchi identity still holds,

$$(R^{ij} - \frac{1}{2}g^{ij})_{;j} = 0. \tag{8}$$

Thus the role of the scalars G and Λ is confined to the effects on the field equation (6) and once g^{ij} is determined, the gravitational phenomena are described in the same ways as in Einstein's equation The covariant derivative of (6), taking into account the Bianchi identity and (7) gives

$$8\pi G_{;j} T^{ij} + \Lambda_{;j} g^{ij} = 0. \tag{9}$$

Here, equations (6) and (9) are considered as the fundamental equations of gravity with G and Λ coupling scalars. The cosmological models based on these equations allow the possibility of investigating different cases for G, as in Dirac's cosmology for example, or to solve some cosmological difficulties (Ozer, M. and Taha M. O. [22], [23]); they may be useful to study the early universe (singular or not) and their relations with particle fields. Sistero [24] found exact solutions for zero pressure models satisfying $G = G_0(\frac{a}{a_0})^m$. Barrow [25] formulated and studied the problem of varying G in Newtonian Gravitation and cosmology. Vishwakarma R. G. ([26], [27]) also studied cosmological models with variable G and Λ.

In this paper, we discuss field equations in section 2, solution of the field equations in section 3 and we conclude in section 4.

2. Model

We assume a flat cosmological model which is homogeneous and isotropic and it is natural to consider the metric tensor of this gravitational field to be of the type

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)] \tag{10}$$

where $a(t)$ the scale factor (the speed of light c and signature + - - - are used). For perfect fluid cosmology we have the energy-momentum tensor

$$T^{ij} \quad = \quad -pg^{ij} + (p+\rho)u^i u^j. \tag{11}$$

We set $u^\nu = (1,0,0,0)$ in (11) and with the metric (5), Einstein's equations (6) gives

$$3\ddot{a} = -4\pi G a(3p + \rho - \frac{\Lambda}{4\pi G}), \tag{12}$$

$$3\dot{a}^2 = 8\pi G a^2(\rho + \frac{\Lambda}{8\pi G}). \tag{13}$$

Elimination of $\ddot{a}$ gives

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p+\rho) = -(\frac{\dot{G}\rho}{G} + \frac{\dot{\Lambda}}{8\pi G}). \tag{14}$$

The conservation of energy momentum yields

$$\dot{\rho} + 3\frac{\dot{a}}{a}(p+\rho) = 0. \tag{15}$$

Using (15) in (14)

$$8\pi\dot{G}\rho + \dot{\Lambda} = 0. \tag{16}$$

Equations (12), (13), and (16) are the fundamental equations. They reduce to standard Freidmann Cosmology when G and Λ are constants. The equations (12) and (13) may be written as

$$8\pi G\rho = -2\frac{\ddot{a}}{a} - (\frac{\dot{a}}{a})^2 + \Lambda, \tag{17}$$

$$8\pi G\rho = 3(\frac{\dot{a}}{a})^2 - \Lambda. \tag{18}$$

Eliminating Λ between (17) and (18) $\ddot{a}$ with the derivative of (18) and using (16), it is found that

$$\frac{d(\rho a^3)}{dt} + p\frac{da^3}{dt} = 0, \tag{19}$$

(17) and (18) are formally identical to those of usual cosmology with G and Λ constants as must be, since the LHS of (6) depends only on the metric components (10) G and Λ enter algebraically in the RHS of (6). Also (12) is identical to that of usual cosmology despite the fact that it comes from a differential form of (18), and from (10), both involving the time dependent scalars G and Λ, however, their derivatives eliminate with (16), thus leading to (19). Equations (16), (18) and (19) are independent and they will be treated as fundamental in the following. The cosmological problem posed by these equations leaves two degrees of freedom; it may be determined by a physical assumption $p = p(\rho)$, i.e. the 'equation of state', and from an additional explicit adoption on $a(t)$, $\rho(t)$, p, G or Λ in terms of t or a which itself depends on t.

3. Solution

The barotropic equation of state is

$$p = \gamma\rho \tag{20}$$

$0 \leq \gamma \leq 1$. Using eqs.(19) and (20), we get

$$\frac{1}{\psi}\frac{d\psi}{da} + \frac{3\gamma}{a} = 0 \tag{21}$$

where

$$\psi = \rho a^3 \tag{22}$$

and ψ can be determined from eq.(21)

$$\psi = \psi_0 \exp\left[-\int \frac{3\gamma}{a} da\right] \tag{23}$$

The Friedmann eq.(18) with (22) becomes

$$3\dot{a}^2 = \frac{8\pi G\psi}{a} + \Lambda a^2. \tag{24}$$

Eq. (16) and (22) with $\frac{d}{dt} = \dot{a}\frac{d}{da}$

$$8\pi\frac{dG}{da} + \frac{a^3}{\psi}\frac{d\Lambda}{da} = 0. \tag{25}$$

If $G = G(a)$ is given, (26) integrates to give $\Lambda = \Lambda(a)$, (25) determines $a = a(t)$ and the problem is solved; if $\Lambda = \Lambda(a)$ is given instead then also we obtain $G(a)$ from (26) giving in turn $a(t)$ from integration of (25). We assume

$$G = G_0\left(\frac{a}{a_0}\right)^m. \tag{26}$$

Let us treat the universe as an open thermodynamic system with initially N particles and assume that a random fluctuation in curvature induces a transformation of gravitational energy into matter energy, producing an additional number of particle dN . This increase in the number of particle from N to $N + dN$ gives rise to a negative supplementary pressure to the thermodynamic pressure and this negative pressure drives the expansion of the universe. Hence, the perfect fluid pressure should be replaced by an effective pressure of the cosmic fluid which is given by

$$p_{eff} = p + p_c. \tag{27}$$

Using (20) and (27) the conservation equation (15) reduces to

$$\dot{\rho} + 3H\rho(1+\gamma) = -3Hp_c. \tag{28}$$

With the help of (3), eq. (28) after integration yields

$$N(t) = N_0 a^3 \rho^{(1+\gamma)} \tag{29}$$

where N_0 is an integration constant. Eq. (29) give rise to a relation between the particle number density (n) and energy density (ρ) as

$$n = n_1 \rho^{(1+\gamma)} \tag{30}$$

where n_1 is the constant of proportionality. According to Gibbs integrability condition, one cannot independently specify an equation of state for the pressure and temperature (Maartens [29]). If we consider one barotropic relation then the other relation must be barotropic and hence we get $T \propto \exp \int \frac{dp}{\rho(p)+p}$. Now using eq. (20), we get

$$T = T_0 \rho^{\frac{\gamma}{1+\gamma}}, \tag{31}$$

T_0 is the integration constant. We consider two cases of the equation of state $p = \gamma\rho$.

Case 1. Radiation dominated model ($\gamma = \frac{1}{3}$)

Putting $\gamma = \frac{1}{3}$ in eq. (20), we get $p = \rho/3$. Then using (22), we get

$$\psi = \frac{\psi_0}{a} \tag{32}$$

and the expression of Λ as $\Lambda = -\dfrac{8\pi m G_0 \psi_0 a^{(m-4)}}{(m-4)a_0^m}$.

We solve Λ for particular value of $m = 3$

$$\Lambda = \frac{24\pi G_0 \psi_0}{a_0^3 a} \tag{33}$$

Using the values of G, ψ and Λ, we get the expression of $a(t)$ as

$$a(t) \quad = \quad Bt^2 \tag{34}$$

where $B = \dfrac{8\pi G_0 \psi_0}{3a_0^3}$. B is positive as scale factor cannot be negative. Now we can evaluate the value of the physical parameters and they are obtained as

$$\rho = \frac{\psi_0}{B^4 t^8} \tag{35}$$

$$N(t) = \frac{N_0 \psi_0^{\frac{4}{3}}}{B^{\frac{7}{3}} t^{\frac{14}{3}}} \tag{36}$$

$$T = \frac{T_0 \psi_0^{\frac{1}{4}}}{Bt^2} \tag{37}$$

$$\Lambda = \frac{24\pi G_0 \psi_0}{B a_0^3 t^2} \tag{38}$$

In this case deceleration parameter is given by $q = -1/2$.

The negative deceleration parameter shows that the model represents an accelerating universe. From the expression of the scale factor we know that the universe start with a big bang. All the physical parameters $N(t)$, ρ, T and Λ are all decreasing function of time.

The event horizon r_E is given by the expression

$$r_E = a(t_0) \int_{t_0}^{\infty} \frac{dt}{a(t)}. \tag{39}$$

In this case event horizon exist and is found to be $2/H_0$.

Case 2. Stiff model ($\gamma = 1$)

Putting $\gamma = 1$ in (23), we get

$$\psi = \frac{\psi_0}{a^3}. \tag{40}$$

Using (26) and (39)in (25) and integrating, we get

$$\Lambda = -\frac{8m\pi G_0 \psi_0 a^{(m-6)}}{(m-6) a_0^m}. \tag{41}$$

We use particular value of m say $m = 5$ in order to simplify mathematical calculation. In this case we obtain the expression of Λ

$$\Lambda = \frac{40\pi G_0 \psi_0}{a_0^5 a}. \tag{42}$$

We calculate the value of scale factor $a(t)$ by substituting the values of G, ψ and Λ

$$a(t) = Ct^2 \tag{43}$$

where $C = \dfrac{4\pi G_0 \psi_0}{a_0^5}$ is positive as scale factor cannot be negative.

Putting $\gamma = 1$, the physical parameters are obtained as a function of time as

$$\rho = \frac{\psi_0}{C^6 t^{12}} \tag{44}$$

$$N(t) = \frac{N_0 \psi_0^2}{C^9 t^{18}} \tag{45}$$

$$n(t) = \frac{n_1 \psi_0^2}{C^6 t^{12}} \tag{46}$$

$$T = \frac{T_0 \psi_0^{\frac{1}{2}}}{C^3 t^6} \tag{47}$$

$$\Lambda = \frac{40\pi G_0 \psi_0}{C a_0^5 t^2} \tag{48}$$

In both the models, deceleration parameter is $q = -1/2$ showing that the universe is accelerating. From the above expressions we observe that in stiff fluid model energy density ρ, particle density $N(t)$, temperature T and cosmological constant Λ are decreasing functions of time. Interestingly event horizon for this model is found to be $2/H_0$.

4. Conclusion

We observed that the scale factor increases with the age of universe in both the models. The scale factor has an initial singularity which supports the start of universe with a big bang. Data obtained by WMAP satellite strongly supports an accelerating universe and the negative deceleration parameter obtain in the paper explains the congruence of the true nature of the universe with our model. Since the number of particles $N(t)$ is a decreasing function of time in the above two cases. The presence of a positive cosmological constant always produces a repulsive effect and we obtain a positive cosmological constant in all the cases. So either the big bang impulse or the presence of time decaying positive cosmological constant in this paper may account for the observed acceleration. Other physical parameters are also decreasing function of cosmic time which are well supported by present observations. Event horizon always exist in both the models.

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