J. T. S.
Vol. 8 (2014), pp.139-158
https://doi.org/10.56424/jts.v8i01.10550

Propagation Equations of Electric and Magnetic Parts of Weyl Tensor in Einstein-Cartan Theory

L. N. Katkar and A. S. Patil

Department of Mathematics
Shivaji University, Kolhapur-416004 (M. S.)
e-mail: lnk 1000@yahoo.com
(Received: November 11, 2013)

Abstract

The electric part and magnetic part of the Weyl tensor with respect to the Kerr-Newman metric in Einstein-Cartan theory of gravitation are obtained, and their propagation equations are studied. The necessary and sufficient conditions for the electric part and the magnetic part to be propagated along the null vector field l_i are obtained. For particular types of free-gravitational field we tried to explore the meaning of these propagation equations with the help of Newman-Penrose, Jogia and Griffiths null tetrad formalism.

1. Introduction

The electric and magnetic parts represent the free gravitational field enabling gravitational action at a distance and describing tidal forces and gravitational waves. This study has a lot of current relevance in cosmology explained by Maartens et al. [12]. Haddow [4] has shown that all vacuum purely magnetic solutions are Petrov-type 1. Ahsan [1] has investigated the examples of space-times having purely magnetic and purely electric part of Weyl tensor. Hasmani and Katkar [5] have shown that the parameter related to vorticity of the fluid causes Gdel universe to be purely electric. Herrera et al. [8] have obtained the electric and magnetic parts of Weyl tensor for the Bondi vacuum metric. They have shown that the necessary and sufficient condition for the space time to be purely electric that the space-time be static. For Minkowski space-time the electric part vanishes. Greenberg [3] derived the propagation equations for the kinematical quantities in relativistic hydrodynamics. Ellis [2] presented the generalized form of propagation and constraint equations of kinematical parameters and Raychaudhuri equation in general relativity. Katkar and Patil [11] have obtained the propagation equations for the expansion and rotation in Einstein-Cartan theory of gravitation.

In the section 2 we give a brief summary of Einstein-Cartan theory of gravitation with its field equations. The section 3 presents the study of Kerr-Newman space-time in Einstein-Cartan theory of gravitation through Jogia-Griffiths tetrad formalism. The electric and magnetic parts of the Weyl tensor for Kerr-Newman space-time are derived in the section 4. The propagation equations of the electric and magnetic parts of the Weyl tensor are presented in the section 5.

2. Einstein-Cartan Theory of Gravitation

Einstein's general relativity is a generalization of his special relativity and Newton's theory of gravitation. This theory deals with all types of motions and also explains the phenomenon of gravitation through geometry. It is supposed to be the most successful theory of gravitation. Still the theory has known flaws that it cannot describe the intrinsic feature of spin of gravitating matter, it cannot prevent singularities and has not been able to account for Mach's principle.

Einstein-Cartan theory of gravitation is a natural generalization of Einstein theory of gravitation by relaxing the assumption that the metric is torsion free. The Einstein-Cartan theory of gravitation formulated in 1973 by Trautman [13] postulates that the spin of matter is the source of torsion of the space-time geometry. This theory is based on non-Riemannian geometry and the non-Riemannian part is described by the affine connections and are defined as

$$\omega^l_{ij} = \Gamma^l_{ij} - K^l_{ij}, \tag{2.1}$$

where $\Gamma^l_{ij} = \Gamma^l_{ji}$ and K_{ijk} is the contortion tensor satisfying

$$K_{i(jk)} = 0. \tag{2.2}$$

The relation between the torsion tensor and contortion tensor is given by

$$Q^k_{ij} = -\frac{1}{2}(K^k_{ij} - K^k_{ji}). \tag{2.3}$$

The relevant field equations are given by Helh et al. [6, 7]

$$R_{ij} - \frac{1}{2}Rg_{ij} = -Kt_{ij}, \tag{2.3}$$

where R_{ij} is the Ricci tensor and is no longer symmetric containing the information about the torsion tensor. Consequently the right hand side of the equation (2.4) cannot be symmetric either, so t_{ij} must also contain information about the spin tensor.

The relation between torsion tensor and spin angular momentum tensor is given by

$$Q^k_{ij} + \delta^k_i Q^l_{jl} - \delta^k_j Q^l_{il} = K S^k_{ij}. \tag{2.5}$$

Spin angular momentum tensor S^k_{ij} is anti-symmetric and is expressed in terms of spin tensor S_{ij} as

$$S^k_{ij} = S_{ij} u^k. \tag{2.6}$$

Assuming the notations of Jogia and Griffiths [9] and Katkar [10], the tetrad components of the field equations (2.5) are given by

$$\begin{aligned}
&\pi_1 = \tau_1 = \sigma_1 = \lambda_1 = 0,\\
&\mu_1 = 2\gamma_1 = \rho_1 = 2\epsilon_1 = -\sqrt{2} K s_1,\\
&\overline{\alpha}_1 = \beta_1 = -\frac{1}{\sqrt{2}} K s_0,\\
&\kappa_1 = \overline{\nu}_1 = -\sqrt{2} K s_0.
\end{aligned} \tag{2.7}$$

Here s_0 and s_1 are tetrad components of spin tensor and assumed to be functions of r alone.

3. Kerr-Newman Space-time

$$\begin{aligned}
ds^2 = \;& [1 - R^{-2}(2mr - e^2)]dt^2 + 2aR^{-2}(2mr - e^2)\sin^2\theta dt\, d\phi\\
& - \frac{R^2}{\triangle_1} dr^2 - R^2 d\theta^2 - [(r^2+a^2)^2 - \triangle_1 a^2 \sin^2\theta]\, R^{-2} \sin^2\theta\, d\phi^2,
\end{aligned} \tag{3.1}$$

where

$R^2 = \overline{R}\overline{R}^* = r^2 + a^2\cos^2\theta,$ $\quad\triangle_1 = r^2 - 2mr + a^2 + e^2,$
$\overline{R} = r + ia\cos\theta,$ $\quad\overline{R}^* = r - ia\cos\theta,$
$a =$ angular momentum per unit mass, $\quad m =$ mass
$e =$ the charge of the gravitating body.

The Kerr-Newman metric is expressed in terms of basis 1-form as

$$ds^2 = 2\theta^1\theta^2 - 2\theta^3\theta^4, \tag{3.2}$$

where

$$\begin{aligned}
\theta^1 &= \frac{\triangle_1}{2R^2}dt + \frac{1}{2}dr - \frac{a\triangle_1}{2R^2}\sin^2\theta\, d\phi,\\
\theta^2 &= dt - \frac{R^2}{\triangle_1}dr - a\sin^2\theta\, d\phi,\\
\theta^3 &= \frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^*}dt + \frac{R^2}{\sqrt{2}\,\overline{R}^*}d\theta - \frac{i(r^2+a^2)\sin\theta}{\sqrt{2}\,\overline{R}^*}d\phi,\\
\theta^4 &= -\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}}dt + \frac{R^2}{\sqrt{2}\,\overline{R}}d\theta + \frac{i(r^2+a^2)\sin\theta}{\sqrt{2}\,\overline{R}}d\phi.
\end{aligned} \tag{3.3}$$

The definition of 1-form $\theta^\alpha = e_i^{(\alpha)}dx^i$ and the equation (3.3) gives the vector fields of the NP complex null tetrad as,

$$\begin{aligned}
l_i &= \frac{1}{\triangle_1}(\triangle_1, -R^2, 0, -a\triangle_1\sin^2\theta),\\
n_i &= \frac{1}{2R^2}(\triangle_1, R^2, 0, -a\triangle_1\sin^2\theta)\\
m_i &= \frac{1}{\sqrt{2}\,\overline{R}}(ia\sin\theta, 0, -R^2, -i\,(r^2+a^2)\,\sin\theta),\\
\overline{m}_i &= \frac{1}{\sqrt{2}\,\overline{R}^*}(-ia\sin\theta, 0, -R^2, -i\,(r^2+a^2)\,\sin\theta).
\end{aligned} \tag{3.4}$$

Comparing the corresponding coefficients of basis 2-form of the equations obtained by taking exterior derivative of basis 1-forms θ^α in equation (3.3) and the Cartan's first equations of structure we readily obtain on using equation (2.7)

$$\begin{aligned}
&\nu^0 = \kappa^0 = \sigma^0 = \lambda^0 = \epsilon^0 = 0,\\
&\rho^0 = -\frac{1}{\overline{R}^*}, \qquad \overline{\beta^0} = \frac{\cot\theta}{2\sqrt{2}\,\overline{R}^*}, \qquad \tau^0 = \frac{-ia\sin\theta}{\sqrt{2}\,R^2},\\
&\gamma^0 = \frac{R^2(r-m) - \triangle_1\overline{R}}{2R^4}, \qquad \mu^0 = \frac{-\triangle_1}{2R^2\overline{R}^*}, \qquad \overline{\pi^0} = \frac{-ia\sin\theta}{\sqrt{2}\,(\overline{R})^2},\\
&\alpha^0 = \frac{-\cot\theta}{2\sqrt{2}\,\overline{R}^*} + \frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}.
\end{aligned} \tag{3.5}$$

Cartan's equations of structure are used to find the tetrad components of curvature tensor and consequently the tetrad components of the Weyl tensor are obtained by using the relation

$$C_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} + \frac{1}{2}(\eta_{\alpha\gamma}R_{\beta\delta} + \eta_{\beta\delta}R_{\alpha\gamma} - \eta_{\beta\gamma}R_{\alpha\delta} - \eta_{\alpha\delta}R_{\beta\gamma}) + \frac{R}{6}(\eta_{\alpha\delta}\eta_{\beta\gamma} - \eta_{\alpha\gamma}\eta_{\beta\delta}).$$

These components are enumerated below to study the electric and magnetic parts of Weyl tensor.

$$C_{1212} = \frac{-4mr+e^2}{R^4} + \frac{(3r^2-a^2\cos^2\theta)(2mr-e^2)}{R^6} - \frac{Ks_0 iar\sin\theta}{R^2\overline{R}^*} + $$
$$+ \frac{K\,\overline{s}_0 iar\sin\theta}{R^2\overline{R}} - 4K^2 s_1^2 - \sqrt{2}Ks_1\left[\frac{2ia\cos\theta}{R^2}\left(1+\frac{\triangle_1}{2R^2}\right)\right] + \frac{R}{6},$$

$$C_{1213} = \sqrt{2}Ks_0,r\left(\frac{\triangle_1}{4R^2}\right) + \frac{\sqrt{2}Ks_0}{R^4}[R^2(r-m) - \triangle_1(5r+3ia\cos\theta)] -$$
$$-2K^2 s_0 s_1 + \frac{2Ks_1 iar\sin\theta}{R^2\overline{R}},$$

$$C_{1214} = \sqrt{2}K\overline{s}_0,r\left(\frac{\triangle_1}{4R^2}\right) + \frac{\sqrt{2}K\overline{s}_0}{R^4}[R^2(r-m) - \triangle_1(5r-3ia\cos\theta)] +$$
$$+2K^2\overline{s}_0 s_1 + \frac{2Ks_1 iar\sin\theta}{R^2\overline{R}^*},$$

$$C_{1223} = \frac{-Ks_0,r}{\sqrt{2}} - 2K^2 s_0 s_1 + \frac{Ks_0}{\sqrt{2}R} + \frac{Ks_1 ia\sin\theta(2r+ia\cos\theta}{R^2\overline{R}},$$

$$C_{1224} = \frac{-K\overline{s}_0,r}{\sqrt{2}} + 2K^2\overline{s}_0 s_1 + \frac{K\overline{s}_0}{\sqrt{2}R^*} + \frac{Ks_1 ia\sin\theta(2r-ia\cos\theta}{R^2\overline{R}^*},$$

$$C_{1234} = \frac{2ia\cos\theta}{R^6}(mR^2 - 2r(2mr-e^2)) - \sqrt{2}Ks_1,r\left(\frac{\triangle_1}{2R^2}+1\right) -$$
$$-\sqrt{2}Ks_1\left[\frac{R^2(r-m)-\triangle_1 r}{R^4}\right],$$

$$C_{1312} = \frac{Ks_0,r}{\sqrt{2}} + \sqrt{2}Ks_0\left[\frac{-ia\cos\theta}{R^2} - \frac{\triangle_1(r+3ia\cos\theta)}{4R^4}\right] - 2K^2 s_0 s_1 -$$
$$-\frac{Ks_1 ia\sin\theta}{(\overline{R})^2},$$

$$C_{1313} = -2K^2 s_0^2 + \frac{Ks_0\cot\theta}{\overline{R}},$$

$$C_{1314} = -\sqrt{2}Ks_1,r - \frac{K\overline{s}_0}{2}\left(\frac{\cot\theta}{\overline{R}} + \frac{2ia\sin\theta}{(\overline{R})^2}\right) - \frac{Ks_0}{2}\left[\frac{-\cot\theta}{\overline{R}^*} + \frac{2ia\sin\theta}{(\overline{R}^*)^2}\right] -$$
$$-\frac{2\sqrt{2}\,Ks_1 r}{R^2},$$

$$C_{1323} = -\frac{Ks_0 ia\sin\theta}{2R^4}[(\overline{R}^*)^2 - R^2],$$

$$C_{1324} = \frac{m}{R^2\overline{R}^*} - \frac{(2mr-e^2)}{R^2(\overline{R}^*)^2} + \frac{Ks_0 ia\sin\theta}{2(\overline{R}^*)^2} - \frac{K\overline{s}_0 ia\sin\theta}{2R^2} + 2K^2 s_0\overline{s}_0 + \frac{R}{6} -$$
$$-\frac{Ks_1,r}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2}+1\right) - \sqrt{2}Ks_1\left[\frac{-2ia\cos\theta+(r-m)}{2R^2} - \frac{\triangle_1}{2R^2\overline{R}^*}\right],$$

$$
\begin{aligned}
C_{1334} =& 2K^2 s_0 s_1 + \frac{K s_0}{2\sqrt{2}R^4}(\triangle_1(5r + 3ia\cos\theta) - 4R^2(r-m)) - \\
&-\frac{K s_{0,r}}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2}\right) - \frac{K s_1 ia\sin\theta}{R^4}(R^2 + \overline{R}^{*2}),\\
C_{1412} =& \frac{K\overline{s}_{0,r}}{\sqrt{2}} + \frac{\sqrt{2}\,K\,\overline{s}_0}{4R^4}[4R^2 is\cos\theta - \triangle_1(r - 3ia\cos\theta)] + 2K^2\overline{s}_0 s_1 - \\
&-\frac{K s_1 ia\sin\theta}{(\overline{R}^*)^2},\\
C_{1413} =& \frac{K s_0}{2}\left[\frac{-\cot\theta}{\overline{R}^*} + \frac{2ia\sin\theta}{(\overline{R}^*)^2}\right] + \sqrt{2}K s_{1,r} + \frac{K\overline{s}_0}{2}\left(\frac{\cot\theta}{\overline{R}} + \frac{2ia\sin\theta}{(\overline{R})^2}\right) + \\
&+\frac{2\sqrt{2}\,K s_1 r}{R^2},\\
C_{1414} =& -2K^2\overline{s}_0^2 + \frac{K\overline{s}_0\cot\theta}{\overline{R}^*},\\
C_{1423} =& \frac{m}{R^2\overline{R}} - \frac{(2mr - e^2)}{R^2(\overline{R})^2} + \frac{K s_0 ia\sin\theta}{2R^2} - \frac{K\overline{s}_0 ia\sin\theta}{2\overline{R}^2} + 2K^2 s_0\overline{s}_0 + \\
&+\frac{K s_{1,r}}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2} + 1\right) + \frac{R}{6} + \frac{K s_1}{\sqrt{2}}\left[\frac{2ia\cos\theta + (r-m)}{R^2} - \frac{\triangle_1}{R^2\overline{R}}\right],\\
C_{1424} =& \frac{K\overline{s}_0 ia\sin\theta}{2R^4}(\overline{R}^2 - R^2),\\
C_{1434} =& 2K^2\overline{s}_0 s_1 + \frac{K\overline{s}_0}{\sqrt{2}\,R^4}\left[2R^2(r-m) - \frac{\triangle_1(5r - 3ia\cos\theta)}{2}\right] + \\
&+\sqrt{2}\,K\overline{s}_{0,r}\left(\frac{\triangle_1}{4R^2}\right) + \frac{K s_1 ia\sin\theta}{R^4}(R^2 + \overline{R}^2),\\
C_{2312} =& -\frac{K s_{0,r}\triangle_1}{2\sqrt{2}\,R^2} - 2K^2 s_0 s_1 + \frac{K s_1 ia\sin\theta(-R^2 + iar\cos\theta)}{R^4} + \\
&+\frac{K s_0}{\sqrt{2}\,R^2}(\overline{R}^* - 2ia\sin\theta),\\
C_{2313} =& \frac{K s_0 ia\sin\theta}{2R^4}[(\overline{R}^*)^2 - R^2],\\
C_{2314} =& \frac{m}{R^2\overline{R}} - \frac{(2mr - e^2)}{R^2(\overline{R})^2} + \frac{K s_0 ia\sin\theta}{2R^2} - \frac{K\overline{s}_0 ia\sin\theta}{2\bar{R}^2} + 2K^2 s_0\overline{s}_0 + \\
&+\frac{K s_{1,r}}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2} + 1\right) + \frac{R}{6} + \frac{K s_1}{\sqrt{2}}\left[\frac{2ia\cos\theta + (r-m)}{R^2} - \frac{\triangle_1}{R^2\overline{R}}\right],\\
C_{2323} =& -2K^2 s_0^2 - \frac{2K s_0 ia\sin\theta}{\bar{R}^2} - \frac{K s_0\cot\theta}{\bar{R}} - \frac{K s_0 2a^2\cos\theta\sin\theta}{R^2\bar{R}},\\
C_{2324} =& -\frac{K s_0}{2}\left[\frac{\cot\theta}{\overline{R}^*} - \frac{2a^2\cos\theta\sin\theta}{R^2\overline{R}^*}\right] + \frac{K\overline{s}_0}{2}\left[\frac{\cot\theta}{\overline{R}} - \frac{2a^2\cos\theta\sin\theta}{R^2\overline{R}}\right] + \\
&+\frac{K s_1}{\sqrt{2}\,R^4}[3r\triangle_1 - 2R^2(r-m)] + \frac{K s_{1,r}\triangle_1}{\sqrt{2}\,R^2},
\end{aligned}
$$

$$C_{2334} = \frac{-Ks_0,r}{\sqrt{2}} - 2K^2 s_0 s_1 + \frac{Ks_0}{\sqrt{2}\,\overline{R}} + \frac{Ks_1 ia\sin\theta(2r+ia\cos\theta)}{R^2\,\overline{R}},$$

$$C_{2412} = -\frac{K\overline{s}_0,r\triangle_1}{2\sqrt{2}\,R^2} + 2K^2\overline{s}_0 s_1 + \frac{K\overline{s}_0}{\sqrt{2}\,R^2}(\overline{R}+2ia\sin\theta) - \\ -\frac{Ks_1 ia\sin\theta(R^2+iar\cos\theta)}{R^4},$$

$$C_{2413} = \frac{m}{R^2\overline{R}^*} - \frac{(2mr-e^2)}{R^2(\overline{R}^*)^2} + \frac{Ks_0 ia\sin\theta}{2(\overline{R}^*)^2} - \frac{K\overline{s}_0 ia\sin\theta}{2R^2} + 2K^2 s_0\overline{s}_0 - \\ -\frac{Ks_1,r}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2}+1\right) + \frac{R}{6} - \sqrt{2}\,Ks_1\left[\frac{-2ia\cos\theta+(r-m)}{2R^2} - \frac{\triangle_1}{2R^2\,\overline{R}^*}\right],$$

$$C_{2414} = -\frac{K\overline{s}_0 ia\sin\theta}{2R^4}(\overline{R}^2-R^2),$$

$$C_{2423} = -\frac{Ks_0}{2}\left[\frac{-\cot\theta}{\overline{R}^*} + \frac{2a^2\cos\theta\sin\theta}{R^2\,\overline{R}^*}\right] - \frac{K\overline{s}_0}{2}\left[\frac{\cot\theta}{\overline{R}} - \frac{2a^2\cos\theta\sin\theta}{R^2\,\overline{R}}\right] + \\ +\frac{Ks_1}{\sqrt{2}\,R^4}[-3r\triangle_1+2R^2(r-m)] - \frac{Ks_1,r\triangle_1}{\sqrt{2}\,R^2},$$

$$C_{2424} = -2K^2\overline{s}_0^2 + \frac{2K\overline{s}_0 ia\sin\theta}{(\overline{R}^*)^2} - \frac{K\overline{s}_0\cot\theta}{\overline{R}^*} - \frac{K\overline{s}_0 2a^2\cos\theta\sin\theta}{R^2\,\overline{R}^*}$$

$$C_{2434} = \frac{K\overline{s}_0,r}{\sqrt{2}} - 2K^2\overline{s}_0 s_1 - \frac{K\overline{s}_0}{\sqrt{2}\overline{R}^*} - \frac{Ks_1 ia\sin\theta(2r-ia\cos\theta)}{R^2\,\overline{R}^*},$$

$$C_{3412} = -\frac{2ia\cos\theta}{R^6}[2r(2mr-e^2)-mR^2] - Ks_0\left(\frac{\cot\theta}{\overline{R}^*} - \frac{ia\sin\theta}{(\overline{R}^*)^2}\right) + \\ +K\overline{s}_0\left(\frac{\cot\theta}{\overline{R}} + \frac{ia\sin\theta}{(\overline{R})^2}\right) + \frac{\sqrt{2}\,Ks_1 r(\triangle_1+2R^2)}{R^4},$$

$$C_{3413} = 2K^2 s_0 s_1 - \frac{Ks_0,r}{\sqrt{2}} + \sqrt{2}\,Ks_0\left[\frac{ia\cos\theta}{R^2} + \frac{\triangle_1(r+3ia\cos\theta)}{4R^4}\right] + \\ +\frac{Ks_1 ia\sin\theta}{(\overline{R})^2},$$

$$C_{3414} = 2K^2\overline{s}_0 s_1 + \frac{K\overline{s}_0,r}{\sqrt{2}} + \sqrt{2}\,K\overline{s}_0\left[\frac{ia\cos\theta}{R^2} + \frac{\triangle_1(3ia\cos\theta-r)}{4R^4}\right] - \\ -\frac{Ks_1 ia\sin\theta}{(\overline{R}^*)^2},$$

$$C_{3423} = -2K^2 s_0 s_1 - \frac{Ks_0,r}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2}\right) + \frac{\sqrt{2}\,Ks_0}{2R^2}(-2ia\sin\theta+\overline{R}^*) - \\ -\frac{Ks_1 ia\sin\theta}{R^4}(r^2+a^2-iar\cos\theta),$$

$$C_{3424} = -2K^2\overline{s}_0 s_1 + \frac{K\overline{s}_0,r}{\sqrt{2}}\left[\frac{\triangle_1}{2R^2}\right] + \frac{\sqrt{2}\,K\,\overline{s}_0}{2R^2}(-2ia\sin\theta-\overline{R}) + \\ +\frac{Ks_1 ia\sin\theta}{R^4}(r^2+a^2+iar\cos\theta),$$

$$\begin{aligned}C_{3434} = &\frac{-e^2}{R^4} + \frac{(r^2 - 3a^2\cos^2\theta)(2mr - e^2)}{R^6} - \frac{Ks_0 iar\sin\theta}{R^2\overline{R}^*} - 4K^2 s_1^2 + \\ &+ \frac{K\overline{s}_0 iar\sin\theta}{R^2\,\overline{R}} + \frac{\sqrt{2}\,Ks_1}{R^4}[ia\cos\theta(2R^2 - \triangle_1) - 4R^2] + \frac{R}{6}.\end{aligned} \tag{3.6}$$

The expressions for C_{1214}, C_{1224}, C_{1412}, C_{1413}, C_{1414}, C_{1423}, C_{1424}, C_{1434}, C_{2412}, C_{2413}, C_{2414}, C_{2423}, C_{2424}, C_{2434}, C_{3414}, C_{3424} are obtained by taking complex conjugate of C_{1213}, C_{1223}, C_{1312}, C_{1314}, C_{1313}, C_{1324}, C_{1323}, C_{1334}, C_{2312}, C_{2314}, C_{2313}, C_{2324}, C_{2323}, C_{2334}, C_{3413} and C_{3423} respectively and replacing 3 by 4 and 4 by 3 therein.

The tetrad components of the dual of Weyl tensor are also obtained from the relation

$$C^*_{\alpha\beta\gamma\delta} = \frac{i}{2}[\epsilon^{\sigma\rho}_{\gamma\delta}\, C_{\alpha\beta\sigma\rho}]. \tag{3.7}$$

4. Electric and Magnetic Parts of the Weyl Tensor

The electric part E_{ij} and the magnetic part H_{ij} are respectively given by

$$E_{ij} = C_{ikjl}u^k u^l, \qquad H_{ij} = C^*_{ikjl}u^k u^l,$$

where C^*_{ikjl} is the dual of C_{ikjl}.

The Expression for electric part of the Weyl tensor in terms of the basis of the tetrad given by Hasmani et al. [5] is exploited to get the electric part of Weyl tensor.

$$\begin{aligned}E_{ij} = &\frac{1}{2} + \Bigg[\frac{-4mr + e^2}{R^4} + \frac{(3r^2 - a^2\cos^2\theta)(2mr - e^2)}{R^6} - \\ &- \frac{Ks_0 iar\sin\theta}{R^2\,\overline{R}^*} + \frac{K\overline{s}_0 iar\sin\theta}{R^2\,\overline{R}} - \sqrt{2}\,Ks_1\left(\frac{2ia\cos\theta}{R^2}\left[1 + \frac{\triangle_1}{2R^2}\right]\right) - \\ &-4K^2 s_1^2 + \frac{R}{6}\Bigg][l_i l_j - 2l_{(i}n_{j)} + n_i n_j] + \\ &+\Bigg\{ -\frac{1}{2}\Bigg[\frac{\sqrt{2}\,K\overline{s}_0}{R^4}\left(R^2(r - m) - \frac{\triangle_1(5r - 3ia\cos\theta)}{4} + \frac{R^2\,\overline{R}}{2}\right) + 4K^2\overline{s}_0 s_1 + \\ &+ \frac{Ks_1 ia\sin\theta(4r - ia\cos\theta)}{R^2\,\overline{R}^*} + \frac{K\,\overline{s}_0,r}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2} - 1\right)\Bigg][l_i m_j - n_i m_j] - \\ &- \frac{1}{2}\Bigg[4K^2\overline{s}_0 s_1 + \frac{K\overline{s}_0,r}{\sqrt{2}}\left(1 - \frac{\triangle_1}{2R^2}\right) - \frac{Ks_1 ia\sin\theta(2r^2 + 3iar\cos\theta)}{R^4} + \\ &+ \frac{K\overline{s}_0}{\sqrt{2}\,R^2}\left(2ia(\cos\theta + \sin\theta) - \frac{\triangle_1(r - 3ia\cos\theta)}{2R^2} + \overline{R}\right)\Bigg][m_i l_j - m_i n_j] +\end{aligned}$$

$$
+\frac{1}{2}\left[\frac{2mr}{R^4}-\frac{2(2mr-e^2)(r^2-a^2\cos^2\theta)}{R^6}+\frac{\sqrt{2}\,Ks_1}{R^4}\left(\triangle_1 ia\cos\theta+\frac{3\triangle_1 r}{2}\right)-\right.
$$

$$
-\frac{2K\overline{s}_0 ia\sin\theta}{\overline{R}^2}-\frac{\sqrt{2}\,Ks_1}{R^2}(2\overline{R}^*+(r-m))+\frac{R}{3}+4K^2 s_0\overline{s}_0-
$$

$$
\left.-\frac{Ks_1,r}{\sqrt{2}}\left(2-\frac{\triangle_1}{R^2}\right)+\frac{2Ks_0a^2\sin\theta\cos\theta}{R^2\overline{R}^*}\right]\overline{m}_i m_j+
$$

$$
\left.+\frac{1}{2}\left[-4K\overline{s}_0^2+\frac{2K\overline{s}_0 ia\sin\theta}{R^2\,\overline{R}^*}(r+2ia\cos\theta)\right]m_i m_j\right\}+\{c.c.\}. \tag{4.1}
$$

where c.c. indicates the complex conjugate of the previous term.

The magnetic part of the Weyl tensor is obtained by replacing the tetrad components of Weyl tensor by its dual and is derived as

$$
\begin{aligned}
H_{ij} = -\frac{i}{2}&\left[\frac{2ia\cos\theta(mR^2-2r(2mr-e^2))}{R^6}-\sqrt{2}\,Ks_1,r\left(1+\frac{\triangle_1}{2R^2}\right)-\right.\\
&\left.-\sqrt{2}\frac{\sqrt{2}\,Ks_1}{R^4}(R^2(r-m)-r\triangle_1)\right][l_il_j-2l_{(i}n_{j)}+n_in_j]+\\
&+\left\{\frac{i}{2}\left[\frac{\sqrt{2}\,K\overline{s}_0}{R^2}\left(\frac{\overline{R}^*-2m}{2}-\frac{\triangle_1(5r-3ia\cos\theta)}{4R^2}\right)-\right.\right.\\
&\left.-\frac{Ks_1a^2\sin\theta\cos\theta}{R^2\,\overline{R}^*}+\frac{K\overline{s}_0,r}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2}+1\right)\right](l_im_j-n_im_j]+\\
&+\frac{i}{2}\left[\frac{K\overline{s}_0}{\sqrt{2}\,R^2}\left(\overline{R}^*-2m-\frac{\triangle_1(5r-3ia\cos\theta)}{2R^2}\right)-\right.\\
&\left.-\frac{Ks_1a^2\sin\theta\cos\theta}{R^2\,\overline{R}^*}+\frac{K\overline{s}_0,r}{\sqrt{2}}\left(\frac{\triangle_1}{2R^2}+1\right)\right][m_il_j-m_in_j]+\\
&+\frac{1}{2}\left[\frac{2ia\cos\theta}{R^6}(mR^2-2r(2mr-e^2))+K\overline{s}_0\left(\frac{\cot\theta}{\overline{R}}+\frac{ia\sin\theta}{(\overline{R})^2}\right)-\right.\\
&\left.-Ks_0\left[\frac{\cot\theta}{\overline{R}^*}-\frac{ia\sin\theta}{(\overline{R}^*)^2}\right]+\frac{\sqrt{2}\,Ks_1}{2R^4}(-4mR^2+5r\triangle_1)\right]\overline{m}_im_j+\\
&\left.+\frac{i}{2}\left[\frac{2K\overline{s}_0}{R^2\overline{R}^*}[-\cot\theta\,R^2+ia\sin\theta(r+3ia\cos\theta)]\right]m_im_j\right\}+\{c.c.\}.
\end{aligned} \tag{4.2}
$$

5. Propagation Equations of Electric and Magnetic Parts of the Weyl Tensor

Propagation equation of E_{ij} is given by

$$
E_{ij;k}\,l^k=0, \tag{5.1}
$$

where $E_{ij;k}\,l^k$ is the covariant derivative of the electric part (E_{ij}) of Weyl tensor in the direction of the tetrad vector field l^k. Differentiating equation (4.1) covariantly in the direction of the tetrad vector field l^k and using the intrinsic derivatives of the null tetrad vector fields, their projections and divergence (refer appendix) and the spin coefficients in the equation (5.1), we get

$$
\begin{aligned}
E_{ij;k}l^k = &\frac{1}{2}\Bigg\{\Bigg[D(C_{1212}) + (C_{1214} + C_{1224} + C_{1412} + C_{2412})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} -\\
&-(C_{1213} + C_{1223} + C_{1312} + C_{2312})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\Bigg]l_il_j +\\
&+\Bigg[-D(C_{1212}) - (C_{1214} + C_{1224}\sqrt{2}\,Ks_0 - (C_{1412} + C_{2412})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} -\\
&-(C_{1213} + C_{1223})\sqrt{2}K\overline{s}_0 + (C_{1312} + C_{2312})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\Bigg]l_in_j +\\
&+\Bigg[-D(C_{1212}) - (C_{1214} + C_{1224})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} - (C_{1412} + C_{2412})\sqrt{2}\,Ks_0 +\\
&+(C_{1213} + C_{1223})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2} - (C_{1312} + C_{2312})\sqrt{2}\,K\overline{s}_0\Bigg]n_il_j +\\
&+[D(C_{1212}) + (C_{1214} + C_{1224} + C_{1412} + C_{2412})\sqrt{2}\,Ks_0 +\\
&+(C_{1213} + C_{1223} + C_{1312} + C_{2312})\sqrt{2}\,K\overline{s}_0]n_in_j +\\
&+\Bigg\{\Bigg[-D(C_{1214} + C_{1224}) + (C_{1214} + C_{1224})\sqrt{2}\,Ks_1 + (C_{1314} + C_{1324} +\\
&+C_{2314} + C_{2324})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2} - (C_{1414} + C_{1424} + C_{2414} + C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} +\\
&+C_{1212}\left(\sqrt{2}Ks_0 - \frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)\Bigg]l_im_j + \Bigg[D(C_{1214} + C_{1224}) -\\
&-\Bigg[(C_{1214} + C_{1224})\sqrt{2}Ks_1 + (C_{1314} + C_{1324} + C_{2314} + C_{2324})\sqrt{2}K\overline{s}_0 +\\
&+(C_{1414} + C_{1424} + C_{2414} + C_{2424})\sqrt{2}\,Ks_0 -\\
&-C_{1212}\left(\sqrt{2}K\overline{s}_0 - \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right)\Bigg]n_im_j +\\
&+\Bigg[-(C_{1214} + C_{1224} + C_{1412} + C_{2412})\left(\sqrt{2}K\overline{s}_0 - \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right) +\\
&+D(C_{1414} + C_{1424} + C_{2414} + C_{2424}) -
\end{aligned}
$$

$$
\begin{aligned}
&-(C_{1414}+C_{1424}+C_{2414}+C_{2424})2\sqrt{2}\,Ks_1\Big]m_im_j+\\
&+\Big[-D(C_{1412}+C_{2412})-(C_{1414}+C_{1424}+C_{2414}+C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+\\
&+C_{1212}\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)+(C_{1412}+C_{2412})\sqrt{2}\,Ks_1+\\
&+(C_{1413}+C_{1423}+C_{2413}+C_{2423})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\Big]m_il_j+\\
&+\Big[-C_{1212}\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right)+D(C_{1412}+C_{2412})-\\
&-(C_{1412}+C_{2412})\sqrt{2}\,Ks_1+(C_{1413}+C_{1423}+C_{2413}+C_{2423})\sqrt{2}\,K\overline{s}_0+\\
&+(C_{1414}+C_{1424}+C_{2414}+C_{2424})\sqrt{2}Ks_0\Big]m_in_j+\\
&+\Big[-(C_{1412}+C_{2412})\left(\sqrt{2}Ks_0+\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}\right)-\\
&-(C_{1213}+C_{1223})\left(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right)+\\
&+D(C_{1413}+C_{1423}+C_{2413}+C_{2423})\Big]m_i\overline{m}_j\Big\}+\{c.c.\}\Big\}.
\end{aligned}
\tag{5.2}
$$

Now the propagation equation for electric part of the Weyl tensor yields

$$E_{ij;k}l^k=0\leftrightarrow A=B=C+D+E+F+G=H+I=J=0,$$

where

$$A=D(C_{1212}+\left[(C_{1214}+C_{1224}+C_{1412}+C_{2412}\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+\{c.c.\}\right],\tag{5.3a}$$

$$
\begin{aligned}
B=&-D(C_{1212}-C_{1214}+C_{1224})\sqrt{2}\,Ks_0-(C_{1412}+C_{2412})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}-\\
&-(C_{1213}+C_{1223})\sqrt{2}\,K\overline{s}_0+(C_{1312}+C_{2312})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2},
\end{aligned}
\tag{5.3b}
$$

$$
\begin{aligned}
C=&-D(C_{1212})-(C_{1214}+C_{1224})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}-(C_{1412}+C_{2412})\sqrt{2}\,Ks_0+\\
&+(C_{1213}+C_{1223})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}-(C_{1312}+C_{2312})\sqrt{2}\,K\overline{s}_0,
\end{aligned}
\tag{5.3c}
$$

$$
\begin{aligned}
D=&D(C_{1212})+(C_{1214}+C_{1224}+C_{1412}+C_{2412})\sqrt{2}\,Ks_0+\\
&+(C_{1213}+C_{1223}+C_{1312}+C_{2312})\sqrt{2}\,K\overline{s}_0,
\end{aligned}
\tag{5.3d}
$$

$$\begin{aligned}E =&-D(C_{1214}+C_{1224})+(C_{1214}+C_{1224})\sqrt{2}\,Ks_1+(C_{1314}+C_{1324}+\\&+C_{2314}+C_{2324})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^{*})^2}-(C_{1414}+C_{1424}+C_{2414}+\\&+C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+C_{1212}\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^{*})^2}\right),\end{aligned} \tag{5.3e}$$

$$\begin{aligned}F =&D(C_{1214}+C_{1224})-(C_{1214}+C_{1224})\sqrt{2}\,Ks_1+(C_{1314}+C_{1324}+\\&+C_{2314}+C_{2324})\sqrt{2}\,K\overline{s}_0+(C_{1414}+C_{1424}+C_{2414}+\\&+C_{2424})\sqrt{2}\,K\overline{s}_0-C_{1212}\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^{*})^2}\right),\end{aligned} \tag{5.3f}$$

$$\begin{aligned}G =&-(C_{1214}+C_{1224}+C_{1412}+C_{2412})\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^{*})^2}\right)+\\&+D(C_{1414}+C_{1424}+C_{2414}+C_{2424})-\\&-(C_{1414}+C_{1424}+C_{2414}+C_{2424})\sqrt{2}\,Ks_1,\end{aligned} \tag{5.3g}$$

$$\begin{aligned}H =&-D(C_{1412}+C_{2412})-(C_{1414}+C_{1424}+C_{2414}+C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+\\&+C_{1212}\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^{*})^2}\right)+(C_{1412}+C_{2412})\sqrt{2}\,Ks_1+\\&+(C_{1413}+C_{1423}+C_{2413}+C_{2423})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^{*})^2},\end{aligned} \tag{5.3h}$$

$$\begin{aligned}I =&-C_{1212}\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^{*})^2}\right)+D(C_{1412}+C_{2412})-(C_{1412}+\\&+C_{2412})\sqrt{2}\,Ks_1+(C_{1413}+C_{1423}+C_{2413}+C_{2423})\sqrt{2}K\overline{s}_0+\\&+(C_{1414}+C_{1424}+(C_{2414}+C_{2424})\sqrt{2}Ks_0,\end{aligned} \tag{5.3i}$$

$$\begin{aligned}J =&-(C_{1412}+C_{2412})\left(\sqrt{2}\,Ks_0+\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}\right)-(C_{1213}+\\&+C_{1223})\left(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^{*})^2}\right)+\\&+D(C_{1413}+C_{1423}+(C_{2413}+C_{2423}).\end{aligned} \tag{5.3j}$$

Similarly the propagation equation of the magnetic part of the Weyl tensor in the direction of tetrad vector field l^k is obtained as

$$H_{ij;k}l^k = 0$$

where

$$H_{ij;k}l^k = \frac{i}{2}\Bigg\{\Bigg[D(-C_{1234}) - (C_{1214} - C_{1224} + C_{1434} + C_{2434})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} -$$

$$-(C_{1213} - C_{1223} + C_{1343} + C_{2343})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\Bigg]l_il_j +$$

$$+\Bigg[D(-C_{1234}) - (-C_{1214} + C_{1224}\sqrt{2}\,Ks_0 + (C_{1434} + C_{2434})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} -$$

$$-(C_{1213} - C_{1223})\sqrt{2}K\overline{s}_0 + (C_{1343} + C_{2343})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\Bigg]l_in_j +$$

$$+\Bigg[D(-C_{1234}) - (-C_{1214} + C_{1224})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} + (C_{1434} + C_{2434})\sqrt{2}\,Ks_0 -$$

$$-(C_{1213} - C_{1223})\frac{-ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2} + (-C_{1343} - C_{2343})\sqrt{2}\,K\overline{s}_0\Bigg]n_il_j +$$

$$+[D(-C_{1234}) + (-C_{1214} + C_{1224})\sqrt{2}\,Ks_0 - (C_{1434} + C_{2434})\sqrt{2}\,Ks_0 +$$

$$+(C_{1213} - C_{1223})\sqrt{2}\,K\overline{s}_0 + (C_{1343} + C_{2343})\sqrt{2}\,K\overline{s}_0]n_in_j +$$

$$+\Bigg\{\Bigg[-D(-C_{1214} + C_{1224}) + (-C_{1214} + C_{1224})\sqrt{2}\,Ks_1 + (-C_{1314} +$$

$$+C_{1324} - C_{2314} + C_{2324})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2} - (-C_{1414} + C_{1424} - C_{2414} +$$

$$+C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} + C_{1243}\left(\sqrt{2}K\overline{s}_0 - \frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)\Bigg]l_im_j +$$

$$+\Bigg[D(-C_{1214} + C_{1224}) + (-C_{1214} + C_{1224})\sqrt{2}Ks_1 +$$

$$+(-C_{1314} + C_{1324} - C_{2314} + C_{2324})\sqrt{2}K\overline{s}_0 +$$

$$+(-C_{1414} + C_{1424} - C_{2414} + C_{2424})\sqrt{2}\,Ks_0 -$$

$$-C_{1243}\left(\sqrt{2}K\overline{s}_0 - \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right)\Bigg]n_im_j +$$

$$+\Bigg[(-C_{1214} + C_{1224} + C_{1343} + C_{2343})\left(-\sqrt{2}K\overline{s}_0 + \frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\right) +$$

$$+D(-C_{1414} + C_{1424} - C_{2414} + C_{2424}) -$$

$$-(-C_{1414} + C_{1424} - C_{2414} + C_{2424})2\sqrt{2}\,Ks_1\Bigg]m_im_j +$$

$$+\Bigg[D(C_{1434} + C_{2434}) - (-C_{1414} + C_{1424} - C_{2414} + C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2} +$$

$$+C_{1243}\left(\sqrt{2}\,K\overline{s}_0 - \frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right) - (C_{1434} + C_{2434})\sqrt{2}\,Ks_1 +$$

$$+(C_{1413} - C_{1423} + C_{2413} - C_{2423})\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\Bigg]m_il_j +$$

$$\begin{aligned}
&+\Big[-C_{1243}\Big(\sqrt{2}\,K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}(\overline{R}^*)^2}\Big)+D(-C_{1434}-C_{2434})+\\
&+(-C_{1434}-C_{2434})\sqrt{2}\,Ks_1+(C_{1413}-C_{1423}+C_{2413}-C_{2423})\sqrt{2}\,K\overline{s}_0+\\
&+(-C_{1414}+C_{1424}-C_{2414}+C_{2424})\sqrt{2}Ks_0\Big]m_in_j+\\
&+\Big[-(C_{1213}-C_{1223})\Big(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\Big)+\\
&+D(C_{1413}-C_{1423}+C_{2413}-C_{2423})+\\
&+(C_{1434}+C_{2434})\Big(\sqrt{2}Ks_0+\frac{ia\sin\theta}{\sqrt{2}\overline{R}^2}\Big)\Big]m_i\overline{m}_j\Big\}+\{c.c.\}\Big\}.
\end{aligned}\tag{5.4}$$

Thus

$$H_{ij;k}l^k=0\Leftrightarrow P=Q=R=S=L=M=N=X=Y=Z=0,$$

where

$$\begin{aligned}
P=&D(-C_{1234})-(C_{1214}-C_{1224}+C_{1434}+C_{2434})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}-\\
&-(C_{1213}-C_{1223}+C_{1343}+C_{2343})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2},
\end{aligned}\tag{5.5p}$$

$$\begin{aligned}
Q=&D(-C_{1234})-(-C_{1214}+C_{1224})\sqrt{2}Ks_0+(C_{1434}+C_{2434})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}-\\
&-(C_{1213}-C_{1223})\sqrt{2}K\overline{s}_0+(C_{1343}+C_{2343})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2},
\end{aligned}\tag{5.5q}$$

$$\begin{aligned}
R=&D(-C_{1234})-(-C_{1214}+C_{1224})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+(C_{1434}+C_{2434})\sqrt{2}Ks_0-\\
&-(C_{1213}-C_{1223})\frac{-ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}+(-C_{1343}-C_{2343})\sqrt{2}K\overline{s}_0,
\end{aligned}\tag{5.5r}$$

$$\begin{aligned}
S=&D(-C_{1234})+(-C_{1214}+C_{1224})\sqrt{2}Ks_0-(C_{1434}+C_{2434})\sqrt{2}Ks_0+\\
&+(C_{1213}-C_{1223})\sqrt{2}K\overline{s}_0+(C_{1343}+C_{2343})\sqrt{2}K\overline{s}_0,
\end{aligned}\tag{5.5s}$$

$$\begin{aligned}
L=&-D(-C_{1214}+C_{1224})+(-C_{1214}+C_{1224})\sqrt{2}Ks_1+(-C_{1314}+\\
&+C_{1324}-C_{2314}+C_{2324})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}-(-C_{1414}+C_{1424}-\\
&-C_{2414}+C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+C_{1243}\Big(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\Big),
\end{aligned}\tag{5.5l}$$

$$\begin{aligned} M =&D(-C_{1214}+C_{1224})+(-C_{1214}+C_{1224})\sqrt{2}Ks_1+(-C_{1314}+\\ &+C_{1324}-C_{2314}+C_{2324})\sqrt{2}K\overline{s}_0+(-C_{1414}+C_{1424}-\\ &-C_{2414}+C_{2424})\sqrt{2}Ks_0-C_{1243}\left(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right), \end{aligned} \tag{5.5m}$$

$$\begin{aligned} N =&(-C_{1214}+C_{1224}+C_{1343}+C_{2343})\left(-\sqrt{2}K\overline{s}_0+\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)+\\ &+D(-C_{1414}+C_{1424}-C_{2414}+C_{2424})-\\ &-(-C_{1414}+C_{1424}-C_{2414}+C_{2424})2\sqrt{2}Ks_1, \end{aligned} \tag{5.5n}$$

$$\begin{aligned} X =&D(C_{1434}+C_{2434})-(-C_{1414}+C_{1424}-C_{2414}+C_{2424})\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}+\\ &+C_{1243}\left(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)-(C_{1434}+C_{2434})\sqrt{2}Ks_1+\\ &+(C_{1413}-C_{1423}+C_{2413}-C_{2423})\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}, \end{aligned} \tag{5.5x}$$

$$\begin{aligned} Y =&-C_{1243}\left(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)+D(-C_{1434}-C_{2434})+\\ &+(C_{1434}-C_{2434})\sqrt{2}Ks_1+(C_{1413}-C_{1423}+C_{2413}-\\ &-C_{2423})\sqrt{2}K\overline{s}_0+(-C_{1414}+C_{1424}-C_{2414}+C_{2424})\sqrt{2}Ks_0, \end{aligned} \tag{5.5y}$$

$$\begin{aligned} Z =&-(C_{1213}-C_{1223})\left(\sqrt{2}K\overline{s}_0-\frac{ia\sin\theta}{\sqrt{2}\,(\overline{R}^*)^2}\right)+(C_{1434}+C_{2434})\times\\ &\left(\sqrt{2}Ks_0+\frac{ia\sin\theta}{\sqrt{2}\,\overline{R}^2}\right)+D(C_{1413}-C_{1423}+C_{2413}-C_{2423}). \end{aligned} \tag{5.5z}$$

A special case of Petrov-type D gravitational field has been considered which is characterized by

$$\psi_0=0,\quad \psi_1=0,\quad \psi_3=0,\quad \psi_4=0\quad \text{and}\quad \psi_2\neq 0, \tag{5.6}$$

where

$$\begin{aligned} &\psi_0=-C_{1313}, && \psi_1=-\frac{1}{2}(C_{1213}+C_{1312}), \qquad \psi_2=C_{1324}\\ &\psi_3=\frac{1}{2}(C_{1224}+C_{2412}), && \psi_4=-C_{2424}. \end{aligned}$$

$$\text{Equations (5.6)}\Rightarrow\quad \begin{aligned} &C_{1313}=0, && (C_{1213}+C_{1312})=0,\\ &(C_{1224}+C_{2412})=0, && C_{2424}=0. \end{aligned} \tag{5.7}$$

We assume $s_0 = \dfrac{ia\sin\theta}{2K\overline{R}^2}$.

Consequently,

$$\begin{aligned}
&E_{ij;k}l^k = 0 \Leftrightarrow DC_{1212} = 0,\\
&DJ + L\left(\frac{\sqrt{2}\,ia\sin\theta}{\overline{R}^2}\right) + c.c. = 0\\
&4C_{1212} + C_{1324} + C_{2314} = 0,\\
&DL - C_{1314} - C_{2324} - 2\sqrt{2}Ks_1L = 0,
\end{aligned} \tag{5.8}$$

where

$$J = C_{1413} + C_{1423} + C_{2413} + C_{2423}, \quad L = C_{1214} - C_{2412}.$$

The necessary and sufficient condition for

$$\begin{aligned}
&H_{ij;k}l^k = 0 \Leftrightarrow DC_{1434} = 0,\\
&Re(C_{1214} + C_{2412}) = 0,\\
&s_1 = 0,\\
&DL + \frac{ia\sin\theta}{\sqrt{2}}\left[C_{1424} + C_{1243}\left(\frac{2}{\overline{R}^2}\right)M\frac{1}{(\overline{R}^*)^2}\right] = 0,\\
&DC_{1424} - N\frac{ia\sin\theta}{\sqrt{(\overline{R}^*)^2}} = 0,\\
&DM = 0,
\end{aligned} \tag{5.9}$$

where

$$M = C_{1314} - C_{1324} + C_{2314} - C_{2324}, \quad N = C_{1214} + C_{2412} - C_{1213} + C_{1223}.$$

Due to large number of components of Weyl tensor and their lengthy and complicated expressions in the equations (3.6) the interpretation of the propagation equations of electric part and magnetic part of the Weyl tensor is difficult hence avoided.

Appendix

The covariant derivatives of null tetrad vector fields in terms of its tetrad components are expressed below:

$$\begin{aligned}
l_{i;j} = &(\gamma^0 + \gamma_1 + \overline{\gamma^0} + \overline{\gamma_1})l_il_j - (\alpha^0 + \alpha_1 + \overline{\beta^0} + \overline{\beta_1})l_im_j -\\
&-(\overline{\alpha^0} + \overline{\alpha_1} + \beta^0 + \beta_1)l_i\overline{m}_j + (\epsilon^0 + \epsilon_1 + \overline{\epsilon^0} + \overline{\epsilon_1})l_in_j - (\overline{\tau^0} + \overline{\tau_1})m_il_j +
\end{aligned}$$

$$+(\overline{\sigma^0}+\overline{\sigma_1})m_im_j+(\overline{\rho^0}+\overline{\rho_1})m_i\overline{m}_j-(\overline{\kappa^0}+\overline{\kappa_1})m_in_j-(\tau^0+\tau_1)\overline{m}_il_j+$$
$$+(\rho^0+\rho_1)\overline{m}_im_j+(\sigma^0+\sigma_1)\overline{m}_i\overline{m}_j-(\kappa^0+\kappa_1)\overline{m}_in_j,$$
$$n_{i;j}=-(\epsilon^0+\epsilon_1+\overline{\epsilon^0}+\overline{\epsilon_1})n_in_j-(\gamma^0+\gamma_1+\overline{\gamma^0}+\overline{\gamma_1})n_il_j-(\overline{\alpha^0}+\overline{\alpha_1}+$$
$$+\beta^0+\beta_1)n_i\overline{m}_j-(\alpha^0+\alpha_1+\overline{\beta^0}+\overline{\beta_1})n_im_j-(\overline{\pi^0}+\overline{\pi_1})\overline{m}_in_j+$$
$$+(\overline{\nu^0}+\overline{\nu_1})\overline{m}_il_j-(\overline{\lambda^0}+\overline{\lambda_1})\overline{m}_i\overline{m}_j-(\overline{\mu^0}+\overline{\mu_1})\overline{m}_im_j+(\pi^0+\pi_1)m_in_j+$$
$$+(\nu^0+\nu_1)m_il_j-(\mu^0+\mu_1)m_i\overline{m}_j-(\lambda^0+\lambda_1)m_im_j,$$
$$m_{i;j}=-(\kappa^0+\kappa_1)n_in_j-(\tau^0+\tau_1)n_il_j+(\sigma^0+\sigma_1)n_i\overline{m}_j+(\rho^0+\rho_1)n_im_j+$$
$$+(\overline{\pi^0}+\overline{\pi_1})l_in_j+(\overline{\nu^0}+\overline{\nu_1})l_il_j-(\overline{\lambda^0}+\overline{\lambda_1})l_i\overline{m}_j-(\overline{\mu^0}+\overline{\mu_1})l_im_j+$$
$$+(\epsilon^0+\epsilon_1-\overline{\epsilon^0}-\overline{\epsilon_1})m_in_j+(\gamma^0+\gamma_1-\overline{\gamma^0}-\overline{\gamma_1})m_il_j+$$
$$+(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1)m_i\overline{m}_j-(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1})m_im_j,$$
$$\overline{m}_{i;j}=-(\overline{\kappa^0}+\overline{\kappa_1})n_in_j-(\overline{\tau^0}+\overline{\tau_1})n_il_j+(\overline{\rho^0}+\overline{\rho_1})n_i\overline{m}_j+(\overline{\sigma^0}+\overline{\sigma_1})n_im_j+$$
$$+(\pi^0+\pi_1)l_in_j+(\nu^0+\nu_1)l_il_j-(\mu^0+\mu_1)l_i\overline{m}_j-(\lambda^0+\lambda_1)l_im_j-$$
$$-(\epsilon^0+\epsilon_1-\overline{\epsilon^0}-\overline{\epsilon_1})\overline{m}_in_j-(\gamma^0+\gamma_1-\overline{\gamma^0}-\overline{\gamma_1})\overline{m}_il_j-$$
$$-(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1)\overline{m}_i\overline{m}_j+(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1})\overline{m}_im_j. \tag{A1.1}$$

The intrinsic derivatives l_i in the directions of the tetrad vector fields are given by

$$l_{i;j}l^j=(\epsilon^0+\epsilon_1+\overline{\epsilon^0}+\overline{\epsilon_1})l_i-(\kappa^0+\kappa_1)\overline{m}_i-(\overline{\kappa^0}+\overline{\kappa_1})m_i,$$
$$l_{i;j}n^j=(\gamma^0+\gamma_1+\overline{\gamma^0}+\overline{\gamma_1})l_i-(\tau^0+\tau_1)\overline{m}_i-(\overline{\tau^0}+\overline{\tau_1})m_i,$$
$$l_{i;j}m^j=(\overline{\alpha^0}+\overline{\alpha_1}+\beta^0+\beta_1)l_i-(\sigma^0+\sigma_1)\overline{m}_i-(\overline{\rho^0}+\overline{\rho_1})m_i,$$
$$l_{i;j}\overline{m}^j=(\alpha^0+\alpha_1+\overline{\beta^0}+\overline{\beta_1})l_i-(\rho^0+\rho_1)\overline{m}_i-(\overline{\sigma^0}+\overline{\sigma_1})m_i,$$
$$n_{i;j}l^j=-(\epsilon^0+\epsilon_1+\overline{\epsilon^0}+\overline{\epsilon_1})n_i+(\overline{\pi^0}+\overline{\pi_1})\overline{m}_i+(\pi^0+\pi_1)m_i,$$
$$n_{i;j}n^j=-(\gamma^0+\gamma_1+\overline{\gamma^0}+\overline{\gamma_1})n_i+(\overline{\nu^0}+\overline{\nu_1})\overline{m}_i-(\nu^0+\nu_1)m_i,$$
$$n_{i;j}m^j=(\overline{\alpha^0}+\overline{\alpha_1}+\beta^0+\beta_1)n_i+(\overline{\lambda^0}+\overline{\lambda_1})\overline{m}_i+(\mu^0+\mu_1)m_i,$$

$$
\begin{aligned}
n_{i;j}\overline{m}^j &= (\alpha^0+\alpha_1+\overline{\beta^0}+\overline{\beta_1})n_i+(\overline{\mu^0}+\overline{\mu_1})\overline{m}_i-(\lambda^0+\lambda_1)m_i,\\
m_{i;j}l^j &= -(\kappa^0+\kappa_1)n_i+(\overline{\pi^0}+\overline{\pi_1})l_i+(\epsilon^0+\epsilon_1-\overline{\epsilon^0}-\overline{\epsilon_1})m_i,\\
m_{i;j}n^j &= -(\tau^0+\tau_1)n_i+(\overline{\nu^0}+\overline{\nu_1})l_i+(\gamma^0+\gamma_1-\overline{\gamma^0}-\overline{\gamma_1})m_i,\\
m_{i;j}m^j &= -(\sigma^0+\sigma_1)n_i+(\overline{\lambda^0}+\overline{\lambda_1})l_i-(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1)m_i,\\
m_{i;j}\overline{m}^j &= -(\rho^0+\rho_1)n_i+(\overline{\mu^0}+\overline{\mu_1})l_i+(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1})m_i,\\
\overline{m}_{i;j}l^j &= -(\overline{\kappa^0}+\overline{\kappa_1})n_i+(\pi^0+\pi_1)l_i-(\epsilon^0+\epsilon_1-\overline{\epsilon^0}-\overline{\epsilon_1})\overline{m}_i,\\
\overline{m}_{i;j}n^j &= -(\overline{\tau^0}+\overline{\tau_1})n_i+(\nu^0+\nu_1)l_i+(\gamma^0+\gamma_1-\overline{\gamma^0}-\overline{\gamma_1})\overline{m}_i,\\
\overline{m}_{i;j}m^j &= -(\overline{\rho^0}+\overline{\rho_1})n_i+(\mu^0+\mu_1)l_i+(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1)\overline{m}_i,\\
\overline{m}_{i;j}\overline{m}^j &= -(\overline{\sigma^0}+\overline{\sigma_1})n_i+(\lambda^0+\lambda_1)l_i-(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1})\overline{m}_i.
\end{aligned}
\tag{A1.2}
$$

Similarly, the projections of the intrinsic derivatives of l_i are given by

$$
\begin{aligned}
l_{i;j}l^i &= 0,\\
l_{i;j}n^i &= (\gamma^0+\gamma_1+\overline{\gamma^0}+\overline{\gamma_1})l_j-(\alpha^0+\alpha_1+\overline{\beta^0}+\overline{\beta_1})m_j-\\
&\quad -(\overline{\alpha^0}+\overline{\alpha_1}+\beta^0+\beta_1)\overline{m}_j+(\epsilon^0+\epsilon_1+\overline{\epsilon^0}+\overline{\epsilon_1})n_j,\\
l_{i;j}m^i &= (\tau^0+\tau_1)l_j-(\rho^0+\rho_1)m_j-(\sigma^0+\sigma_1)\overline{m}_j+(\kappa^0+\kappa_1)n_j,\\
l_{i;j}\overline{m}^i &= (\overline{\tau^0}+\overline{\tau_1})l_j-(\overline{\sigma^0}+\overline{\sigma_1})m_j-(\overline{\rho^0}+\overline{\rho_1})\overline{m}_j+(\overline{\kappa^0}+\overline{\kappa_1})n_j,\\
n_{i;j}l^i &= -(\epsilon^0+\epsilon_1+\overline{\epsilon^0}+\overline{\epsilon_1})n_j-(\gamma^0+\gamma_1+\overline{\gamma^0}+\overline{\gamma_1})l_j-\\
&\quad -(\overline{\alpha^0}+\overline{\alpha_1}+\beta^0+\beta_1)\overline{m}_j-(\alpha^0+\alpha_1+\overline{\beta^0}+\overline{\beta_1})m_j,\\
n_{i;j}n^i &= 0,\\
n_{i;j}m^i &= -(\overline{\pi^0}+\overline{\pi_1})n_j-(\overline{\nu^0}+\overline{\nu_1})l_j+(\overline{\lambda^0}+\overline{\lambda_1})\overline{m}_j+(\overline{\mu^0}+\overline{\mu_1})m_j,\\
n_{i;j}\overline{m}^i &= -(\pi^0+\pi_1)n_j-(\nu^0+\nu_1)l_j+(\mu^0+\mu_1)\overline{m}_j+(\lambda^0+\lambda_1)m_j,\\
m_{i;j}l^i &= -(\kappa^0+\kappa_1)n_j-(\tau^0+\tau_1)l_j+(\sigma^0+\sigma_1)\overline{m}_j+(\rho^0+\rho_1)m_j,\\
m_{i;j}n^i &= (\overline{\pi^0}+\overline{\pi_1})n_j+(\overline{\nu^0}+\overline{\nu_1})l_j-(\overline{\lambda^0}+\overline{\lambda_1})\overline{m}_j-(\overline{\mu^0}+\overline{\mu_1})m_j,
\end{aligned}
$$

$$
\begin{aligned}
m_{i;j}m^i =& 0,\\
m_{i;j}\overline{m}^i =& -(\epsilon^0+\epsilon_1-\overline{\epsilon^0}-\overline{\epsilon_1})n_j-(\gamma^0+\gamma_1-\overline{\gamma^0}-\overline{\gamma_1})l_j-\\
&-(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1)\overline{m}_j-(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1})m_j,\\
\overline{m}_{i;j}l^i =& -(\overline{\kappa^0}+\overline{\kappa_1})n_j-(\overline{\tau^0}+\overline{\tau_1})l_j+(\overline{\rho^0}+\overline{\rho_1})\overline{m}_j+(\overline{\sigma^0}+\overline{\sigma_1})m_j,\\
\overline{m}_{i;j}n^i =& (\pi^0+\pi_1)n_j+(\nu^0+\nu_1)l_j-(\mu^0+\mu_1)\overline{m}_j-(\lambda^0+\lambda_1)m_j,\\
\overline{m}_{i;j}m^i =& (\epsilon^0+\epsilon_1-\overline{\epsilon^0}-\overline{\epsilon_1})n_j+(\gamma^0+\gamma_1-\overline{\gamma^0}-\overline{\gamma_1})l_j+\\
&+(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1)\overline{m}_j-(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1})m_j,\\
\overline{m}_{i;j}\overline{m}^i =& 0.
\end{aligned}
\tag{A1.3}
$$

Divergence of the tetrad vector fields are given by

$$
\begin{aligned}
l^i{}_{;i} =& (\epsilon^0+\epsilon_1+\overline{\epsilon^0}+\overline{\epsilon_1})-(\rho^0+\rho_1)-(\overline{\rho^0}+\overline{\rho_1}),\\
n^i{}_{;i} =& -(\gamma^0+\gamma_1+\overline{\gamma^0}+\overline{\gamma_1})+(\overline{\mu^0}+\overline{\mu_1})+(\mu^0+\mu_1),\\
m^i{}_{;i} =& -(\tau^0+\tau_1)+(\overline{\pi^0}+\overline{\pi_1})-(\overline{\alpha^0}+\overline{\alpha_1}-\beta^0-\beta_1),\\
\overline{m}^i{}_{;i} =& -(\overline{\tau^0}+\overline{\tau_1})+(\pi^0+\pi_1)-(\alpha^0+\alpha_1-\overline{\beta^0}-\overline{\beta_1}).
\end{aligned}
\tag{A1.4}
$$

References

[1] **Ahsan, Z. :** Electric and magnetic Weyl tensors, Indian J. of Pure and Applied Math., 30 no. 9 (1999), 863-869.

[2] **Ellis, G F R. :** Relativistic cosmology, Cargse Lectures in Physics vol. VI, ed E Schatzmann (New York: Gordon and Breach), (1973).

[3] **Greenberg, P. J. :** Propagation equations for the set of natural geometrical invariants in relativistic hydrodynamics, J. math. Analysis and Appli., 29 no. 3 (1970), 647-667.

[4] **Haddow, B. M. :** Purely magnetic spacetimes, J. Math. Phys., 36 no. 10 (1995), 5848-5854.

[5] **Hasmani, A. H. and Katkar, L. N. :** A study of electric part of Weyl tensor in the Gdel universe, Aligarh bulletin of mathematics, 27 no. 1 (2008), 47-52.

[6] **Helh, F. W., Von der Heyde, P. and Kerlik, G. D. :** General relativity with spin and torsion and its deviations from Einstein's theory, Phys., Rev. D.10, (1974), 1066-1069.

[7] **Helh, F. W., Von der Heyde, P., Kerlik, G. D. and Nester, J. M. :** General relativity with spin and torsion: Foundations and prospects, Rev. Mod. Phys., 48 (1976), 393-416.

[8] **Herrera, L. A., Santos, N. O. and Carot, J. :** Gravitational radiation, vorticity and the electric and magnetic part of the Weyl tensor, J. Math Phys., 47 no. 5 (2006), 052502.

[9] **Jogia, S. and Griffiths, J. B. :** A Newman-Penrose type formalism for space-times with torsion, Gen. Relativ. Gravit., 12, no. 8 (1980), 597-617.

[10] **Katkar, L. N. :** Structure equations in U4 theory of gravitation, Int. J. Theor. Phys., 48 no. 3 (2009), 874-884.

[11] **Katkar, L. N. and Patil, V. K. :** On spacelike congruences in Riemann-Cartan space-time, Int J. Theor. Phys., 48 no. 11 (2009), 3035-3043.

[12] **Maartens, R., Ellis, G F. R. and Siklos, S. T. C. :** Local freedom in the gravitational field, Class Quant. Grav., 14 (1997), 1927.

[13] **Trautman, A. :** On the structure of the Einstein-Cartan equations, Symp. Math., 12 (1973), 139- 162.