J. T. S.
Special Vol. 8 (2014), pp.53-63
https://doi.org/10.56424/jts.v8i01.10558

A Note on Transversal hypersurfaces of Lorentzian para-Sasakian manifolds with a Semi-Symmetric Non-Metric Connection

Arjumand Ahmad and Shamsur Rahman

Department of Mathematics
Maulana Azad National Urdu University
Polytechnic, Darbhanga (Branch)- 846001, Bihar, India
e-mail: aarrjjuummaanndd@gmail.com, shamsur@rediffmail.com
(Received: November 11, 2013)

Abstract

Transversal hypersurfaces of Lorentzian para-Sasakian manifold are defined. It is proved that the fundamental 2-form on the transversal hypersurfaces of Lorentzian para-Sasakian manifold with (f, g, u, v, λ)-structure are closed. In this paper it is shown that transversal hypersurfaces of Lorentzian para-Sasakian manifold admits a product structure with a semi symmetric non metric connection. It is shown that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection are closed.

Keywords and Phrases : Lorentzian almost paracontact manifolds, Almost product metric structure, Transversal hypersurfaces, Lorentzian almost paracontact Sasakian manifolds, Quarter Semi-symmetric non metric connection.
2000 AMS Subject Classification : 53C25, 53C40.

1. Introduction

Let ∇ be a linear connection in an n-dimensional differentiable manifold M. The torsion tensor T and the curvature tensor R of ∇ are respectively given by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y]$$

$$R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$$

The connection ∇ is symmetric if the torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is metric if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection. In ([1], [5]) A. Friedmann and J.A. Schouten introduced the idea of a semi-symmetric linear connection in a differentiable manifold. A linear connection is said to be a semi-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = u(Y)\phi X - u(X)\phi Y,$$

where u is a 1-form. M. M. Tripathi [10] studied the semi-symmetric metric connection in a Kenmotsu manifolds. In [11], the semi-symmetric non-metric connection in a Ken-motsu manifold was studied by M. M. Tripathi and N. Nakkar. Also in [12], M. M. Tripathi proved the existence of a new connection and he showed that in particular cases, this connection reduces to semi-symmetric connections; even some of them are not introduced so far. On the other hand, there is a class of almost paracontact metric manifolds, namely Lorentzian para-Sasakian manifolds. In 1989, K. Matsumoto[6], introduced the notion of Lorentzian para-Sasakian manifold. Then I. Mihai and R. Rosca [3] introduced the same notion independently and they obtained several results on this manifolds. Lorentzian para-Sasakian manifolds have also been studied by K. Matsumoto and I. Mihai [7], U. C. De and et al.,[16], A. A. Shaikh and S. Biswas [2], M. M. Tripathi and U. C. De [13]. S. Tanno [15] gave a classification for connected almost contact metric manifolds whose automorphism groups have the maximum dimension. For such a manifold, the sectional curvature of plane sections containing xi is a constant, say c. He showed that they can be divided into three classes: (1) Homogenous normal contact Riemannian manifolds with $c > 0$, (2) global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$ and (3) a warped product space $R \times f^{C^n}$ if $c < 0$. It is known that the manifolds of class (1) are characterized by some tensorial relations admitting a contact structure. Kenmotsu [6] characterized the differential geometric properties of the third case by tensor equation $(\bar{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$. The structure so obtained is now known as Kenmotsu structure. In general, these structures are not Sasakian [6]. Yano and Okumura [4] introduced (f, g, u, v, λ) structure on a manifold. In [17], S. Rahman studied Transversal Hypersurfaces of Almost Hyperbolic contact manifolds endowed with semi symmetric non metric connection respectively. Transversal hypersurfaces is a hypersurface which never contain the vector field ξ defining the almost contact structure. It is well known that on a transversal hypersurface of almost contact metric manifold there exist a (f, g, u, v, λ) structure.

The paper is organized as follows: In section 2, we give a brief introduction to Lorentzian para-Sasakian manifolds. In Section 3, It is proved that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection admits an almost product structure and each transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection admits an almost product semi-Riemannian structure. The fundamental 2-form on the transversal hypersurfaces of Lorentzian para-Sasakian manifold with Lorentzian para-contact structure (f, g, u, v, λ) are closed. It is also proved that transversal hypersurfaces of Lorentzian para-Sasakian manifold with a semi symmetric non metric connection admits a product structure. Some properties of transversal hypersurfaces of Lorentzian para-Sasakian manifold are closed.

2. Lorentzian para-Sasakian manifolds

Let $\bar{M}$ be a n-dimensional almost contact metric manifold with almost contact metric stucture (ϕ, ξ, η, g) such that

$$\phi^2 X = X + \eta(X)\xi, \quad \eta(\xi) = -1 \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y) \tag{2.2}$$

$$g(X, \xi) = \eta(X) \tag{2.3}$$

$$g(\phi X, Y) = g(X, \phi Y) = \psi(X, Y) \tag{2.4}$$

for vector fields X,Y tangent to $\bar{M}$. Then the structure (ϕ, ξ, η, g) is termed as Lorentzian para-contact structure.
Also in a Lorentzian para-contact structure the following relations hold:

$$\phi\xi = 0, \quad \eta(\phi X) = 0, \quad and \quad rank(\phi) = n - 1$$

A Lorentzian para-contact manifold $\bar{M}$ is called Lorentzian para-Sasakian (LP-Sasakian) manifold if

$$(\bar{\bar{\nabla}}_X \phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi \tag{2.5}$$

$$\bar{\bar{\nabla}}_X \xi = \phi X \tag{2.6}$$

for all vector fields X,Y tangent to $\bar{M}$ where $\bar{\bar{\nabla}}$ is the Riemannian connection with respect to g. On other hand, a semi symmetric non metric connection $\bar{\bar{\nabla}}$ on M is defined by

$$\bar{\nabla}_X Y = \bar{\bar{\nabla}}_X Y + \eta(Y)X \tag{2.7}$$

Using (2.7) in (2.5) , we get respectively

$$(\bar{\nabla}_X \phi)Y = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi - \eta(Y)\phi X \tag{2.8}$$

$$\bar{\nabla}_X \xi = \phi X - X \tag{2.9}$$

3. Main results

Let M be a hypersurface of an almost hyperbolic contact manifold $\bar{M}$ equipped with an almost hyperbolic contact structure (ϕ, ξ, η). We assume that the structure vector field ξ never belongs to tangent space of the hypersurface M, such that a hypersurface is called a transversal hypersurface of an almost contact manifold. In this case the structure vector field ξ can be taken as an affine normal to the hypersurface. Vector field X on M and ξ are linearly independent, therefore we may write

$$\phi X = F(X) + \omega(X)\xi \tag{3.1}$$

where F is a $(1,1)$ tensor field and ω is a 1-form on M. From (3.1) we have

$$\phi\xi = F\xi + \omega(\xi)\xi$$

or

$$0 = F\xi + \omega(\xi)\xi$$

$$\phi^2 X = F(\phi X) + \omega(\phi X)\xi \tag{3.2}$$

$$X + \eta(X)\xi = F(FX + \omega(X)\xi) + \omega(FX + \omega(X)\xi)\xi$$

$$X + \eta(X)\xi = F^2 X + (\omega \circ F)(X)\xi \tag{3.3}$$

Taking account of equation (3.3) we get

$$F^2 X = X \tag{3.4}$$

$$F^2 = I \tag{3.5}$$

$$\eta = \omega \circ F$$

Thus we have

Theorem 3.1. Each transversal hypersurface of a Lorentzian almost paracontact manifold endowed with a semi symmetric non metric connection admits an almost product structure F and a 1-form ω.

From (3.4) and (3.5), it follows that

$$\eta = \omega \circ F$$

$$\eta(FX) = (\omega \circ F)FX$$

$$\eta(FX) = \omega(F^2 X)$$

$$(\omega \circ F)X = \omega(X)$$

$$\omega = \eta \circ F \tag{3.6}$$

Now, we assume that $\overline{M}$ admits a Lorentzian almost paracontact metric structure (ϕ, ξ, η, g) endowed with a semi symmetric non metric connection. We denote by g the induced metric on M also. Then for all $X, Y \in TM$, we obtain

$$g(FX, FY) = g(X, Y) + \eta(X)\eta(Y) + \omega(X)\omega(Y) \tag{3.7}$$

We define a new metric G on the transversal hypersurface given by

$$G(X, Y) = g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y) \tag{3.8}$$

So,

$$\begin{aligned} G(FX, FY) &= g(FX, FY) + \eta(FX)\eta(FY) \\ &= g(X, Y) + \eta(X)\eta(Y) - \omega(X)\omega(Y) + (\eta \circ F)(X)(\eta \circ F)(Y) \\ &= g(X, Y) + \eta(X)\eta(Y) - \omega(X)\omega(Y) + \omega(X)\omega(Y) \\ &= g(X, Y) + \eta(X)\eta(Y) = G(X, Y) \end{aligned}$$

Then we get

$$G(FX, FY) = G(X, Y)$$

where equations (3.4), (3.6), (3.7) and (3.8) are used. Then G is Lorentzian metric on M that is (F, G) is an almost product Lorentzian structure on the transversal hypersurface M of $\overline{M}$.

Thus, we are able to state the following

Theorem 3.2. Each transversal hypersurface of Lorentzian almost paracontact manifold endowed with a semi symmetric non metric connection admits an almost product Lorentzian structure.

We now assume that M is orientable and choose a unit vector field N of $\overline{M}$, normal to M. Then Gauss and Weingarten formulae of semi symmetric non metric connection are given respectively by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y)N, \qquad X, Y \in TM, \tag{3.10}$$

$$\overline{\nabla}_X N = -HX + \lambda X \tag{3.11}$$

where $\overline{\nabla}$ and ∇ are respectively the Levi-Civita connections in $\overline{M}$ and M, and h is the second fundamental form related to H by

$$h(X, Y) = g(HX, Y) \tag{3.12}$$

For any vector field X tangent to M, defining

$$\phi X = fX + u(X)N \tag{3.13}$$

$$\phi N = -U \tag{3.14}$$

$$\xi = V + \lambda N \tag{3.15}$$

$$\eta(X) = v(X)$$
$$\lambda = \eta(N) = g(\xi, N) \tag{3.16}$$

for $X \in TM$ we get an induced Lorentzian (f, g, u, v, λ)-structure on the transversal hypersurface such that

$$f^2 = I + u \otimes V + v \otimes U \tag{3.17}$$

$$fU = -\lambda V, \qquad fV = \lambda U \tag{3.18}$$

$$u \circ f = \lambda v, \qquad v \circ f = -\lambda u \tag{3.19}$$

$$u(U) = -1 - \lambda^2, \qquad u(V) = v(U) = 0, \qquad v(V) = -1 - \lambda^2 \tag{3.20}$$

$$g(fX, fY) = -g(X, Y) - u(X)u(Y) - v(X)v(Y) \tag{3.21}$$

$$g(X, fY) = -g(fX, Y), \qquad g(X, U) = u(X), \qquad g(X, V) = v(X) \tag{3.22}$$

for all $X, Y \in TM$ where

$$\lambda = \eta(N) \qquad (3.23).$$

Thus, we see that every transversal hypersurface of an Lorentzian almost paracontact metric manifold endowed with a semi symmetric non metric connection also admits a Lorentzian (f, g, u, v, λ)-structure.

Next we find relation between the induced almost product metric structure (F, G) and the induced Lorentzian (f, g, u, v, λ)-structure on the transversal hypersurface of an Lorentzian almost paracontact metric manifold endowed with a semi symmetric non metric connection. In fact, we have the following

Theorem 3.3. Let M be a transversal hypersurface of an Lorentzian almost para contact manifold $\overline{M}$ equipped with Lorentzian almost para contact with a semi symmetric non metric connection with structure (ϕ, ξ, η, g) and induced almost product metric structure (F, G).

Then we have

$$\lambda\omega = u, \tag{3.24}$$

$$F = f - \frac{1}{\lambda} u \otimes V, \tag{3.25}$$

$$FU = \frac{1}{\lambda} V, \tag{3.26}$$

$$u \circ F = u \circ f = \lambda v, \tag{3.27}$$

$$FV = fV = \lambda U, \tag{3.28}$$

$$u \circ F = \frac{1}{\lambda} u. \tag{3.29}$$

Proof.

$$\phi X = FX + \omega(X)\,\xi$$

$$\xi = V + \lambda N$$
$$\phi X = FX + \omega(X)V + \lambda\omega(X)N, \tag{3.30}$$
$$\phi X = fX + u(X)N. \tag{3.31}$$

From equation (3.30) and (3.31) we have

$$\lambda\omega X = u(X), \qquad \omega(X) = \frac{1}{\lambda}u(X),$$
$$FX = fX - \omega(X)V,$$
$$FX = f - \frac{1}{\lambda}u(X)V$$
$$F = f - \frac{1}{\lambda}u \otimes V$$

which is equation (3.25).

$$(uoF)(X) = (uof)(X) - \frac{1}{\lambda}u(X)u(V), \quad u(V) = 0,$$
$$uoF = uof = \lambda v$$

which gives equation (3.27).

$$FU = fV - \frac{1}{\lambda}u(v)V$$
$$FU = -\lambda V - \frac{1}{\lambda}(-1-\lambda^2)V = \frac{1}{\lambda}V$$
$$FU = \frac{1}{\lambda}V$$

which gives equation (3.26)

$$(uoF)(X) = (uof)(X) - \frac{1}{\lambda}u(X)u(V)$$
$$= (uof)(X) - \frac{1}{\lambda}u(X)\left(-1-\lambda^2\right)$$
$$= -\lambda u(X) + \frac{1}{\lambda}u(X) + \lambda(X)$$
$$= \frac{1}{\lambda}u(X)$$
$$uoF = \frac{1}{\lambda}u$$

,

$$FV = fV - \frac{1}{\lambda}u(V)V = fV = \lambda U$$

which is equation (3.28) here equations (3.18),(3.19),(3.20),(3.21),(3.22),(3.23) are used.

Lemma 3.4. Let M be a transversal hypersurface with Lorentzian (f, g, u, v, λ)-structure of a Lorentzian almost para contact manifold $\overline{M}$ endowed with a semi symmetric non metric connection. Then

$$(\overline{\nabla}_X \phi)Y = ((\nabla_X f)Y - u(Y)HX + \lambda X u(Y) + h(X, Y)N + h(X, fY)N \\ +(\nabla_X u)(Y)N \tag{3.32}$$

$$\overline{\nabla}_X \xi = \nabla_X V - \lambda HX + \lambda^2 X + [h(X, V) + X(\lambda)]N \tag{3.33}$$

$$(\overline{\nabla}_X \phi)N = -\nabla_X U + fHX - f\lambda X - [h(X, U) + \lambda u(X) - \mu(HX)]N \tag{3.34}$$

$$(\overline{\nabla}_X \eta)Y = (\nabla_X v)Y + h(X, Y)\lambda \tag{3.35}$$

for all $X, Y \in TM$. The proof is straight forward and hence ommited.

Theorem 3.5. Let M be a transversal hypersurfaces with Lorentzian (f, g, u, v, λ)-structure of a Lorentzian cosymplectic manifold $\overline{M}$ endowed with a semi symmetric non metric connection. Then

$$(\nabla_X f)Y = u(Y)HX - \lambda X u(Y) \tag{3.36}$$

$$(\nabla_X u)Y = -h(X, Y) - h(X, fY)\lambda \tag{3.37}$$

$$\nabla_X V = \lambda HX - \lambda^2 X \tag{3.38}$$

$$h(X, V) = -X\lambda \tag{3.39}$$

$$\nabla_X U = fHX - f\lambda X, \qquad h(X, U) = \mu(HX) - \lambda u(X) \tag{3.40}$$

$$\nabla_X v)Y = -h(X, Y)\lambda \tag{3.41}$$

for all $X, Y \in TM$.

Proof. Using (2.8), (3.12), (3.15) in (3.31), we obtain

$$((\nabla_X f)Y - u(Y)HX + \lambda X u(Y) + h(X, Y)N + h(X, fY)N + (\nabla_X u)(Y)N = 0$$

Equating tangential and normal parts in the above equation, we get (3.36) and (3.37) respectively. Using (2.9) and (3.15) in (3.33), we have

$$\nabla_X V - \lambda HX + \lambda^2 X + [h(X, V) + X(\lambda)]N = 0$$

Equating tangential and normal parts we get (3.38) and (3.39) respectively. Using (2.8), (3.14) and (3.15) in (3.34) Using (2.8), (3.14) and (3.15) in (3.33) and equating tangential, we get (3.40). In the last (3.41) follows from (3.35).

Theorem 3.6. If M be a transversal hypersurface with Lorentzian (f, g, u, v, λ)-structure of a Lorentzian cosymplectic manifold endowed with a semi symmetric non metric connection,then the 2- form Φ on M is given by

$$\Phi(X, Y) \equiv g(X, fY)$$

is closed.

Proof. From (3.36) we get

$$(\nabla_X \Phi)(Y, Z) = h(X, Y)u(Z) - h(X, Z)u(Y),$$

$$(\nabla_X \Phi)(Y, Z) + (\nabla_Y \Phi)(Z, X) + (\nabla_Z \Phi)(X, Y) = 0.$$

Hence the theorem is proved.

Theorem 3.7. Let M be a transversal hypersurface with Lorentzian (f, g, u, v, λ)-structure of a Lorentzian para Sasakian manifold $\overline{M}$ endowed with a semi symmetric non metric connection. Then

$$(\nabla_X f)Y = g(X, Y)V + \eta(Y)X + 2\eta(X)\eta(Y)V - \eta(Y)fX + u(Y)HX - \lambda X u(Y) \tag{3.42}$$

$$(\nabla_X u)Y = g(X, Y)\lambda + 2\eta(X)\eta(Y)\lambda - h(X, fY) - \eta(Y)u(X) - h(X, Y) \tag{3.43}$$

$$\nabla_X V = -\lambda^2 X + HX\lambda - X + fX \tag{3.44}$$

$$h(X, V) = u(X) - X(\lambda) \tag{3.45}$$

$$\nabla_X U = -2\eta(X)\lambda V + fHX - \lambda X \tag{3.46}$$

$$h(X, U) = u(HX) - \lambda u(X) + 2\eta(X)\lambda^2 - u(X)\lambda \tag{3.47}$$

for all $X, Y \in TM$.

Proof. Using (2.8), (3.13), (3.15) in (3.32), we obtain

$$g(X, Y)V + \eta(Y)X + 2\eta(X)\eta(Y)V - \eta(Y)fX$$

$$+[g(X, Y)\lambda + 2\eta(X)\eta(Y)\lambda - u(X)\eta(Y)]N$$

$$= (\nabla_X f)Y - u(Y)HX + \lambda X u(Y) + [h(X, Y) + h(X, fY) + (\nabla_X u)(Y)]N$$

Equating tangential and normal parts in the above equation, we get (3.42) and (3.43) respectively. Using (2.9) and (3.15) in (3.33), we have

$$fX - X + u(X)N = \nabla_X V - HX\lambda + \lambda^2 X + [h(X, V) + X(\lambda)]N$$

Equating tangential and normal parts we get (3.44) and (3.45) respectively. Using (2.8), (3.14) and (3.15) in (3.33) and equating tangential parts, we get (3.46) in the last (3.47) follows from (3.34).

Theorem 3.8. If M is transversal hypersurface with Lorentzian (f, g, u, v, λ)-structure of a Lorentzian para Sasakian manifold endowed with a semi symmetric non metric connection, $\overline{M}$. Then Φ on M given by

$$\Phi(X, Y) = g(X, fY)$$

satisfying

$$\begin{aligned}
&(\nabla_X\Phi)(Y,Z)+(\nabla_Y\Phi)(Z,X)+(\nabla_Z\Phi)(X,Y)\\
&=g(X,Z)v(Y)+g(Y,X)v(Z)+g(Z,Y)v(X)+\eta(Z)g(Y,X)\\
&+\eta(X)g(Z,Y)+\eta(Y)g(X,Z)+2\eta(X)v(Y)\eta(Z)+2\eta(X)\eta(Y)v(Z)\\
&+2v(X)\eta(Y)\eta(Z)-u(Z)g(Y,fX)-u(X)g(Z,fY)-u(Y)g(X,fZ)\\
&+u(Z)h(X,Y)+u(X)h(Y,Z)+u(Y)h(Z,X)-\lambda u(Z)g(Y,X)\\
&-\lambda u(X)g(Z,Y)-\lambda u(Y)g(X,Z).
\end{aligned}$$

We have

$$(\nabla_X\Phi)(Y,Z)=g(Y,(\nabla_X f)Z) \tag{3.48}$$

$$\begin{aligned}
&=g(Y,(g(X,Z)V+\eta(Z)X+2\eta(X)\eta(Z)V-\eta(Z)fX+u(Z)HX-\lambda Xu(Z)\\
&=g(X,Z)g(Y,V)+\eta(Z)g(Y,X)+2\eta(X)\eta(Z)g(Y,V)-\eta(Z)g(Y,fX)\\
&+u(Z)g(Y,HX)-\lambda u(Z)g(Y,X)=g(X,Z)v(Y)+\eta(Z)g(Y,X)\\
&+2\eta(X)\eta(Z)v(Y)-\eta(Z)g(Y,fX)+u(Z)h(X,Y)-\lambda u(Z)g(Y,X).
\end{aligned}$$

Similarly we obtain,

$$\begin{aligned}
(\nabla_Y\Phi)(Z,X)&=g(Y,X)v(Z)+\eta(X)g(Z,Y)+2\eta(Y)\eta(X)v(Z) \qquad (3.49)\\
&\quad-\eta(X)g(Z,fY)+h(Y,Z)u(X)-\lambda u(X)g(Z,Y)\\
(\nabla_Z\Phi)(X,Y)&=g(Z,Y)v(X)+\eta(Y)g(X,Z)+2\eta(X)\eta(Y)v(X) \qquad (3.50)\\
&\quad-\eta(Y)g(X,fZ)+h(Z,X)u(Y)-\lambda u(Y)g(X,Z).
\end{aligned}$$

Adding equations (3.48), (3.49) and (3.50), we obtain

$$\begin{aligned}
&(\nabla_X\Phi)(Y,Z)+(\nabla_Y\Phi)(Z,X)+(\nabla_Z\Phi)(X,Y)\\
&=g(X,Z)v(Y)+g(Y,X)v(Z)+g(Z,Y)v(X)+\eta(Z)g(Y,X)\\
&+\eta(X)g(Z,Y)+\eta(Y)g(X,Z)+2\eta(X)v(Y)\eta(Z)+2\eta(X)\eta(Y)v(Z)\\
&+2v(X)\eta(Y)\eta(Z)-u(Z)g(Y,fX)-u(X)g(Z,fY)-u(Y)g(X,fZ)\\
&+u(Z)h(X,Y)+u(X)h(Y,Z)+u(Y)h(Z,X)-\lambda u(Z)g(Y,X)\\
&-\lambda u(X)g(Z,Y)-\lambda u(Y)g(X,Z).
\end{aligned}$$

References

[1] **Friedmann, A. and Schouten, J. A. :** Über die Geometrie der halbsymmetrischen Ubertragungen, Math. Z., 21 No.1 (1924), 211-223.

[2] **Shaikh, A. A. and Biswas, S. :** On LP-Sasakian manifolds, Bull. of the Malaysian Math. Sci. Soc., 27 (2004), 17-26.

[3] **Mihai,I. and Rosca, R. :** On Lorentzian P-Sasakian manifolds, Classical Analysis, World Scientific Publ., (1992), 155-169.

[4] **Mihai, I., Shaikh, A. A. and De, U. C. :** On Lorentzian para-Sasakian manifolds, Rendi-conti del Seminario Matematico di Messina, Serie II, 1999.

[5] **Schouten, J. A. :** Ricci Calculus, Springer, 1954.

[6] **Matsumoto, K. :** On Lorentzian para contact manifolds, Bull. of Yamagata Univ. Nat. Sci., 12 (1989), 151-156.

[7] **Matsumoto, K. and Mihai, I. :** On a certain transformation in Lorentzian para Sasakian manifold, Tensor N. S., 47 (1988), 189-197.

[8] **Kenmotsu, K. :** A class of almost contact Riemannian manifolds, Tohoku Math. J., 24 (1972) 93-103.

[9] **Yano K. and Okumura, M. :** Kodai Math. Sem. Rep., 22 (1970) 401-23.

[10] **Tripathi, M. M. :** On a semi-symmetric metric connection in a Kenmotsu manifold, J. Pure Math. 16 (1999), 67-71.

[11] **Tripathi, M. M. and Nakkar, N. :** On a semi-symmetric non-metric connection in a Kenmotsu manifold, Bull. Cal. Math. Soc., 16 No. 4 (2001), 323-330.

[12] **Tripathi, M. M. :** A new connection in a Riemannian manifold, International Electronic Journal of Geometry, 1, 1 (2008), 15-24.

[13] **Tripathi, M. M. and De, U. C. :** Lorentzian almost paracontact manifolds and their sub-manifolds, Journal of the Korean Society of Mathematical Education, 2 (2001), 101-125.

[14] **Prasad R. and Tripathi, M. M. :** Transversal hyper surface of Kenmotsu manifold, Indian J. pure appl. Math., 34 No.3 (2003), 443-452.

[15] **Tanno, S. :** The automorphism groups of almost contact Riemannian manifolds, Tohoku Math. J., 21 (1969), 21-38.

[16] **De, U. C., Matsumoto, K. and Shaikh, A. A. :** On Lorentzian Para-Sasakian manifolds, Ren-dicontidel Seminaro Matemetico di Messina, Serie II, Supplemento al, 3 (1999), 149-158.

[17] **Rahman, S. :** Transversal Hypersurfaces of Almost Hyperbolic contact manifolds endowed with semi symmetric non metric connection. Turkic World Mathematical Society Journal of Pure and Applied Mathematics, 3 No. 2 (2012), 210-219.