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τ-Curvature On Kenmotsu Manifold

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Abstract

In the present paper we have obtained the necessary and sufficient condition for a extended generalized τ-ϕ-recurrent Kenmotsu manifold to be a generalized ricci-recurrent manifold. Furthermore, we have studied τ-ϕ-symmetric Kenmotsu manifold, τ-ξ-flat Kenmotsu manifold and a Kenmotsu manifold satisfying $\tau(X,Y)\cdot R = 0$.

Keywords and Phrases : τ-curvature tensor, τ-ϕ-symmetric Kenmotsu manifold, extended generalized τ-ϕ-recurrent Kenmotsu manifold, η-Einstein manifold, τ-ξ-flat Kenmotsu manifold.
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Introduction

As is well known, symmetric spaces plays an important role in differential geometry. The work on local symmetric Riemannian manifolds began by Cartan [2]. This property of a Riemannian manifold has been weakened by many authors [[22], [18], [3], [7], [19], [16], [17]] in several directions such as recurrent manifolds, semi-symmetric manifolds, pseudo-symmetric manifolds, weakly symmetric manifolds. Further, ϕ-recurrent, generalized ϕ-recurrent, extended generalized ϕ-recurrent manifolds were introduced and studied by many geometers.

In 1979 Dubey [8] introduced the notion of generalized recurrent manifold and then such a manifold was studied by De and Guha [6]. The manifold M, $n > 2$, is called generalized recurrent [8] if its curvature tensor R of type (1,3) satisfies the condition $\nabla R = A \otimes R + B \otimes G$, where G is a tensor of type (1,

3) given by $G(X,Y)Z = g(Y,Z)X - g(X,Z)Y$, A and B are nowhere vanishing unique 1-forms, defined by $A(\cdot) = g(\cdot,\rho_1)$ and $B(\cdot) = g(\cdot,\rho_2)$, respectively for all vector fields X, Y, $Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of all smooth vector fields on M and ∇ is the Levi-Civita connection.

In 1952 Patterson [15] introduced Ricci-recurrent manifold. According to Patterson [15] manifold (M,g) of dimension n, is called a Ricci-recurrent if $(\nabla_X S)(Y,Z) = A(X)S(Y,Z)$, for some 1-form A. Ricci-recurrent manifold has been studied by many authors. An extended version of ricci-recurrent manifold is the generalized Ricci-recurrent manifold. A non-flat Riemannian manifold called generalized Ricci-recurrent [5] if its Ricci tensor S of type (0, 2) satisfies the condition $\nabla S = A \otimes S + B \otimes g$, where A and B are two non zero 1-forms. In particular, if $B = 0$, then the manifold reduces to Ricci-recurrent manifold.

Our work is structured as follows: The first section is a very brief review of Kenmotsu manifold and a τ-curvature tensor. The next section is devoted to the study of τ-ϕ-symmetric kenmotsu manifold for two cases $\tau = 0$ and $\tau \neq 0$. For $\tau = 0$, the Kenmotsu manifold is τ-ϕ-symmetric provided either r is a constant or $a_7 = 0$, this is obviously true if the manifold is W_i flat$(i = 0,\cdots 9)$ or W_j^* flat $(i = 0,1)$ or conhormonically flat or projectively flat or M-projectively flat. For $\tau \neq 0$ case it is shown that any two conditions of (i) M^{2n+1} is ϕ-τ-symmetric, (ii) M^{2n+1} is ϕ-symmetric, (iii) either $a_7 = 0$ or r is constant, are true then the remaining statement holds. In Section 3, we have proved that the extended generalized τ-ϕ-recurrent Kenmotsu manifold is generalized Ricci-recurrent manifold with $a_0 + 2na_1 + a_2 + a_3 \neq 0$ and viceversa. Finally, it is shown that τ-ξ-flat Kenmotsu manifold is a η-Einstein manifold provided $a_4 \neq 0$ and has a scalar curvature r if $a_4 = 0$ and $a_7 \neq 0$. Moreover we have proved that Kenmotsu manifold satisfying $\tau(X,Y) \cdot R = 0$ is a η-Einstein manifold.

1. Preliminaries

Kenmotsu manifold has been introduced and studied by K. Kenmotsu in 1972 [9]. They set up one of the three classes of almost contact metric manifolds whose automorphism group attains the maximum dimension [20]. For such a manifold, the sectional curvature of a plane sections containing ξ is a constant, say c. It has been studied as homogeneous normal contact Riemannian manifolds if $c > 0$. Global Riemannian products of a line or a circle with a Kahler manifold of constant holomorphic sectional curvature with $c = 0$, and a warped product space $R \times_f C_n$, if $c < 0$. Kenmotsu [9] characterized the differential geometric properties of manifold for $c < 0$ and the structure so obtained is now known as Kenmotsu structure. A Kenmotsu structure is not Sasakian. Manifolds of $c > 0$ are characterized by some tensor equations, it has a Sasakian structure. Manifolds with $c = 0$ are characterized by a tensorial relation admitting a cosymplectic structure. Kenmotsu obtained some tensorial equations to characterize manifolds of $c < 0$.

An almost contact metric manifold is a differentiable manifold M^{2n+1} endowed with a structure (ϕ, ξ, η, g) given by a tensor field ϕ of type (1,1), a vector field ξ, a 1-form η satisfying

$$\phi^2 = -I + \eta\, o\, \xi, \qquad \eta(\xi) = 1, \tag{1.1}$$

and a Riemannian metric g such that $g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$ for any vector fields X and Y. The fundamental 2-form Φ is defined by $\Phi(X, Y) = g(X, \phi Y)$for any vector fields X and Y. It is well known that contact metric manifolds are almost contact metric manifolds such that $\Phi = d\eta$.

Thus a manifold M^{2n+1} equipped with this structure is called an almost contact manifold and is denoted by $(M^{2n+1}, \phi, \xi, \eta)$. If g is a Riemannian metric on an almost contact manifold M^{2n+1} such that,

$$(\nabla_X \phi)Y = -\eta(Y)\phi X - g(X, \phi Y)\xi, \tag{1.2}$$

$$\nabla_X \xi = (X - \eta(X)\xi), \tag{1.3}$$

holds, then $(M^{2n+1}, \phi, \xi, \eta)$ is called Kenmotsu manifold. Here ∇ denotes the operator of covariant differentiation with respect to g.

In a Kenmotsu manifold M^{2n+1}, the following relations holds;

$$\eta(R(X, Y)Z) = [g(X, Z)\eta(Y) - g(Y, Z)\eta(X)], \tag{1.4}$$

$$\text{(a) } R(\xi, X)Y = [\eta(Y)X - g(X, Y)\xi], \ \text{ (b) } R(X, Y)\xi = [\eta(X)Y - \eta(Y)X], \tag{1.5}$$

$$\text{(a) } S(X, Y) = -2ng(X, Y), \ \text{ (b) } S(X, \xi) = -2n\eta(X), \ \text{ (c) } QX = -2nX, \tag{1.6}$$

$$\text{(a) } S(\xi, \xi) = -2n, \ \text{ (b) } Q\xi = -2n\xi, \tag{1.7}$$

$$(\nabla_W R)(X, Y)\xi = g(W, X)Y - g(W, Y)X - R(X, Y)W, \tag{1.8}$$

$$S(\phi X, \phi Y) = S(X, Y) + 2n\eta(X)\eta(Y). \tag{1.9}$$

In a $(2n + 1)$-dimensional Riemannian manifold M^{2n+1}, the τ-curvature tensor [10] is given by

$$\begin{aligned} \tau(X, Y)Z = {} & a_0 R(X, Y)Z + a_1 S(Y, Z)X + a_2 S(X, Z)Y + a_3 S(X, Y)Z \\ & + a_4 g(Y, Z)QX + a_5 g(X, Z)QY + a_6 g(X, Y)QZ \\ & + a_7 r\{g(Y, Z)X - g(X, Z)Y\}, \end{aligned} \tag{1.10}$$

where R, S, Q and r are the curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature respectively. In particular, τ-curvature tensor is reduces to quasi-conformal curvature tensor C*, conformal curvature tensor C, conharmonic curvature tensor L, concircular curvature tensor V, pseudo-projective curvature tensor P*, projective curvature tensor P, M-projective curvature tensor, W_i-curvature tensors $(i = 0, \cdots, 9)$ and W_j^*-curvature tensors $(j = 0, 1)$, by assigning particular values to $a_i's$ $(i = 0, 1, \cdots, 7)$ in the equation (1.10).

2. τ-ϕ-symmetric Kenmotsu manifold

Definition 2.1. A Kenmotsu manifold M^{2n+1} is said to be ϕ-symmetric [12] if the condition $\phi^2((\nabla_W R)(X, Y)Z) = 0$ holds, for all vector fields $X, Y, Z \in \chi(M)$.

Taking $\tau = 0$ in (1.10) and using (1.6), we get

$$\begin{aligned}-a_0 R(X, Y)Z = &-2n(a_1 + a_4)g(Y, Z)X - 2n(a_2 + a_5)g(X, Z)Y \\ &-2n(a_3 + a_6)g(X, Y)Z + a_7 r\{g(Y, Z)X - g(X, Z)Y\}.\end{aligned}$$

On covariant differentiation of the above equation with respect to W, and assuming that all vector fields X, Y, Z, W are orthogonal to ξ, one can get

$$-a_0((\nabla_W R)(X, Y)Z) = -a_7 dr(W)g(Y, Z)X + a_7 dr(W)g(X, Z)Y.$$

i.e.,

$$((\nabla_W R)(X, Y)Z) = \frac{a_7}{a_0} dr(W)\{g(X, Z)Y - g(Y, Z)X\}.$$

Applying ϕ^2 on both sides of the above equation and using (1.1), we get

$$\phi^2((\nabla_W R)(X, Y)Z) = \frac{a_7}{a_0} dr(W)\{g(X, Z)Y - g(Y, Z)X\}.$$

Therefore we can state;

Theorem 2.1. A τ-flat Kenmotsu manifold is ϕ-symmetric provided either r is constant or $a_7 = 0$.

From Theorem (2.1), we have the following corollary;

Corollary 2.1. A Kenmotsu manifold is ϕ-symmetric if either r is constant or manifold is W_i flat $(i = 0, \cdots 9)$ or W_j^* flat $(i = 0, 1)$ or conhormonically flat or projectively flat or M-projectively flat.

And if $\tau \neq 0$, then we arrive at

$$\phi^2((\nabla_W \tau)(X, Y)Z) = \phi^2((\nabla_W R)(X, Y)Z) + \frac{a_7}{a_0} dr(W)\{g(Y, Z)X - g(X, Z)Y\}.$$

And so we can state;

Theorem 2.2. If in a Kenmotsu manifold M^{2n+1} any two of the following statements hold then the remaining statement holds
(a) M^{2n+1} is ϕ-τ-symmetric.
(b) M^{2n+1} is ϕ-symmetric.
(c) Either $a_7 = 0$ or r is constant.

3. Extended generalized τ-ϕ- recurrent Kenmotsu manifold

Definition 3.1. A Kenmotsu manifold is said to be a extended generalized τ-ϕ-recurrent manifold if there exists non-zero 1-forms A and B such that

$$\phi^2((\nabla_W \tau)(X,Y)Z) = A(W)\phi^2(\tau(X,Y)Z) + B(W)\phi^2(G(X,Y)Z), \tag{3.1}$$

for arbitrary vector fields X, Y, Z, W. If X, Y, Z, W are orthogonal to ξ, then the manifold is called locally ϕ-recurrent manifold. If the 1 -form A vanishes, then the manifold reduces to ϕ-symmetric manifold.

Theorem 3.1. Extended generalized τ-ϕ- recurrent Kenmotsu manifold M^{2n+1}, with $a_0 + 2na_1 + a_2 + a_3 \neq 0$ is generalized Ricci-recurrent if and only if the following relation holds:

$$\begin{aligned} &\frac{\{B(W) - A(W)(a_0 - 2n(a_2 + a_3 + a_5 + a_6) - a_7 r) + a_7 dr(W)\}}{(a_0 + 2na_1 + a_2 + a_3)}\eta(Y)\eta(Z) \\ &\quad - \frac{\{a_2[S(W,Z) + 2ng(W,Z)]\eta(Y) - a_3[S(W,Y) + 2ng(W,Y)]\eta(Z)\}}{(a_0 + 2na_1 + a_2 + a_3)} \\ &\qquad + \frac{a_5\eta(Z)\eta((\nabla_W Q)Y) + a_6\eta(Y)\eta((\nabla_W Q)Z)}{(a_0 + 2na_1 + a_2 + a_3)} = 0. \end{aligned} \tag{3.2}$$

Proof. By taking an account of Definition 3.2 and using (1.1), we obtain

$$\begin{aligned} -(\nabla_W \tau)(X,Y)Z + \eta((\nabla_W \tau)(X,Y)Z)\xi &= A(W)[-\tau(X,Y)Z + \eta(\tau(X,Y)Z)\xi] \\ &\quad + B(W)[-G(X,Y)Z + \eta(G(X,Y)Z)\xi]. \end{aligned}$$

i.e.,

$$\begin{aligned} g((-\nabla_W \tau)(X,Y)Z,U) &+ \eta((\nabla_W \tau)(X,Y)Z)\eta(U) \\ &= A(W)[g(-\tau(X,Y)Z,U) + \eta(\tau(X,Y)Z)\eta(U)] \\ &\quad + B(W)[-g(G(X,Y)Z,U) + \eta(G(X,Y)Z)\eta(U)]. \end{aligned}$$

Let $\{e_i : i = 1,2,3,\cdots,2n+1\}$ be an othonormal basis of the tangent space at any point of the manifold. Setting $X = U = e_i$ in the above and taking summation over i, $1 \leq i \leq 2n+1$ and using (1.10), we have

$$\begin{aligned} -g((\nabla_W \tau)(e_i, Y)Z, e_i) + &\eta((\nabla_W \tau)(e_i, Y)Z)\eta(e_i) \\ &= A(W)[-g(\tau(e_i, Y)Z, e_i) + \eta(\tau(e_i, Y)Z)\eta(e_i)] \quad (3.3) \\ &\quad + B(W)[-g(G(e_i, Y)Z, e_i) + \eta(G(e_i, Y)Z)\eta(e_i)]. \end{aligned}$$

Using the equations (1.3), (1.6), (1.8) and (1.10) and the symmetric property of Ricci-tensor and the relation $g((\nabla_W R)(X, Y)Z, U) = -g((\nabla_W R)(X, Y)U, Z)$, we have

$$\begin{aligned} (\nabla_W S)(Y, Z) = & \\ & \frac{\{B(W) - A(W)(a_0 - 2n(a_2 + a_3 + a_5 + a_6) - a_7 r) + a_7 dr(W)\}}{(a_0 + 2na_1 + a_2 + a_3)}\eta(Y)\eta(Z) \\ & + \frac{\{(2n-1)B(W) - (a_4 + (2n-1)a_7)dr(W) - A(W)((2n-1)a_7 r}{(a_0 + 2na_1 + a_2 + a_3)} \\ & \frac{+(r+2n)a_4 + a_0)\}}{} g(Y, Z) + [A(W) + \frac{A(W)(a_5 + a_6)}{a_0 + 2na_1 + a_2 + a_3}]S(Y, Z) \quad (3.4) \\ & + \frac{a_5 \eta(Z)\eta((\nabla_W Q)Y) + a_6 \eta(Y)\eta((\nabla_W Q)Z)}{(a_0 + 2na_1 + a_2 + a_3)} \\ & - \frac{\{a_2[S(W, Z) + 2ng(W, Z)]\eta(Y) - a_3[S(W, Y) + 2ng(W, Y)]\eta(Z)\}}{(a_0 + 2na_1 + a_2 + a_3)}. \end{aligned}$$

The above relation will reduces to $\nabla S = A^* S + B^* g$ only when the relation (3.2) holds,

where $A^* = [A(W) + \dfrac{A(W)(a_5 + a_6)}{a_0 + 2na_1 + a_2 + a_3}]$

and $B^* = \dfrac{\{(2n-1)B(W) - (a_4 + (2n-1)a_7)dr(W) - A(W)((2n-1)a_7 r}{(a_0 + 2na_1 + a_2 + a_3)}$

$$\frac{+(r+2n)a_4 + a_0)\}}{}.$$

4. τ-ξ-flat Kenmotsu manifold

Putting $Y = Z = \xi$ in (1.10) and taking inner product with U, we obtain

$$\begin{aligned} \tau(X, \xi, \xi, U) = & a_0 R(X, \xi, \xi, U) + a_1 S(\xi, \xi)g(X, U) + a_2 S(X, \xi)g(\xi, U) \\ & + a_3 S(X, \xi)g(\xi, U) + a_4 g(\xi, \xi)g(QX, U) + a_5 g(X, \xi)g(Q\xi, U) \\ & + a_6 g(X, \xi)g(Q\xi, U) + a_7 r(g(\xi, \xi)g(X, U) - g(X, \xi)g(\xi, U)). \quad (4.1) \end{aligned}$$

By virtue of (1.5), (1.6) and the condition of τ-ξ-flat in (4.1), we get

$$0 = a_0\eta(U)\eta(X) - a_0 g(X,U) + (a_7 r - 2na_1)g(X,U)$$
$$-2n(a_2 + a_3 + a_5 + a_6)\eta(X)\eta(U) + a_4 S(X,U) - a_7 r\eta(X)\eta(U). \quad (4.2)$$

Simplifying (4.2), we get

$$S(X,U) = \frac{(a_0 + 2na_1 - a_7 r)}{a_4} g(X,U) + \frac{(a_7 r + 2n(a_2 + a_3 + a_5 + a_6) - a_0)}{a_4}\eta(X)\eta(U).$$

Thus we have the following theorem;

Theorem 4.1. A τ-ξ-flat Kenmotsu manifold is η-Einstein provided $a_4 \neq 0$.

Now suppose $a_4 = 0$ and $a_7 \neq 0$ then from (4.2), we have

$$0 = (a_0 + 2na_1 - a_7 r)g(X,U) + (a_7 r + 2n(a_2 + a_3 + a_5 + a_6) - a_0)\eta(X)\eta(U).$$

Contracting the above, we get

$$r = \frac{(a_2 + a_3 + a_5 + a_6) + a_0 + (2n+1)a_1}{a_7}. \quad (4.3)$$

Therefore,

Theorem 4.2. In a τ-ξ-flat Kenmotsu manifold with $a_4 = 0$ and $a_7 \neq 0$, the scalar curvature r is given by (4.3).

Next if $a_4 = 0$ and $a_7 = 0$ then (4.2) gives

$$0 = -(2na_1 + a_0)g(X,U) + (a_0 - 2n(a_2 + a_3 + a_5 + a_6))\eta(X)\eta(U),$$

on contraction, we have

$$2n(a_2 + a_3 + a_5 + a_6 + a_0 + (2n+1)a_1) = 0. \quad (4.4)$$

Thus we can state;

Theorem 4.3. In τ-ξ-flat Kenmotsu manifold with $a_4 = 0$ and $a_7 = 0$, $2n(a_2 + a_3 + a_5 + a_6 + a_0 + (2n+1)a_1) = 0$.

Theorem 4.4. A Kenmotsu manifold M^{2n+1}, satisfying $\tau(X,Y) \cdot R = 0$, is an Einstein manifold provided $a_1 \neq 0$.

Proof. Suppose $\tau(X,Y) \cdot R = 0$. Then we have

$$\tau(X,Y)R(U,V)W - R(\tau(X,Y)U,V)W - R(U,\tau(X,Y)V)W$$
$$-R(U,V)\tau(X,Y)W = 0. \quad (4.5)$$

Putting $X = \xi$ in (4.5) and then taking inner product with ξ, we obtain

$$\begin{aligned} &g(\tau(\xi,Y)R(U,V)W,\xi) - g(R(\tau(\xi,Y)U,V)W,\xi) \\ &-g(R(U,\tau(\xi,Y)V)W,\xi) - g(R(U,V)\tau(\xi,Y)W,\xi) = 0. \end{aligned} \tag{4.6}$$

By virtue of (1.4), (1.5), (1.6) and (4.6), we get

$$\begin{aligned} &2n(a_2+a_4+a_5)g(U,Y)\eta(V)\eta(W) - 2n(a_2+a_4+a_5)g(V,Y)\eta(U)\eta(W) \\ &-a_1S(Y,U)\eta(V)\eta(W) - a_1S(Y,V)\eta(U)\eta(W) \\ &+4n(a_3+a_6)g(U,W)\eta(Y)\eta(V) - 4n(a_3+a_6)g(V,W)\eta(Y)\eta(U). \end{aligned} \tag{4.7}$$

Contracting (4.7), we have

$$\begin{aligned} S(Y,V) = &\frac{2n(a_2+a_4+a_5)}{a_1}g(Y,V) \\ &+\frac{2n\{a_1-(a_2+a_4+a_5)-4n(a_3+a_6)}{a_1}\eta(Y)\eta(V). \end{aligned}$$

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