

J. T. S.

Special Vol. 8 (2014), pp.17-32

<https://doi.org/10.56424/jts.v8i01.10561>

## **Accelerating bulk viscous LRS Bianchi-II string cosmological models in Sáez-Ballester theory of gravitation**

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(Received: November 11, 2013)

### **Abstract**

In this paper, we obtain a spatially homogeneous and anisotropic Locally Rotationally Symmetric Bianchi type-II string cosmological model of the universe for bulk viscous fluid distribution within the framework of scalar-tensor theory of gravitation proposed by Sáez and Ballester (Phys. Lett. 113:467, 1986). To prevail the deterministic solutions we consider time-dependent deceleration parameter (DP) which provides the value of scale factor as  $a = [\sinh(\alpha t)]^{\frac{1}{n}}$ , where  $\alpha$  and  $n$  are arbitrary positive constants. This acclimates time-dependent DP representing models which generate a transition of the universe from the early decelerated phase to the recent accelerating phase. It is also observed that for  $0 < n \leq 1$ , our model is in accelerating phase but for  $n > 1$ , our model is evolving from decelerating phase to accelerating phase. The modified Einstein's field equations are solved exactly and the derived model is found to be in good concordance with recent observations. Some physical and geometric properties of the models are also discussed.

**Keywords and Phrases :** Cosmic string, Anisotropic universe, Bulk viscosity.

**PACS number :** 04.20.-q, 04.20.Jb.

### **1. Introduction**

One of the outstanding problems in cosmology today is developing a more precise understanding of structure formation in the universe, that is, the origin

of galaxies and other large-scale structures. Existing theories for the structure formation of the Universe fall into two categories, based either upon the amplification of quantum fluctuations in a scalar field during *inflation*, or upon symmetry breaking phase transition in the early Universe which leads to the formation of *topological defects* such as domain walls, cosmic strings, monopoles, textures and other 'hybrid' creatures. Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (see Refs. Zel'dovich et al. [1], Kibble [2, 3], Everett [4], Vilenkin [5]). It is believed that cosmic strings give rise to density perturbations which lead to formation of galaxies (Zel'dovich [6]). These cosmic strings have stress energy and couple to the gravitational field. Therefore, it is interesting to study the gravitational effect which arises from strings. The general treatment of strings was initiated by Letelier [7, 8] and Stachel [9]. Singh and Singh [10] investigated string cosmological models with magnetic field in the context of space-time with  $G_3$  symmetry. Singh [11] has studied string cosmology with electromagnetic fields in Bianchi type-II, -VIII and -IX space-times. Lidsey et al. [12] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Bali and Upadhaya [13], Bali and Anjali [14], Bali et al. [15] have investigated Bianchi type I magnetized string cosmological models. Recently Reddy [16, 17], Reddy and Naidu [18], Reddy et al. [19, 20], Pradhan [21, 22], Pradhan et al. [23–27], Yadav et al. [28], Agarwal et al. [29] and Chawla et al. [30] have studied cosmic string cosmological models in different contexts.

The distribution of galaxies in the universe shows that the matter is satisfactorily described by a perfect fluid. To consider more realistic models one must take into account the viscosity mechanisms, which have already attracted the attention of many researchers. Most studies in cosmology involve a perfect fluid. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest that we should analyze dissipative effects in cosmology. Further, there are several processes which are expected to give rise to viscous effect. The effect of viscosity on the evolution of cosmological models can be important such as counteracting gravitational collapse or expansion, creating a bounded model, levitating the initial singularity, modifying the effect of the energy density and pressure during the cosmological evolution [31]. The study of bulk viscous mechanism in cosmology attracted the attention of

many researchers due to its significant role in the description of high entropy of the present universe [32, 33] and references therein. However, it is conjectured that material distribution behaves like a viscous fluid in the early phase of the evolution of the universe when galaxies were formed [34]. Recently, Bali et al. [35], Yadav et al. [36], Pradhan [37] and Pradhan et al. [38–40] have discussed string cosmological models in presence of bulk viscosity.

In the last few decades several new theories of gravitation, as alternatives to Einstein's theory of gravitation, have been developed. In such theories of gravitation, scalar tensor theories proposed by Brans and Dicke [41], Nordvedt [42], Wagoner [43], Rose [44], Dun [45], Sáez and Ballester [46], Barber [47], La and Steinhardt [48] are some most important among them. There are two categories of gravitational theories involving a classical scalar field  $\phi$ . In first category the scalar field  $\phi$  has the dimension of the inverse of the gravitational constant  $G$  among which the Brans-Decke theory [41] is of considerable importance and the role of the scalar field is confined to its effect on gravitational field equations. Brans and Decke formulated a scalar-tensor theory of gravitation which introduces an additional scalar field  $\phi$  besides the metric tensor  $g_{ij}$  and a dimensionless coupling constant  $\omega$ . This theory goes to general relativity for large values of the coupling constant  $\omega > 500$ . The second category of theories involve a dimensionless scalar field. Sáez and Ballester [46] developed a scalar-tensor theory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields. In spite of the dimensionless character of the scalar field, an anti-gravity regime appears. This theory suggests a possible way to solve the missing-matter problem in non-flat FRW cosmologies. The Scalar-Tensor theories of gravitation play important role to remove the graceful exit problem in the inflation era [49]. In earlier literature, string cosmological models within the framework of Sáez-Ballester scalar-tensor theory of gravitation, have been studied by Adhav et al. [50], Rao et al. [51] and Samantha et al. [52] in presence of bulk viscosity. Recently, Naidu et al. [53, 54] and Reddy et al. [55] have studied LRS Bianchi type-II models in Sáez and Ballester scalar tensor theory of gravitation in different contexts.

The aim of this paper is to investigate a class of new solutions of Locally Rotationally Symmetric Bianchi type-II string universe in presence of bulk viscosity within the framework of scalar-tensor theory of gravitation proposed by Sáez and Ballester.

## 2. The Metric and Field Equations

We consider the LRS Bianchi type-II line element, given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + 2B^2 x dy dz + (B^2 x^2 + A^2) dz^2, \quad (1)$$

where the metric potentials  $A, B$  are functions of  $t$  alone.

The field equations in the scalar-tensor theory proposed by Saez and Ballester [1] are given by

$$G_{ij} - \omega \phi^r \left( \phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -T_{ij}, \quad (2)$$

where  $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$  and  $8\pi G = c = 1$ .

The scalar field  $\phi$  satisfies the equation

$$2\phi^r \phi_{;i}^i + r \phi^{r-1} \phi_{,k} \phi^{,k} = 0. \quad (3)$$

Here  $r$  is an arbitrary constant and  $\omega$  is a dimensionless coupling constant. Comma and semi-colon respectively denote partial and covariant derivative with respect to cosmic time  $t$ .  $T_{ij}$  is the energy-momentum tensor of the matter.

For a cloud of massive string, the energy momentum tensor for bulk viscous fluid is given by

$$T_{ij} = (\rho + p^*) u_i u_j + p^* g_{ij} - \lambda x_i x_j, \quad (4)$$

where

$$p^* = p - 3\xi H. \quad (5)$$

Here  $p$  is the isotropic pressure;  $p^*$  the effective pressure;  $\rho$  the proper energy density for a cloud string with particles attached to them;  $\xi$  the coefficient of bulk viscosity that determines the magnitude of viscous stress relative to expansion;  $\lambda$  the string tension density;  $u^i = (0, 0, 0, 1)$  the four velocity of the particles, and  $x^i$  is a unit space-like vector representing the direction of string. The vectors  $u^i$  and  $x^i$  satisfy the conditions

$$g_{ij} u^i u^j = -g_{ij} x^i x^j = -1, \quad u^i x_i = 0. \quad (6)$$

Choosing  $x^i$  parallel to  $\partial/\partial x$ , we have

$$x^i = (A^{-1}, 0, 0, 0). \quad (7)$$

If the particle density of the configuration is denoted by  $\rho_p$ , then

$$\rho = \rho_p + \lambda. \quad (8)$$

Considering the form of the energy-momentum tensor (4), the Einstein's field equations (2) and scalar field equation (3), for the Bianchi type-II space-time (1) in Saez-Ballester theory, are given explicitly as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = \lambda - p^* + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \quad (9)$$

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = -p^* + \frac{1}{2}\omega\phi^r\dot{\phi}^2, \quad (10)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = \rho - \frac{1}{2}\omega\phi^r\dot{\phi}^2, \quad (11)$$

and

$$\ddot{\phi} + \dot{\phi} \left( \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \right) + \frac{r}{2} \frac{\dot{\phi}^2}{\phi} = 0. \quad (12)$$

Here, and also in what follows, a dot indicates ordinary differentiation with respect to  $t$ .

The energy conservation equation  $T_{;j}^{ij} = 0$ , leads to the following expression:

$$\dot{\rho} + (\rho + p^*) \left( 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right) - \lambda \frac{\dot{A}}{A} = 0. \quad (13)$$

### 3. Solutions of the Field Equations

Equations (9)–(12) are four equations in seven unknowns  $A$ ,  $B$ ,  $\phi$ ,  $p$ ,  $\rho$ ,  $\xi$  and  $\lambda$ . Three additional constraints relating these parameters are required to obtain explicit solutions of the system.

We first assume that the component  $\sigma_1^1$  of the shear tensor ( $\sigma_i^j$ ) is proportional to the expansion scalar ( $\theta$ ), i.e.,  $\sigma_1^1 \propto \theta$ . This condition leads to the following relation between the metric potentials:

$$A = (B)^m, \quad (14)$$

where  $m$  is a positive constant. The motive behind assuming this condition is explained in references [56–58]. Collins et al. [59] have also pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies the condition that  $\frac{\sigma}{\theta}$  is constant.

Following Pradhan [60], and Pradhan et al. [61], we assume the law of variation of scale factor as

$$a(t) = (A^2 B)^{\frac{1}{3}} = (\sinh(\alpha t))^{\frac{1}{n}}, \quad (15)$$

which yields a time dependent DP as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = n [1 - \tanh^2(\alpha t)] - 1, \quad (16)$$

where  $\alpha$  and  $n$  are positive constants.

The motive behind the variable DP is the transition from earlier decelerated expansion to the present accelerating one as observed in recent observations of Type Ia supernova (Riess et al. [62, 63]; Perlmutter et al. [64]; Tonry et al. [65]; Clocchiatti et al. [66]) and CMB anisotropies (Bennett et al. [67]; de Bernardis et al. [68]; Hanany et al. [69]). Therefore, DP must show signature flipping (Riess et al. [70]; Padmanabhan and Roychoudhury [71]; Amendola [72]) instead of being a constant.

From Eq. (16), we observe that  $q > 0$  for  $t < \frac{1}{\alpha} \tanh^{-1}(1 - \frac{1}{n})^{\frac{1}{2}}$  and  $q < 0$  for  $t > \frac{1}{\alpha} \tanh^{-1}(1 - \frac{1}{n})^{\frac{1}{2}}$ . It is also observed that for  $0 < n \leq 1$ , our model is in accelerating phase but for  $n > 1$ , our model is evolving from decelerating phase to accelerating phase. Figure 1 indicates variation of deceleration parameter ( $q$ ) versus time ( $t$ ) which gives the behavior of  $q$  for different values of  $n$ . It is also clear from Fig. 1 that for  $n \leq 1$ , the model is evolving only in accelerating phase whereas for  $n > 1$  the model is evolving from early decelerated phase to present accelerating phase.

Finally to conveniently specify the source, we assume the perfect gas equation of state, which may be written as

$$p = \gamma\rho, \quad 0 \leq \gamma \leq 1. \quad (17)$$

From equations (14) and (15), the metric functions can be explicitly written as

$$A(t) = [\sinh(\alpha t)]^{\frac{3m}{n(2m+1)}}, \quad (18)$$

$$B(t) = [\sinh(\alpha t)]^{\frac{3}{n(2m+1)}}. \quad (19)$$

Therefore, the geometry of universe (1) takes the form

$$ds^2 = -dt^2 + [\sinh(\alpha t)]^{\frac{6m}{n(2m+1)}} dx^2 + [\sinh(\alpha t)]^{\frac{6}{n(2m+1)}} dy^2 + 2[\sinh(\alpha t)]^{\frac{6}{n(2m+1)}} xdydz + \left[ [\sinh(\alpha t)]^{\frac{6}{n(2m+1)}} x^2 + [\sinh(\alpha t)]^{\frac{6m}{n(2m+1)}} \right] dz^2. \quad (20)$$

The physical quantities of observational interest in cosmology such as scalar of expansion ( $\theta$ ), spatial volume ( $V$ ), shear scalar ( $\sigma^2$ ), mean anisotropy parameter ( $A_m$ ) and directional Hubble's parameters ( $H_1$ ,  $H_2$ ) for model (20) are

respectively given by

$$\theta = 3H = \frac{3\alpha}{n} \coth(\alpha t), \quad (21)$$

$$V = (\sinh(\alpha t))^{\frac{3}{n}}, \quad (22)$$

$$\sigma^2 = \frac{1}{3} \left( \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 = 3 \left[ \frac{\alpha(m-1)}{n(2m+1)} \coth(\alpha t) \right]^2, \quad (23)$$

$$A_m = \frac{2\sigma^2}{3H^2} = 2 \left( \frac{m-1}{2m+1} \right)^2, \quad (24)$$

$$H_1 = mH_2 = \frac{3m\alpha}{n(2m+1)} \coth(\alpha t). \quad (25)$$

In our derived model, the present value of DP is estimated as

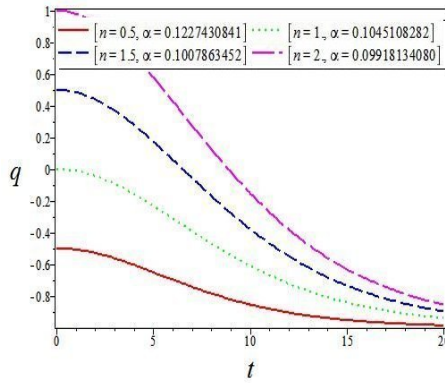
$$q_0 = n - \left( 1 + \frac{\alpha^2}{nH_0^2} \right), \quad (26)$$

where  $H_0$  is the present value of Hubble's parameter.

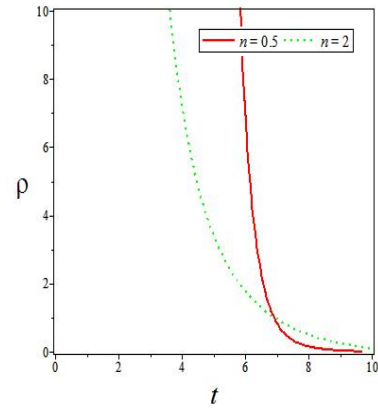
For

$$n = 0.135 \left[ 1 + \sqrt{1 + 10^{37} \alpha^2} \right], \quad (27)$$

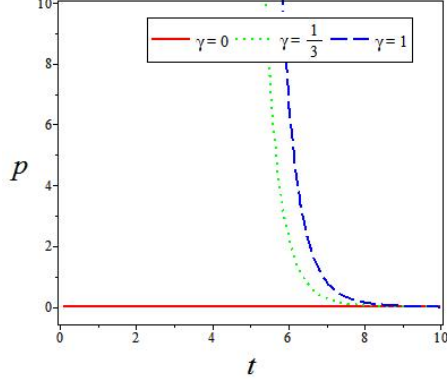
we obtain  $q_0 = -0.73$ , which is exactly matching with the observed value of DP at present epoch (Cunha et al. [73]). Therefore, we restrict  $\alpha = 2 \times 10^{-18}$  and corresponding value of  $n$  (i.e.  $n = 0.5$ ) for graphical representations of the physical parameters.



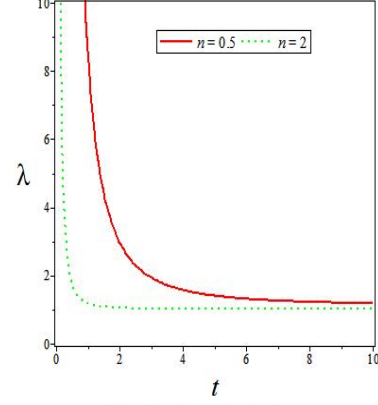
**Fig. 1:** Plot of deceleration parameter  $q$  vs time  $t$ .



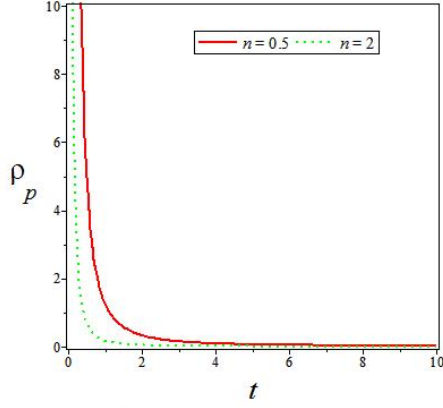
**Fig. 2:** Plot of energy density  $\rho$  vs time  $t$ .



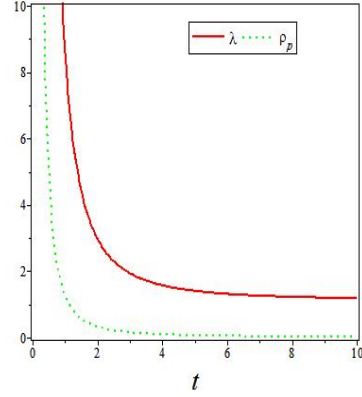
**Fig. 3:** Plot of isotropic pressure ( $p$ ) vs time  $t$ .



**Fig. 4:** Plots of string tension density ( $\lambda$ ) vs time  $t$ .



**Fig. 5:** Plot of particle density ( $\rho_p$ ) vs time  $t$ .



**Fig. 6:** Plots of particle density ( $\rho_p$ ) and string tension density ( $\lambda$ ) vs time  $t$ .

Expressions for scalar field ( $\phi$ ), effective pressure ( $p^*$ ), energy density ( $\rho$ ), isotropic pressure ( $p$ ), coefficient of bulk viscosity ( $\xi$ ), string tension density ( $\lambda$ ) and particle density ( $\rho_p$ ) for model (20) are given by

$$\phi = \left[ \frac{r+2}{2} \left( \phi_0 \int \frac{dt}{(\sinh(\alpha t))^{\frac{3}{n}}} + \phi_1 \right) \right]^{\frac{2}{r+2}}, \quad (28)$$



$$p^* = -\frac{3m(5m-2)\alpha^2}{n^2(2m+1)^2} \coth^2(\alpha t) - \frac{6m\alpha^2}{n(2m+1)} \left[ \left( \frac{1-n}{n} \right) \coth^2(\alpha t) + 1 \right] \\ + \frac{3}{4} (\sinh(\alpha t))^{\frac{6(1-2m)}{n(1+2m)}} + \frac{\omega\phi_0^2}{2(\sinh(\alpha t))^{\frac{6}{n}}}, \quad (29)$$

$$\rho = \frac{9m(m+2)\alpha^2}{n^2(2m+1)^2} \coth^2(\alpha t) - \frac{1}{4} (\sinh(\alpha t))^{\frac{6(1-2m)}{n(1+2m)}} + \frac{\omega\phi_0^2}{2(\sinh(\alpha t))^{\frac{6}{n}}}, \quad (30)$$

$$p = \frac{9m(m+2)\gamma\alpha^2}{n^2(2m+1)^2} \coth^2(\alpha t) - \frac{\gamma}{4} (\sinh(\alpha t))^{\frac{6(1-2m)}{n(1+2m)}} + \frac{\omega\phi_0^2\gamma}{2(\sinh(\alpha t))^{\frac{6}{n}}}, \quad (31)$$

$$\xi = \frac{[m^2(3\gamma+5) + 2m(3\gamma-1)]\alpha}{n(2m+1)^2} \coth(\alpha t) - \frac{n(\gamma+3)}{12\alpha} \tanh(\alpha t) (\sinh(\alpha t))^{\frac{6(1-2m)}{n(1+2m)}} \\ + \frac{(\gamma-1)\omega\phi_0^2 n}{6\alpha} \frac{\tanh(\alpha t)}{(\sinh(\alpha t))^{\frac{6}{n}}} + \frac{2m\alpha}{2m+1} \left[ \left( \frac{1-n}{n} \right) \coth(\alpha t) + \tanh(\alpha t) \right], \quad (32)$$

$$\lambda = \frac{3(1-m)(3-n)\alpha^2}{n^2(2m+1)} \coth^2(\alpha t) + \frac{3(1-m)\alpha^2}{n(2m+1)} + (\sinh(\alpha t))^{\frac{6(1-2m)}{n(1+2m)}}, \quad (33)$$

$$\rho_p = \frac{[m^2(9-2n) + m(3+n) + (n-3)]\alpha^2}{n^2(2m+1)^2} \coth^2(\alpha t) \\ - \frac{5}{4} (\sinh(\alpha t))^{\frac{6(1-2m)}{n(1+2m)}} - \frac{3(1-m)\alpha^2}{n(2m+1)} + \frac{\omega\phi_0^2}{2(\sinh(\alpha t))^{\frac{6}{n}}}. \quad (34)$$

Above solutions satisfy energy conservation Eq. (13) identically. Hence our solution is exact. From Eq. (22), it can be seen that spatial volume is zero at  $t = 0$  and it increases with time. This shows that universe starts evolving with zero volume at  $t = 0$  and expand with cosmic time. From Eq. (21), we observe that when  $t \rightarrow 0$ , expansion scalar  $\theta$  becomes infinity which indicates inflationary scenario. From this analysis we conclude that it is choice of scale factor that makes the model inflationary at early stage of the universe. The model has a point type singularity [74] at  $t = 0$ . The shear scalar diverge at  $t = 0$ . The scale factors  $A(t)$  and  $B(t)$  tend to zero as  $t \rightarrow 0$  and tend to infinity as  $t \rightarrow \infty$ . The anisotropy parameter is uniform throughout the whole expansion of the universe when  $m \neq \frac{1}{2}$  but for  $m = -\frac{1}{2}$ , it tends to infinity. This shows that the universe is expanding with increase of cosmic time but the rate of expansion and shear scalar decrease to zero and tend to isotropic. Cosmological evolution of LRS Bianchi type-II space-time is expansionary, with all three scale factors monotonically increasing function of time.

From Eq. (30), it is observed that the rest energy density  $\rho$  is a decreasing function of time and  $\rho > 0$  always. The rest energy density has been graphed versus time in Figure 2. It is evident that the rest energy density remains positive in mode of evolution.

Figure 3 depicts the variation of pressure versus time for  $\gamma = 0, \frac{1}{3}, 1$ . We observe that in empty space the pressure is zero. In radiating dominated model ( $\gamma = \frac{1}{3}$ ) and in stiff fluid model ( $\gamma = 1$ ), pressure is a positive decreasing function of time and becomes negligible at late time.

From Eq. (33), it is observed that the string tension density  $\lambda$  is a decreasing function of time and  $\lambda > 0$  always. Figure 4 shows the plots of string tension density versus time for  $n = 0.5$  and 2. It is evident that the  $\lambda$  remains positive in both cases but it decreases more sharply with the cosmic time for greater value of  $n$ . In early phase of universe, the string tension density of both cases will dominate the dynamics and later time it approaches to zero.

From Eq. (34), it is evident that the particle density  $\rho_p$  is a decreasing function of time and  $\rho_p > 0$  for all time. Figure 5 shows the plots of particle density versus time for  $n = 0.5$  and 2. It is evident that the  $\rho_p$  remains positive in both cases but it decreases more sharply with the cosmic time for greater value of  $n$ .

Figure 6 demonstrates the comparative behaviour of variation of  $\rho_p$  and  $\lambda$  versus the cosmic time for  $n = 0.5$ . In this case, we observe that  $\rho_p < \lambda$ . Therefore, according to Kibble [2] and Krori *et al.* [75], strings dominate the universe evolving with acceleration. If this is so, we should have some signature of massive string at present epoch of the observations. However, it is not been seen so far. Further it is observed that for sufficiently large times, the  $\rho_p$  and  $\lambda$  tend to zero. Therefore, the strings disappear from the universe at late time (i.e. present epoch). The same is predicted by the current observations.

#### 4. Concluding Remarks

In this paper, a spatially homogeneous and anisotropic L R S Bianchi-II models representing massive strings in general relativity has been studied. Generally the model represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic. The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. We also observe that Murphy's conclusion [76] about the absence of a big bang type singularity in the infinite past in models with bulk viscous

fluid, in general, is not true. The results obtained in Myung and Cho [77] also show that, it is, in general, not valid, since for some cases big bang singularity occurs in finite past.

The strings dominate in the early universe and eventually disappear from the universe for sufficiently large times. At early universe, the possible occupation of cosmic strings is not allowed to exceed over 10% due to constraints of latest CMB data. At late time evolution, the strings become negligible even then still play an important role in astronomical experiments. Recent results from the PAMELA [78] and ATIC [79] experiments have indicated an excess power of cosmic ray positron flux compared to what is predicted from astrophysical backgrounds alone. Recently, Brandenberger *et al.* [80] have studied cosmic ray positron from cosmic strings showing that very few leptonic cosmic strings could decay into leptons and may be applied to explain recently discovered positron anomaly by Pamela data.

For different choice of  $n$ , we can generate a class of string models in L R S Bianchi type-II universe. It is observed that such string models are also in good harmony with current observations.

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