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Mathematical Model for Malaria Transmission and Biological Control

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Abstract

In this paper, a nonlinear mathematical model for the control of vector borne diseases, like malaria is proposed and analyzed. In the modeling process it is assumed that the mosquito population is controlled by using larvivorous fish, which partially depends on the larva of mosquito population. It is further assumed that the mosquito population grows logistically. The equilibria of the model are obtained and their stability is discussed by using stability theory of differential equations. Further numerical simulation is performed to verify the analytically obtained results.

Keywords : Mathematical model, human population, mosquito population, Larvivorous fish population.

1. Introduction

Vector transmitted diseases are known to induce major behavioral and economic changes in tropical and subtropical region, in which mosquito-transmitted diseases are most prevalent illness and are responsible for many life-threatening diseases, e.g. malaria, yellow fever, dengue fever and chikangunya etc. Out of these mosquito borne serious illness, the malaria is a highly complex disease in humans caused by several species of mosquito-borne parasite (Plasmodium falciparum, vivax, malariae, knowlesi and ovale) and it is endemic in many parts of the World, infecting between 350-500 million people per year. Malaria is transmitted exclusively by genus Anopheles. In the last several years, so many efforts have been made to reduce the incidence of malaria focusing on reducing the number of mosquitoes and preventing mosquito bites. However, most of these efforts, especially the ones that use pesticides, have been banned by the environmental protection organizations as they have an adverse effect on non-targeted population. Despite more than hundred years of a serious efforts for its control, malaria still ranks as one of the most widespread and prevalent infectious diseases. Also due to continuous application of pesticides, mosquitoes have developed resistance to these chemicals and now they are not so effective. This suggests us that looking for an alternative method of control of mosquitoes and biological control seems to be environmental friendly method to control mosquito population. Biological control means introduction or manipulation of organisms to subdue vector population. Biological control, particularly using Larvivorous fish plays a very positive role in controlling mosquitoes. The method of control of mosquito using Larvivorous fish is not new, it has been implemented since 1937 in many parts of the world. But control of mosquitoes using pesticides was fast so it suppressed this conventional method of control of mosquitoes. Now again this method of larval control is accepted and successfully implemented in many part of world.

Although there are lots of experimental studies to evaluate the efficiency of this method of control but not many researchers have explored mathematical modeling of this method of control of vectors.

2. Mathematical model

Let $N(t)$ be the total human population density in the region under consideration at any time t. Which is divided into two subclasses namely, $X(t)$ as susceptible class and infective class with density as $Y(t)$. $M_s(t)$ as density of susceptible mosquitoes, $M_i(t)$ as density of infective mosquitoes. $B(t)$ as population density of Larvivorous fish. A is the constant immigration rate, the constant β represents the transmission rate of susceptible to the infective class, constant d and ν denote the natural death rate and recovery rate of human population respectively. The constant α represents disease induced death rate and σ denotes growth rate of mosquito population. L is carrying capacity of mosquitoes. The constant θ is death rate of mosquito due to crowding, constant θ_0 is death rate of mosquitoes and θ_1 is the depletion rate of mosquitoes due to Larvivorous fish. λ is the transmission rate of mosquito population. Constant γ is growth rate of Larvivorous fish and constant γ_0 is the growth rate of fish due to mosquito population. γ_1 is the growth rate of Larvivorous fish due to uptake of Larva of mosquito population.

$$
\begin{aligned}
\frac{dX}{dt} &= A - \beta X M_i - dX + \nu Y, \\
\frac{dY}{dt} &= \beta X M_i - (d + \nu + \alpha) Y, \\
\frac{dM_s}{dt} &= \sigma M - \frac{\theta M^2}{L} - \theta_1 BM - \theta_0 M_s - \lambda M_s Y, \\
\frac{dM_i}{dt} &= \lambda M_s Y - \theta_0 M_i, \\
\frac{dB}{dt} &= \gamma B - \frac{\gamma_0 B^2}{K} + \gamma_1 MB.
\end{aligned} \tag{2.1}
$$

Using $N = X + Y$, $M = M_s + M_i$ and $\sigma - \theta_0 = \theta$, the reduced model is

$$
\begin{aligned}
\frac{dY}{dt} &= \beta(N - Y) M_i - (d + \nu + \alpha) Y, \\
\frac{dN}{dt} &= A - dN - \alpha Y, \\
\frac{dM_i}{dt} &= \lambda (M - M_i) Y - \theta_0 M_i, \\
\frac{dM}{dt} &= \theta M (1 - \frac{M}{L}) - \theta_1 BM, \\
\frac{dB}{dt} &= \gamma B - \frac{\gamma_0 B^2}{K} + \gamma_1 MB.
\end{aligned} \tag{2.2}
$$

It suffices to study the model system (2.2).

The region of attraction for all solutions initiating in the positive orthant is given by

$$\Omega := \left\{ (Y, N, M_i, M, B) : 0 \le Y \le N \le \frac{A}{d}, 0 \le M_i \le M \le M_m, 0 \le B \le B_m \right\},$$

where $M_m = \dfrac{L\sigma}{\theta}$ and $B_m = \dfrac{K}{\gamma_0} \left\{ \gamma + \dfrac{\gamma_1 L \sigma}{\theta} \right\}$.

3. Equilibrium analysis

The model system (2.2) has the following six non-negative equilibria:

(i) $E_0 \left(0, \frac{A}{d}, 0, 0, 0\right)$ which always exists.
(ii) $E_1 \left(0, \frac{A}{d}, 0, L, 0\right)$ which always exists.
(iii) $E_2 \left(0, \frac{A}{d}, 0, 0, \frac{K\gamma}{\gamma_0}\right)$ which always exists.

(iv) $E_3\left(0, \frac{A}{d}, 0, M_3, B_3\right)$ exists provided

$$\theta - \frac{\theta_1 K\gamma}{\gamma_0} > 0. \tag{3.1}$$

(v) $E_4(Y_4, N_4, M_{i_4}, M_4, 0)$ exists provided

$$\frac{\beta A\lambda L}{d\theta_0(d+\nu+\alpha)} > 1. \tag{3.2}$$

(vi) $E^*(Y^*, N^*, M_i^*, M^*, B^*)$ exists provided

$$\frac{\beta A\lambda\left[\theta - \frac{\theta_1 K\gamma}{\gamma_0}\right]}{d\theta_0(d+\nu+\alpha)\left(\frac{\theta}{L} + \frac{\theta_1 K\gamma_1}{\gamma_0}\right)} > 1, \tag{3.3}$$

$$\theta - \frac{\theta_1 K\gamma}{\gamma_0} > 0. \tag{3.4}$$

Now, it is sufficient to show the existence of equilibria E_4 and E^*.

Existence of E_4

In the equilibrium $E_4(Y_4, N_4, M_{i_4}, M_4, 0)$, the values of Y_4, N_4, M_{i_4} and M_4 may be obtained by solving the following algebraic equations:

$$\begin{aligned}
\beta(N-Y)M_i - (d+\nu+\alpha)Y &= 0, && (3.5)\\
A - dN - \alpha Y &= 0, && (3.6)\\
\lambda(M-M_i)Y - \theta_0 M_i &= 0, && (3.7)\\
\theta(1-\frac{M}{L}) &= 0. && (3.8)
\end{aligned}$$

From equation (3.6), we have

$$N = \frac{A-\alpha Y}{d}. \tag{3.9}$$

From equation (3.7), we have

$$M_i = \frac{\lambda M Y}{\theta_0 + \lambda Y}. \tag{3.10}$$

From equation (3.8), we have

$$M = L. \tag{3.11}$$

From equation (3.5), we have

$$\beta\left[\frac{A-(\alpha+d)Y}{d}\right]\frac{\lambda L Y}{(\theta_0+\lambda Y)} - (d+\nu+\alpha)Y = 0. \tag{3.12}$$

Let $Y \neq 0$, we have

$$f(Y) = \beta \left[\frac{A - (\alpha + d)Y}{d} \right] \frac{\lambda L}{(\theta_0 + \lambda Y)} - (d + \nu + \alpha). \tag{3.13}$$

From equation (3.13), we note the following:

(i) $f(0) > 0$ provided condition (3.2) is satisfied.

(ii) $f\left(\dfrac{A}{\alpha + d}\right) < 0$.

(iii) $f'(Y) < 0$.

Thus there exists a unique positive root, say Y_2^* of equation (3.13) in the interval $\left(0, \frac{A}{\alpha+d}\right)$. Using this value of Y_4 in equations (3.9)-(3.11) we can get the values of N_4, M_{i_4} and M_4, respectively. Thus the equilibrium $E_4(Y_4, N_4, M_{i_4}, M_4, 0)$, exists if condition (3.2) holds.

Existence of E^*

This equilibrium E^* may be obtained by solving the following set of algebraic equations:

$$\beta(N - Y)M_i - (d + \nu + \alpha)Y = 0, \tag{3.14}$$

$$A - dN - \alpha Y = 0, \tag{3.15}$$

$$\lambda(M - M_i)Y - \theta_0 M_i = 0, \tag{3.16}$$

$$\theta - \frac{\theta M}{L} - \theta_1 B = 0, \tag{3.17}$$

$$\gamma - \frac{\gamma_0 B}{K} + \gamma_1 M = 0. \tag{3.18}$$

From equation (3.15), we have

$$N = \frac{A - \alpha Y}{d}. \tag{3.19}$$

From equation (3.16), we have

$$M_i = \frac{\lambda M Y}{\theta_0 + \lambda Y}. \tag{3.20}$$

From equation (3.18), we have

$$B = \frac{K}{\gamma_0}(\gamma + \gamma_1 M). \tag{3.21}$$

From equation (3.17), we have

$$M = \left[\frac{\theta - \frac{\theta_1 K\gamma}{\gamma_0}}{\frac{\theta}{L} + \frac{\theta_1 K\gamma_1}{\gamma_0}}\right] \tag{3.22}$$

From equation (3.14), we have

$$\beta\left[\frac{A-(\alpha+d)Y}{d}\right]\left[\frac{\lambda Y}{\theta_0+\lambda Y}\right]\left[\frac{\theta - \frac{\theta_1 K\gamma}{\gamma_0}}{\frac{\theta}{L} + \frac{\theta_1 K\gamma_1}{\gamma_0}}\right] - (d+\nu+\alpha)Y = 0. \tag{3.23}$$

Let $Y \neq 0$ then we have

$$F(Y) = \beta\left[\frac{A-(\alpha+d)Y}{d}\right]\left[\frac{\lambda}{\theta_0+\lambda Y}\right]\left[\frac{\theta - \frac{\theta_1 K\gamma}{\gamma_0}}{\frac{\theta}{L} + \frac{\theta_1 K\gamma_1}{\gamma_0}}\right] - (d+\nu+\alpha). \tag{3.24}$$

From equation (3.24), we note the following

(i) $F(0) > 0$, provided condition (3.3) is satisfied.

(ii) $F\left(\dfrac{A}{\alpha+d}\right) < 0.$

(iii) $F'(Y) < 0.$

Thus there exists a unique positive root Y^* of equation (3.24) if condition (3.3) is satisfied. Using this value of Y^* in equation (3.22), we get positive value of $M(\text{say}, M^*)$ if the condition (3.4) is satisfied. From equations (3.19)-(3.21), we can get positive values of N^*, M_i^* and B^*. Thus the equilibrium $E^*(Y^*, N^*, M_i^*, M^*, B^*)$ exists if the conditions (3.3) and (3.4) are satisfied.

4. Stability analysis

In this section, we investigate the local stability of the equilibria E_0, E_1, E_2, E_3, E_4 and E^* by determining the sign of the eigenvalues of Jacobian matrix corresponding to each equilibria. The Jacobian matrix for the model system (2.2) is given as follows:

$$J = \begin{pmatrix} -a_{11} & \beta M_i & \beta(N-Y) & 0 & 0 \\ -\alpha & -d & 0 & 0 & 0 \\ \lambda(M-M_i) & 0 & -a_{33} & \lambda Y & 0 \\ 0 & 0 & 0 & a_{44} & -\theta_1 M \\ 0 & 0 & 0 & \gamma_1 B & a_{55} \end{pmatrix},$$

where $a_{11} = \beta M_i + (d+\nu+\alpha)$, $a_{33} = \lambda Y + \theta_0$, $a_{44} = \theta - \dfrac{2\theta M}{L} - \theta_1 B$ and $a_{55} = \gamma - \dfrac{2\gamma_0 B}{K} + \gamma_1 M.$

Let J_0, J_1, J_2, J_3, J_4 and J^* be the Jacobian matrices evaluated at equilibria E_0, E_1, E_2, E_3, E_4 and E^*, respectively, which are quite obvious.

The stability behavior of interior equilibrium E^* is not obvious from the corresponding Jacobian matrix. The following theorems give sufficient conditions for local and non-linear stability of equilibrium E^*.

Theorem 1. *Let the following inequality hold:*

$$\beta^2\lambda^2(N^* - Y^*)^2(M^* - M_i^*)^2 < \frac{4}{9}(\lambda Y^* + \theta_0)^2(\beta M_i^* + d + \nu + \alpha)^2, \qquad (4.1)$$

then E^* is locally asymptotically stable.

Proof. Linearizing the model system (2.2) about E^* by using the following transformations

$$Y = Y^* + y,\ N = N^* + n,\ M_i = M_i^* + m_i,\ M = M^* + m \text{ and}$$
$$B = B^* + b.$$

where y, n, m_i, m and b are small perturbations around the equilibrium E^*. Now using the following positive definite function:

$$V = \frac{1}{2}y^2 + \frac{k_1}{2}n^2 + \frac{k_2}{2}m_i^2 + \frac{k_3}{2M^*}m^2 + \frac{k_4}{2B^*}b^2, \qquad (4.2)$$

(where k_1, k_2, k_3 and k_4 are positive constants to be chosen appropriately)

Differentiating above equation with respect to t along the solutions of linearized system of (2.2), we get

$$\begin{aligned}\dot{V} &= -(\beta M_i^* + d + \nu + \alpha)y^2 - k_1 d n^2 - k_2(\lambda Y^* + \theta_0)m_i^2 - \frac{k_3\theta}{L}m^2 \\ &\quad -\frac{k_4\gamma_0}{K}b^2 + [\beta M_i^* - k_1\alpha]yn + [\beta(N^* - Y^*) + \lambda(M^* - M_i^*)k_2]ym_i \\ &\quad +k_2\lambda Y^* m_i m + [k_4\gamma_1 - k_3\theta_1]bm.\end{aligned}$$

Choose $k_1 = \frac{\beta M_i^*}{\alpha}$ and $k_4 = \frac{k_3\theta_1}{\gamma_1}$, we have

$$\begin{aligned}\dot{V} &= -(\beta M_i^* + d + \nu + \alpha)y^2 - \frac{\beta M_i^* d}{\alpha}n^2 \\ &\quad -k_2(\lambda Y^* + \theta_0)m_i^2 - \frac{k_3\theta}{L}m^2 \\ &\quad -\frac{k_3\theta_1\gamma_0}{\gamma_1 K}b^2 + \beta(N^* - Y^*)ym_i \\ &\quad +k_2\lambda(M^* - M_i^*)ym_i + k_2\lambda Y^* m_i m.\end{aligned}$$

$\dot{V}$ will be negative definite if the following conditions are satisfied

$$\beta^2(N^*-Y^*)^2 < \frac{2}{3}k_2(\lambda Y^*+\theta_0)(\beta M_i^*+d+\nu+\alpha) \quad (4.3)$$

$$k_2\lambda^2(M^*-M_i^*)^2 < \frac{2}{3}(\lambda Y^*+\theta_0)(\beta Y^*+d+\nu+\alpha) \quad (4.4)$$

$$k_2\lambda^2 Y^{*2} < \frac{2k_3\theta}{3L}(\lambda Y^*+\theta_0) \quad (4.5)$$

From inequalities (4.3) and (4.4), we can get a positive value of k_2 provided the condition (4.1) holds. Further from the inequalities (4.5), we can get a positive values of k_3, provided the condition (4.1) holds. Hence the proof.

Furthermore, to establish the non-linear stability of the equilibrium E^*, we employ Liapunov's stability theory. Thus, we obtain following results regarding the global stability of equilibrium E^*,

Theorem 2. *The equilibrium E^*, if exists, is globally asymptotically stable in Ω, provided*

$$\beta^2\lambda^2(N^*-Y^*)^2(M^*-M_i^*)^2 < \frac{4}{9}\theta_0^2(d+\nu+\alpha)^2. \quad (4.6)$$

Proof. Consider the following positive definite function:

$$\begin{aligned} W &= \frac{1}{2}(Y-Y^*)^2+\frac{m_1}{2}(N-N^*)^2+\frac{m_2}{2}(M_i-M_i^*)^2 \\ &\quad +m_3\left(M-M^*-M^*\ln\frac{M}{M^*}\right)+m_4\left(B-B^*-B^*\ln\frac{B}{B^*}\right). \end{aligned}$$

where m_1, m_2, m_3 and m_4 are positive constants to be chosen appropriately.

Now differentiating W with respect to t along the solution of model system (2.2), we get

$$\begin{aligned} \dot{W} &= -(\beta M_i+d+\nu+\alpha)(Y-Y^*)^2-m_1 d(N-N^*)^2 \\ &\quad -m_2(\theta_0+\lambda Y)(M_i-M_i^*)^2-\frac{m_3\theta}{L}(M-M^*)^2 \\ &\quad -\frac{m_4\gamma_0}{K}(B-B^*)^2+\beta(N^*-Y^*)(Y-Y^*)(M_i-M_i^*) \\ &\quad +m_2\lambda(M^*-M_i^*)(Y-Y^*)(M_i-M_i^*) \\ &\quad +m_2\lambda Y(M-M^*)(M_i-M_i^*) \\ &\quad +[\beta M_i-m_1\alpha](Y-Y^*)(N-N^*) \\ &\quad +[m_4\gamma_1-m_3\theta_1](M-M^*)(B-B^*). \end{aligned}$$

Choosing $m_1 = \frac{\beta M_i^*}{\alpha}$ and $m_4 = \frac{m_3\theta_1}{\gamma_1}$, we have

$$\begin{aligned}\dot{W} &= -(\beta M_i + d + \nu + \alpha)(Y - Y^*)^2 - m_1 d(N - N^*)^2 \\ &\quad -m_2(\theta_0 + \lambda Y)(M_i - M_i^*)^2 - \frac{m_3\theta}{L}(M - M^*)^2 \\ &\quad -\frac{m_4\gamma_0}{K}(B - B^*)^2 + \beta(N^* - Y^*)(Y - Y^*)(M_i - M_i^*) \\ &\quad +m_2\lambda(M^* - M_i^*)(Y - Y^*)(M_i - M_i^*) \\ &\quad +m_2\lambda Y(M - M^*)(M - M_i^*).\end{aligned}$$

$\dot{W}$ can be made negative definite inside the region of attraction Ω if the following condition are satisfied

$$\beta^2(N^* - Y^*)^2 < \frac{2}{3}m_2\theta_0(d + \nu + \alpha), \tag{4.7}$$

$$m_2\lambda^2(M^* - M_i^*)^2 < \frac{2}{3}\theta_0(d + \nu + \alpha), \tag{4.8}$$

$$m_2\lambda^2\frac{A^2}{d^2} < \frac{2}{3L}m_3\theta\theta_0, \tag{4.9}$$

From inequalities (4.7) and (4.8), we can get a positive value of m_2 provided the condition (4.6) holds. Further, from the inequalities (4.9), we can obtain positive value of m_3. provided the condition (4.6) holds. Hence the proof.

5. Numerical simulation

To confirm the analytically obtained results and to illustrate the dynamical behavior of the system, numerical simulation has been carried out using MATLAB 7.0.5. We have taken the following set of parameter values in model system (2.2):

$$\begin{aligned}&A = 10,\ \beta = 0.000001,\ d = 0.00004,\ \nu = 0.2,\ \alpha = 0.001, \lambda = 0.00001, \\ &\theta_0 = 0.2,\ \sigma = 0.4,\ \theta = 0.00001,\ L = 100,\ \theta_1 = 0.004,\ \gamma = 0.01, \\ &K = 1000,\ \gamma_0 = 1.2,\ \gamma_1 = 0.000001.\end{aligned}$$

For the above set of parameter values it may be checked that the condition of existence of endemic equilibrium E^* and the global stability condition (i.e. 4.6) are satisfied. The equilibrium components are found as follows:

$Y^* = 5546.238293,\ N^* = 111344.0427,\ M^* = 48543.68932,\ {M_i}^* = 10539.11991,$

$B^* = 48.78640777.$

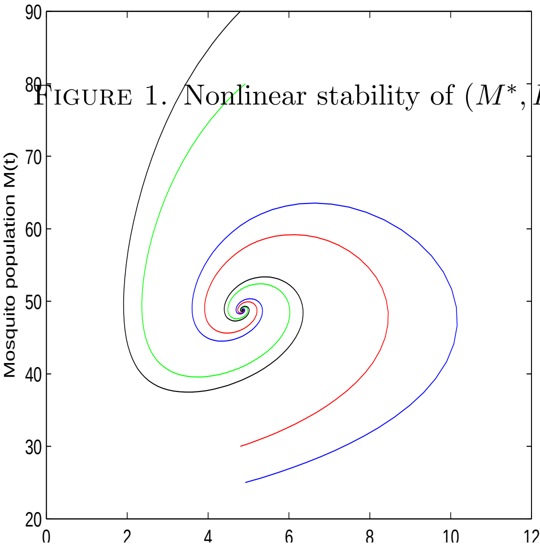

Figure 1. Nonlinear stability of (M^*, B^*) in $M - B$ plane.

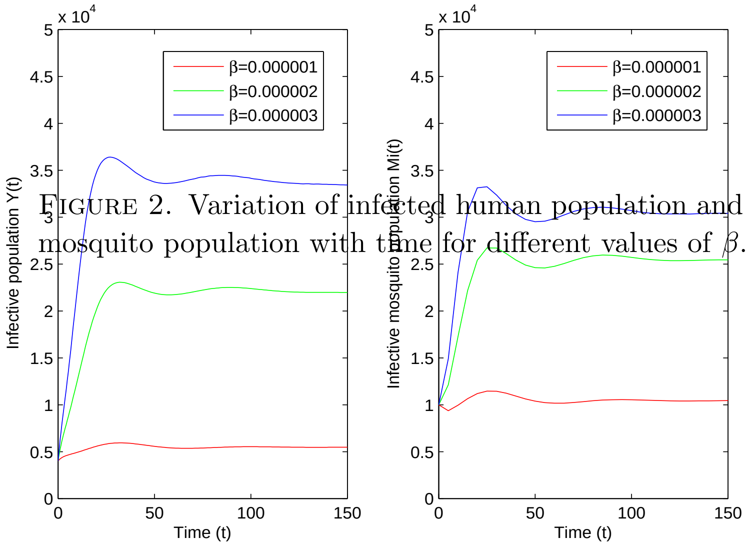

Figure 2. Variation of infected human population and infected mosquito population with time for different values of β.

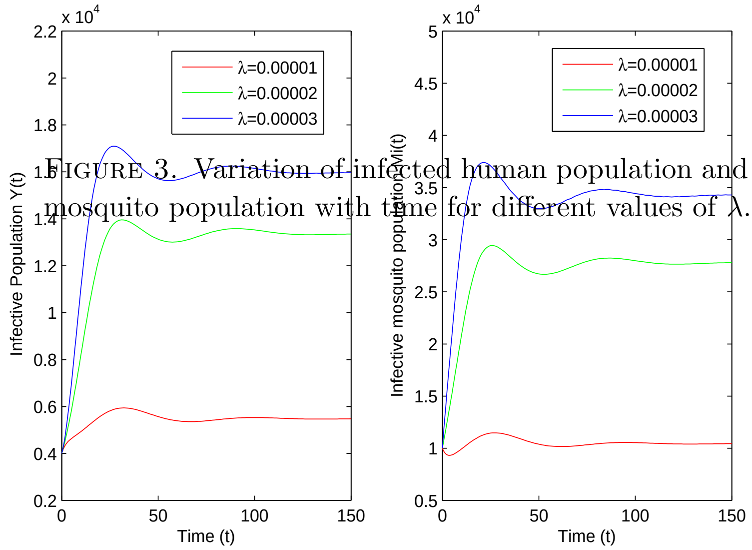

FIGURE 3. Variation of infected human population and infected mosquito population with time for different values of λ.

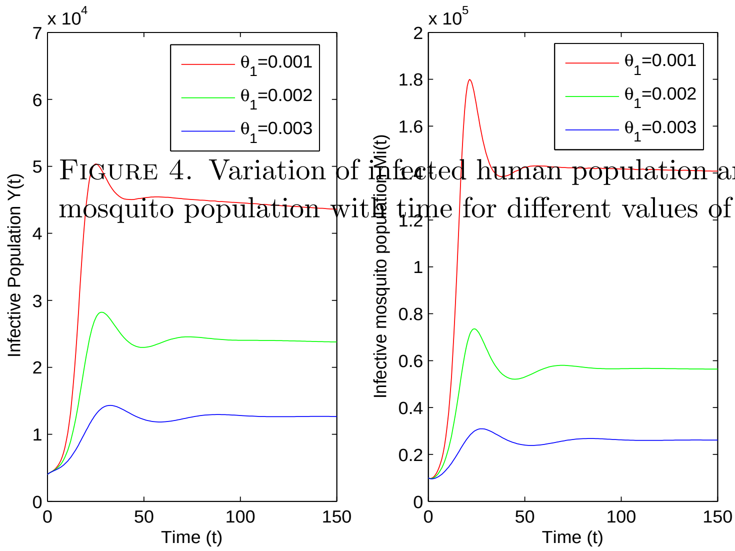

FIGURE 4. Variation of infected human population and infected mosquito population with time for different values of θ_1.

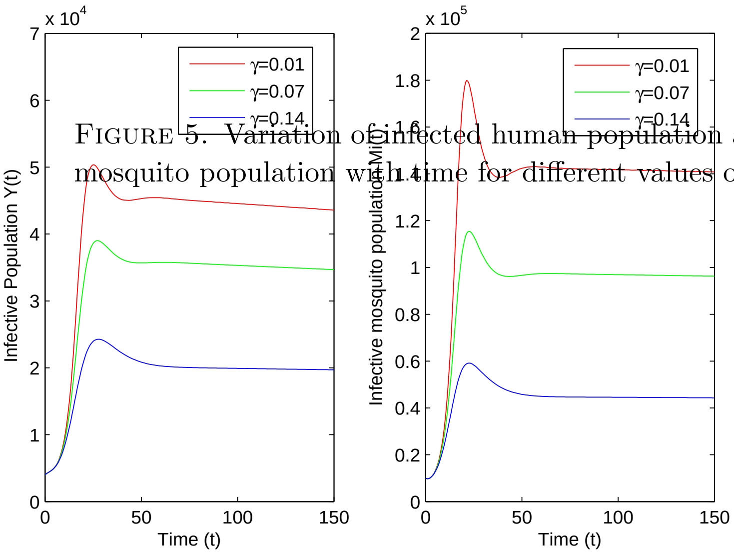

Figure 5. Variation of infected human population and infected mosquito population with time for different values of γ.

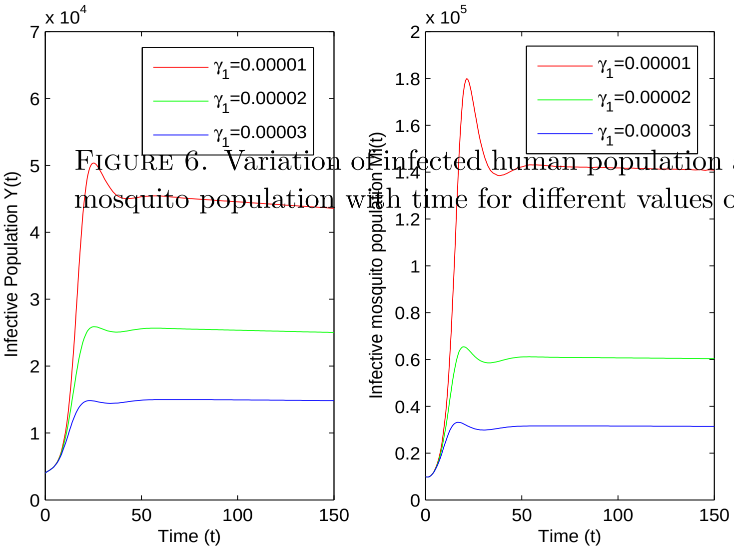

Figure 6. Variation of infected human population and infected mosquito population with time for different values of γ_1.

The eigenvalues of the Jacobian matrix corresponding to the equilibrium E^* for the model system (2.2) are -0.4352, -0.0316, -0.0002, $-0.0317 + 0.0936i$ and $-0.0317 - 0.0936i$. We note that the three eigenvalues of J_{E^*} are negative and the other two eigenvalues have negative real part. Hence, for the above set of parameter values the endemic equilibrium E^* is locally asymptotically stable.

With these parameter values, the solution trajectories of the model system (2.2) have been drawn in figure 1 with different initial starts. From this figure, we may see that all the trajectories initiating inside the region of attraction are approaching towards the equilibrium point (M^*, B^*). This shows the non-linear stability behavior of the endemic equilibrium E^* in $M - B$ plane.

The variation of infective human population '$Y(t)$' and infective mosquito population '$M_i(t)$' with respect to time 't' for different values of rate of transmission of susceptible human population to infective human population 'β' and the rate of transmission of susceptible mosquito population to infective mosquito population 'λ' are shown in figures 2 and 3, respectively. These figures, illustrate that as the rate of transmission of susceptible human population to infective human population and the rate of transmission of susceptible mosquito population to infective mosquito population increase, infective human population '$Y(t)$' and infective mosquito population '$M_i(t)$' both increase. Further, the variations of infective human population '$Y(t)$' and infective mosquito population '$M_i(t)$' with respect to time 't' for different values of the depletion rate coefficient of mosquito population due to Larvivorous fish 'θ_1', growth rate of Larvivorous fish 'γ', growth rate coefficient of Larvivorous fish due to uptake of larva of mosquito population 'γ_1' are shown in figures 4, 5 and 6, respectively. From these figures, it is apparent that as the depletion rate coefficient of mosquito population due to Larvivorous fish 'θ_1', growth rate coefficient of Larvivorous fish due to mosquito population 'γ_1' increase. infective human population '$Y(t)$' and infective mosquito population '$M_i(t)$' decrease.

6. Conclusion

In this paper, A nonlinear mathematical model for malaria is proposed and analyzed. Equilibria of the model are found and stability behavior of these equilibria are discussed using variational matrix method. It is found that under some conditions the mosquito population may present in the atmosphere but the infected human population is zero. This suggest that under some conditions,the malaria can be eradicated from the community. Also it is observed that with the introduction of predatory fish, the equilibrium level of larvae population decreases which causes the decrease in the equilibrium level of adult mosquito population. So introduction of Larvivorous fish has positive impact in controlling the transmission of malaria. Further, numerical simulation is performed to demonstrate the analytical results.

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