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Common Fixed Point Theorems for Quadruple Mappings satisfying Property E. A using Inequality involving Quadratic Terms

Savita Gupta[1] and Rakesh Tiwari

[1]Department of Mathematics,
Shri Shankaracharya Institute Of Technology And Management
Bhilai (C.G.), 492001 India
Department of Mathematics,
Govt. V. Y. T. PG. Autonomous College
Durg (C. G.), 491001 India
e-mail: savita.gupta17@gmail.com, rakeshtiwari66@gmail.com
([1]Corresponding Author)
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Abstract

The aim of this paper is establish common fixed point theorems for quadruple of occasionally weakly compatible mapping satisfying properties E.A using inequality involving quadratic terms.

Keywords and Phrases : Point of coincidence Property (E. A), Common property (E. A), Occasionally weakly compatible maps, Common fixed points.
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1. Introduction

The concept of weakly commuting mappings of Sessa [19] is sharpened by Rhoades [2] and further generalized by Jungck and Rhoades [2]. Similarly, noncompatible mapping is generalized by AAamri and Moutawakil [1] called property (E. A). Noncompatibility is also important to study the fixed point theory. There may be pairs of mappings which are noncompatible but weakly compatible. Imdad and Ali [6], Liu et al. [8], Pathak et al. [9] used this concept to prove existence results in common fixed point theory. Throughout this paper (X, d) is a metric space which we denote simply by X; and A and T are selfmaps of X.

Definition (1.1). (Jungck and Rhoades [10]). Let A and T be selfmaps of a set X. If $Ax = Tx = w$ (say), $w \in X$, for some x in X, then x is called a coincidence point of A and T and the set of coincidence points of A and T in X is denoted by $C(A, T)$, and w is called a point of coincidence of A and T.

Definition (1.2). The pair (A, T) is said to

(i) satisfy property (E. A) [1] if there exists a sequence x_n in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = t$, for some t in X be compatible [11] if $\lim_{n\to\infty} d(ATx_n, TAx_n) = 0$, whenever x_n is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Txn = t$, for some t in X.

(ii) be occasionally weakly compatible (owc) [5] if $TAx = ATx$ for some $x \in C(A, T)$.

(iii) be compatible [11] if $\lim_{n\to\infty} d(ATx_n, TAx_n) = 0$, whenever x_n is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Txn = t$, for some t in X.

(iv) be weakly compatible[12] if $TAx = ATx$ whenever $Ax = Tx$, $x \in X$.

(v) be noncompatible if there is at least one sequence x_n in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = t$, for some t in X, but $\lim_{n\to\infty} d(ATx_n, TAx_n)$ is either non-zero or non-existent.

Definition (1.3). (Liu et al. [8]). Let (X, d) be a metric space and A, B, S and T be four selfmaps on X. The pairs (A, S) and (B, T) are said to satisfy common property (E. A) if there exist two sequences x_n and y_n in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$, for some t in X. In 1996, Tas et al.[14] proved the following theorem.

Theorem (1.4). (Tas et al. [14]). Let A, B, S and T be selfmaps of a complete metric space (X, d) such that $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ and satisfying the inequality,

$$\begin{aligned}[d(Ax, By)]^2 \leq & c_1 \max\{[d(Sx, Ax)]^2, [d(Ty, By)]^2, [d(Sx, Ty)]^2\} \\ & + c_2 \max\{d(Sx, Ax)d(Sx, By), d(Ty, Ax)d(Ty, By)\} \\ & + c_3\, d(Sx, By)d(Ty, Ax)\end{aligned} \tag{1.1}$$

for all $x, y \in X$, where $c_1, c_2, c_3 \geq 0$, $c_1 + 2c_2 < 1, c_1 + c_3 < 1$. Further, assume that the pairs (A, S) and (B, T) are compatible on X. If one of the mappings A, B, S and T is continuous then A, B, S and T have a unique common fixed point in X.

Babu and Kameswari [[15], Theorem 2.1] generalized Theorem 1.4.1 by relaxing the continuity of A, B, S and T; and replacing the compatible property of (A, S) and (B, T) by weakly compatible. In fact, Kameswari [13] proved the following theorem.

Theorem (1.5). (Kameswari [13]). Let A, B, S and T be selfmaps of a complete metric space (X, d) such that $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$; and satisfying the inequality (1.1). Further, assume that the pairs (A, S) and (B, T) are weakly compatible on X. If either of $A(X)$ or $B(X)$ or $S(X)$ or $T(X)$ is a complete subspace of X, then A, B, S and T have a unique common fixed point in X.

Theorem (1.6). (G.V.R. Babu* et al.[7] proof). Let A, B, S, and T be four selfmaps of a metric space (X, d) satisfying the inequality

$$\begin{aligned}[d(Ax, By)]^2) \leq & c_1 \max\{[d(Sx, Ax)]^2, [d(Ty, By)]^2, [d(Sx, Ty)]^2\} \\ & + c_2 \max\ \{d(Sx, Ax)d(Sx, By)d(Ty, By)d(Ty, Ax)\} \\ & + c_3 d(Sx, By)d(Ty, Ax)\end{aligned}$$

for all $x, y \in X$, where $c_1, c_2, c_3 \geq 0$ and $c_1 + c_3 < 1$. Suppose that either (i) $B(X) \subseteq S(X)$, the pair (B, T) satisfies property (E.A) and $T(X)$ is a closed subspace of X; or (ii) $A(X) \subseteq T(X)$, the pair (A, S) satisfies property (E.A) and $S(X)$ is a closed subspace of X, holds. Then $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$.

Most recently Savita Gupta et al. [18] proof, Some common fixed point theorems in metric spaces satisfying an implicit relation involving quadratic terms.

Theorem (1.7). Let A and S be two self-mappings of a metric space (X, d) such that

1. $\overline{A(X)} \subseteq S(X)$,
2. for all $x, y \in X$ and some $\psi \in \Psi$,

$$\begin{aligned}&\psi\Big(d^2(Ax, Ay), d^2(Sx, Sy), d(Sx, Ax)d(Ax, Sy), d(Sy, Ay)d(Sy, Ax), \\ &d(Sx, Ay)d(Sy, Ax), d^2(Sy, Ax)\Big) \leq 0,\end{aligned} \tag{1.2}$$

3. $\overline{A(X)}$ is a complete subspace of X.

Moreover, the mappings A and S have a unique common fixed point in X provided the pair (A, S) is weakly compatible.

In this paper, we prove the existence of common fixed points for two pairs of occasionally weakly compatible selfmaps satisfying property (E. A)/common property (E. A) using an inequality involving quadratic terms.

2. Main Results

Proposition (2.1). Let A, B, S, and T be four selfmaps of a metric space (X, d) satisfying the inequality

$$\begin{aligned}&[d(Ax, By) + p\ d(Sx, Ty)]d(Ax, By)\\&\leq c_1\max\{[d(Sx, Ax)]^2, [d(Ty, By)]^2, [d(Sx, Ty)]^2\}\\&\ + c_2\max\ \{d(Sx, Ax)d(Sx, By)d(Ty, By)d(Ty, Ax)\}\\&\ + c_3 d(Sx, By)d(Ty, Ax),\end{aligned} \tag{2.1}$$

for all $x, y \in X$, where $0 \leq p < 1, c_1, c_2, c_3 \geq 0$ and $c_1 + c_3 < 1$. Suppose that either

(i) $B(X) \subseteq S(X)$, the pair (B, T) satisfies property (E.A) and $T(X)$ is a closed subspace of X; or (ii) $A(X) \subseteq T(X)$, the pair (A, S) satisfies property (E.A) and $S(X)$ is a closed subspace of X holds. Then $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$.

Proof. Suppose (i) holds. Since the pair (B, T) satisfies property (E. A), there exists a sequence x_n in X such that

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z \text{ for some } z \in X. \tag{2.2}$$

Since $B(X) \subseteq S(X)$, there exists a sequence y_n in X such that

$$Bx_n = Sy_n.$$

Hence,

$$\lim_{n\to\infty} Sy_n = z. \tag{2.3}$$

First, we claim that $\lim_{n\to\infty} Ayn = z$. For this purpose, we consider

$$\begin{aligned}&[d(Ay_n, Bx_n) + p\ d(Sy_n, Tx_n)]d(Ay_n, Bx_n)\\&\leq c_1 \max\{[d(Sy_n, Ay_n)]^2, [d(Tx_n, Bx_n)]^2, [d(Sy_n, Tx_n)]^2\}\\&+ c_2 \max\{d(Sy_n, Ay_n)d(Sy_n, Bx_n), d(Tx_n, Bx_n)d(Tx_n, Ay_n)\}\\&+ c_3 d(Sy_n, Bx_n)d(Tx_n, Ay_n)\\&= c_1 \max\{[d(Bx_n, Ay_n)]^2, [d(Tx_n, Bx_n)]^2\}\\&+ c_2 d(Tx_n, Bx_n)d(Tx_n, Ay_n).\end{aligned} \tag{2.4}$$

On taking the limit superior in (2.4), using (2.2) and (2.3), we get,

$$\begin{aligned}\lim_{n\to\infty} \sup[d(Ay_n, Bx_n)]^2 &\leq c_1 \lim_{n\to\infty} \sup[d(Ay_n, Sy_n)]^2\\&= c_1 \lim_{n\to\infty} \sup[d(Ay_n, Bx_n)]^2\end{aligned}$$

so that, $\lim_{n\to\infty} sup[d(Ay_n, Bx_n)]^2 = 0$ and hence $\lim_{n\to\infty}[d(Ay_n, Bx_n)]^2 = 0$.

Hence,

$$\lim_{n\to\infty} Ay_n = z. \tag{2.5}$$

Since $T(X)$ is a closed subspace of X, by (2.2), we have

$$z = Tv \text{ for some } v \in X. \tag{2.6}$$

If $Bv \neq z$, then

$$\begin{aligned}
&[d(Ay_n, Bv) + p\ d(Sy_n, Tv)]d(Ay_n, Bv)\\
&\leq c_1 \max\,[d(Sy_n, Ay_n)]^2, [d(Tv, Bv)]^2, [d(Sy_n, Tv)]^2\\
&+ c_2 \max d(Sy_n, Ay_n)d(Sy_n, Bv), d(Tv, Bv)d(Tv, Ay_n)\\
&+ c_3 d(Sy_n, Bv)d(Tv, Ay_n).
\end{aligned} \tag{2.7}$$

On letting $n \to \infty$ (2.7), using (2.2), (2.3), (2.5) and (2.6), we have

$$[d(z, Bv)]^2 \leq c_1[d(z, Bv)]^2,$$

a contradiction. Hence,

$$Bv = z. \tag{2.8}$$

Hence, from (2.6) and (2.8), we get

$$Bv = Tv = z. \tag{2.9}$$

Hence,

$$C(B, T) \neq \phi. \tag{2.10}$$

Since $B(X) \subseteq S(X)$ and $z \in B(X)$, there exists a $u \in X$ such that

$$z = Su. \tag{2.11}$$

If $z \neq Au$, then from (2.9) and (2.11), we get

$$\begin{aligned}
&[d(Au, z) + p\ d(Su, Tv)]d(Au, z)\\
&= [d(Au, Bv) + pd(Su, Tv)]d(Au, Bv)\\
&\leq c_1 \max\,[d(Su, Au)]^2, [d(Tv, Bv)]^2, [d(Su, Tv)]^2\\
&+ c_2 \max d(Su, Au)d(Su, Bv), d(Tv, Bv)d(Tv, Au)\\
&+ c_3 d(Su, Bv)d(Tv, Au)\\
&= c_1[d(Au, z)]^2,
\end{aligned}$$

a contradiction. Hence,

$$Au = z. \tag{2.12}$$

Hence, from (2.11) and (2.12), we get

$$Au = Su = z. \tag{2.13}$$

Hence, $C(A, S) \neq \phi$.

Similarly, the assertion of the theorem holds under assumption (ii). Hence, Proposition 2.1 follows.

Theorem (2.2). In addition to the hypotheses of Proposition 2.1 on A, B, S and T, if both the pairs (A, S) and (B, T) are owc on X, then the maps A, B, S and T have a unique common fixed point in X.

Proof. By Proposition 2.1, $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$. Since the pair (A, S) is owc, there exists $u^{'} \in C(A, S)$ such that

$$Au^{'} = Su^{'} = z^{'} \text{ (say)} \tag{2.14}$$

and

$$ASu^{'} = SAu^{'}. \tag{2.15}$$

Hence, from (2.14) and (2.15), we get

$$Az^{'} = Sz^{'} = z^{''} \text{ (say)}. \tag{2.16}$$

Since the pair (B, T) is owc, there exists $v^{'} \in C(B, T)$ such that

$$Bv^{'} = Tv^{'} = w \text{ (say)} \tag{2.17}$$

and,

$$BTv^{'} = TBv^{'}. \tag{2.18}$$

Hence, from (2.17) and (2.18), we get

$$Bw = Tw = w' \text{ (say)}. \tag{2.19}$$

Next we claim that

$$z^{''} = w^{'}.$$

If $z^{''} = w^{'}$, then from (2.16) and (2.19), we have

$$\begin{aligned}
&[d(z^{''}, w^{'}) + pd(Sz^{'}, Tw)]d(z^{''}, w^{'}) \\
&= [d(Az^{'}, Bw) + pd(Sz^{'}, Tw)]d(Az^{'}, Bw) \\
&\leq c_1 \max [d(Sz^{'}, Az^{'})]^2, [d(Tw, Bw)]^2, [d(Sz^{'}, Tw)]^2 \\
&+ c_2 \max d(Sz^{'}, Az^{'})d(Sz^{'}, Bw), d(Tw, Bw)d(Tw, Az^{'}) \\
&+ c_3 d(Sz^{'}, Bw)d(Tw, Az^{'}) = (c_1 + c_3)[d(z^{''}, w^{'})]^2,
\end{aligned}$$

a contradiction. Hence,

$$z^{''} = w^{'}. \tag{2.20}$$

Hence, from (2.16) and (2.20), we get

$$Az^{'} = Sz^{'} = w^{'}. \tag{2.21}$$

Next we show that $w' = z'$.

If $w' \neq z'$, then from (2.14) and (2.21), we obtain

$$\begin{aligned} d(z^{'}, w^{'}) =& [d(Au^{'}, Bw) + pd(Su^{'}, Tw)]d(z^{'}, w^{'}) \\ &\leq c_1 \max [d(Su^{'}, Au^{'})]^2, [d(Tw, Bw)]^2, [d(Su^{'}, Tw)]^2 \\ &+ c_2 \max d(Su^{'}, Au^{'})d(Su^{'}, Bw), d(Tw, Bw)d(Tw, Au^{'}) \\ &+ c_3 d(Su^{'}, Bw)d(Tw, Au^{'}) = (c_1 + c_3)[d(z^{'}, w^{'})]^2, \end{aligned}$$

a contradiction. Hence,

$$w^{'} = z^{'}. \tag{2.22}$$

Hence, from (2.19), (2.21) and (2.22), we get

$$Az^{'} = Sz^{'} = z^{'}, \tag{2.23}$$

and

$$Bw = Tw = z^{'}. \tag{2.24}$$

Next we claim that $w = z^{'}$.

If $w \neq z^{'}$, then from (2.17) and (2.24), we have

$$\begin{aligned} d(z^{'}, w) =& [d(Az^{'}, Bv^{'}) + pd(Sz^{'}, Tv^{'})]d(Az^{'}, Bv^{'}) \\ &\leq c_1 \max [d(Sz^{'}, Az^{'})]^2, [d(Tv^{'}, Bv^{'})]^2, [d(Sz^{'}, Tv^{'})]^2 \\ &+ c_2 \max d(Sz^{'}, Az^{'})d(Sz^{'}, Bv^{'}), d(Tv^{'}, Bv^{'})d(Tv', Az') \\ &+ c_3 d(Sz', Bv')d(Tv', Az') = (c_1 + c_3)[d(z^{'}, w)]^2, \end{aligned}$$

a contradiction.
Hence,

$$w = z^{'}. \tag{2.25}$$

Hence, from (2.24) and (2.25), we get

$$Bz^{'} = Tz^{'} = z^{'}. \tag{2.26}$$

Therefore, from (2.23) and (2.26), we obtain

$$Az^{'} = Bz^{'} = Sz^{'} = Tz^{'} = z^{'}.$$

The uniqueness of 'z'' follows from the inequality (2.1).

Thus we conclude the theorem.

Proposition (2.3). Let A, B, S and T be four selfmaps of a metric space (X,d) satisfying the inequality (2.1) of Proposition 2.1. Suppose that (A,S) and (B,T) satisfy a common property (E. A) and $S(X)$ and $T(X)$ are closed subspaces of X. Then $C(A,S)=\phi$ and $C(B,T)=\phi$.

Proof. Suppose that the pairs (A,S) and (B,T) satisfy a common property (E. A). Then there exist two sequences x_n and y_n in X such that

$$\liminf Ay_n = \liminf Sy_n = \liminf Bx_n = \liminf Tx_n = z \quad \text{for some } z \in X. \tag{2.27}$$

Assume that $S(X)$ and $T(X)$ are closed subspaces of X. Then,

$$z = Su = Tv \text{ for some } u, v \in X. \tag{2.28}$$

If $Bv \neq z$, then from (2.27) and (2.28), we have

$$\begin{aligned}
&[d(Ay_n,Bv)+pd(Sy_n,Tv)]d(Ay_n,Bv)\\
&\le c_1 \max\{[d(Sy_n,Ay_n)]^2,[d(Tv,Bv)]^2,[d(Syn,Tv)]^2\}\\
&+c_2 \max\{d(Sy_n,Ay_n)d(Sy_n,Bv),d(Tv,Bv)d(Tv,Ay_n)\}\\
&+c_3 d(Sy_n,Bv)d(Tv,Ay_n).
\end{aligned} \tag{2.29}$$

On letting $\lim\limits_{n\to\infty}$ (2.29), using (2.27) and (2.28), we have, $[d(z,Bv)]^2 \le c_1[d(z,Bv)]^2$, a contradiction. Hence,

$$Bv = z.k \tag{2.30}$$

Hence, from (2.28) and (2.30), we get

$$Bv = Tv = z. \tag{2.31}$$

Again, if $Au \neq z$, then from (2.28) and (2.31), we get

$$\begin{aligned}
d(Au,z) =&[d(Au,Bv)+d(Su,Tv)]d(Au,Bv)\\
&\le c_1 \ \max\{[d(Su,Au)]^2,[d(Tv,Bv)]^2,[d(Su,Tv)]^2\}\\
&+\ c_2 \ \max\{d(Su,Au)d(Su,Bv),d(Tv,Bv)d(Tv,Au)\}\\
&+\ c_3 \ d(Su,Bv)d(Tv,Au) = c_1[d(Au,z)]^2,
\end{aligned}$$

a contradiction. Hence,

$$Au = z. \tag{2.32}$$

Hence, from (2.28) and (2.32), we get

$$Au = Su = z. \tag{2.33}$$

Hence, from (2.31) and (2.33), it follows that

$$C(A, S) \neq \phi \text{ and } C(B, T) \neq \phi.$$

Hence, Proposition 2.3 follows.

Theorem (2.4). In addition to the hypotheses of Proposition 2.3 on A, B, S and T, if both the pairs (A, S) and (B, T) are owc on X, then the maps A, B, S and T have a unique common fixed point in X.

Proof. By Proposition 2.3, $C(A, S) \neq \phi$ and $C(B, T) \neq \phi$. The rest of the proof runs as that of Theorem 2.2.

Example (2.5). Let $X = [\frac{1}{3}, 1]$ with the usual metric. We define mappings A, B, S and T on X by

$$A(x) = \begin{cases} \frac{1}{3}, & if\ \frac{1}{3} \leq x \leq \frac{2}{3}; \\ \frac{2}{3}, & if\ \frac{2}{3} \leq x \leq 1 \end{cases},$$

$$B(x) = \begin{cases} \frac{11}{12}, & if\ \frac{1}{3} \leq x \leq \frac{2}{3}; \\ \frac{2}{3}, & if\ \frac{2}{3} \leq x \leq 1 \end{cases},$$

$$S(x) = \begin{cases} \frac{5}{5}, & if\ \frac{1}{3} \leq x \leq \frac{2}{3}; \\ \frac{1}{3}, & if\ \frac{2}{3} \leq x \leq 1 \end{cases} \text{ and } T(x) = \begin{cases} \frac{1}{2}, & if\ \frac{1}{3} \leq x \leq \frac{2}{3}; \\ 1 - \frac{1}{2}x, & if\ \frac{2}{3} \leq x \leq 1 \end{cases}$$

Here we observe that both $S(X)$ and $T(X)$ are closed; and neither $A(X) \subseteq T(X)$ nor $B(X) \subseteq S(X)$. The inequality (2.1) holds with $c_1 = \frac{1}{3}$, $c_2 = 3\frac{3}{5}$ $c_3 = \frac{1}{2}$ and $p = 0$. Also, we note that the inequality (2.1) fails to hold for any $c_1 + 2c_2 < 1$ and $c_1 + c_3 < 1$ and $0 \leq\ p < 1$ when $x, y \in [\frac{1}{3}, \frac{2}{3}]$. Further, the sequence $\{x_n\}$, $x_n = \frac{2}{3} + \frac{1}{n+3}$, $n = 1, 2, 3, ...$ is in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \frac{2}{3}$, so that the pairs (A, S) and (B, T) satisfy common property (E. A). Clearly, the pairs (A, T) and (B, T) are owc. Hence, the selfmaps A, B, S and T satisfy all the conditions of Theorem 2.4 and $\frac{2}{3}$ is the unique common fixed point of A, B, S and T.

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