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Anisotropic Bianchi Type-V Model and the Conharmonically Flatness

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Abstract

The study of Conharmonic curvature tensor is well known in the differential geometry and general relativity. The present study deals with spatially homogeneous and anisotropic Bianchi type-V universe with Conharmonically flatness when the source of gravitation is the viscous fluid. To get deterministic solutions of the Einstein's field equation it is considered a relationship between the metric coefficient $C = B^n$, n is a constant. We discuss the different cases to describe the geometrical, kinematical and physical properties of the model.

Keywords : Bianchi type V model; Conharmonically flatness.

1. Introduction

The modern cosmology is concern with nothing less than through the understanding and explanation of past history, the present state and the future evolution of the universe. The observations show that our universe at large scale is homogeneous and isotropic and in accelerating phase (Gasperini et al. [5], Vishwakarma [19]). In fact, there are theoretical arguments from the recent experimental data which support the existence of an anisotropic phase approaching to isotropic phase leading to consider the model of universe with anisotropic background. Spatially homogeneous and anisotropic cosmological models play significant role in the description of universe at its early stage of evolution. Bianchi type-V universe, which is homogeneous and anisotropic having richer structure than others Bianchi models and is interesting to study ([11, 12, 13, 16, 21] and reference therein). Its interest may be understood by recalling that the Bianchi type V universes are the natural generalization of open Robertson-Walker space time which eventually become isotropic. A numbers of authors (Pradhan and Rai [11], Ram et al. [12], Singh [17], Singh et

al. [18], Singh and Chaubey [16]) have studied Bianchi type V space time in different physical contexts. Camci et al.[21] have studied Curvature and Weyl collineations of Bianchi type V space time.

The rationale behind the present work is to investigate that- what consequences do emerges on cosmological models for vanishing conharmonic curvature tensor. Conharmonic curvature tensor is an invariant geometrical object under conharmonic transformation have been defined by Ishii [6] who introduced the conharmonic transformation as a subgroup of the conformal transformation satisfying $\phi_{;i}^i + \phi_{,i}\phi^{,i} = 0$, where ϕ is a real valued differentiable function of coordinates. This tensor plays significant role in the study of differential geometry of certain F-structure (e.g, complex, almost complex, Kahler, almost Kahler, Hermitian, almost Hermitian structures, etc.)([3, 15] and references their in) and genera relativity([7], [2], [1], [3]).

The conharmonic tensor L_{hijk} for the four dimensional space time manifold, is

$$L_{hijk} = R_{hijk} - \frac{1}{2}(g_{ij}R_{hk} - g_{ik}R_{hj} + g_{hk}R_{ij} - g_{hj}R_{ik}) \quad (1)$$

where R_{hijk} is the Riemann curvature tensor and R_{ij} , the Ricci tensor. A space time manifold for which this tensor vanishes at each point is said to be Conharmonically flatness; thus this tensor represents the deviation of the manifold from conharmonic flatness. Abdussatar [1] have studied conharmonic transformation in general relativity. Abdussattar and Dwivedi [2] have obtained the conditions for the symmetries of the anisotropic fluid space time admitting a conharmonic killing vectors to be inherited. Kumar and Srivastava [7] have studied FRW cosmology for conharmonically flat space time.

Motivated by the above works, in this paper it has studied Bianchi type-V cosmological model for conharmonically flatness when the source of matter distribution is viscous fluid. The paper is organized as follows : after introductory part, section (2) contains some basic equations. In section(3) we discuss the solutions of field equations in different cases-(3.1) when shear viscosity is constant, (3.2) when shear viscosity is proportional to scalar expansion and (3.3) in the absence of shear viscosity. In last section (4), the discussions and concluding remarks of the paper are carried out.

2. Basic equations

Consider the spatially homogeneous and anisotropic Bianchi type V metric of the form

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2mx}(B^2 dy^2 + C^2 dz^2) \quad (2)$$

where the cosmic scale factors A, B, C are function of t only and m is a constant.

The Einstein's field equation reads as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi GT_{ij} \quad (3)$$

where T_{ij} is the energy momentum tensor of the viscous fluid given by

$$T_{ij} = (\rho + p')u_i u_j + p'g_{ij} + \eta(u_{i;j} + u_{j;i} - u_i u^k u_{k;j} - u_j u^k u_{k;i}) \quad (4)$$

$$\text{where } p' = p - (\xi - \frac{2}{3}\eta)\theta \quad (5)$$

Here semicolon (;) denotes the covariant differentiation and ρ, p, η, ξ are respectively the energy density, pressure, coefficient of shear viscosity, coefficient of bulk viscosity and θ is the scalar expansion defined by

$$\theta = u^i_{;i} \quad (6)$$

u^i is the fluid four velocity satisfying $u_i u^i = -1$. We choose the comoving coordinate so that

$$u^i = (1, 0, 0, 0) \quad (7)$$

For the metric(2) field equation (3) for conharmonically flatness yield the following system of equations(see for more details, [7] and references their in)

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = \frac{8}{3}\pi G\rho \quad (8)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} - 2\frac{\dot{m}^2}{A^2} = -\frac{8}{3}\pi G[p - 2\eta\frac{\dot{A}}{A} - (\xi - \frac{2}{3}\eta)\theta] \quad (10)$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} - 2\frac{\dot{m}^2}{A^2} = -\frac{8}{3}\pi G[p - 2\eta\frac{\dot{B}}{B} - (\xi - \frac{2}{3}\eta)\theta] \quad (11)$$

$$\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - 2\frac{\dot{m}^2}{A^2} = -\frac{8}{3}\pi G[p - 2\eta\frac{\dot{C}}{C} - (\xi - \frac{2}{3}\eta)\theta] \quad (12)$$

Here over dot denotes differentiation with respect to t .

The parameters such as average scale factor(R), Hubble's parameter(H) and shear scalar (σ) for the metric(2) are given by

$$R = (ABC)^{\frac{1}{3}} \quad (13)$$

$$H = \frac{1}{3}(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}) \quad (14)$$

$$\sigma = \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} \right) \quad (15)$$

3. Solutions of the field equations

The system of equations (8 - 12) are employed to obtain the cosmological solution. These five system of equations involves seven unknowns A , B , C , ρ , p , ξ and η . In order to obtain the complete solution of the field equation, it is require two additional equations. Let us consider

$$C = B^n \quad (16)$$

where n is a constant. In view of equ.(9) and (16), we have

$$A^2 = BC \quad (17)$$

Now, subtracting equ.(11) from (10) and its first integral is

$$ABC \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = c_1 e^{\int k \eta dt} \quad (18)$$

where c_1 is constant of integration and $k = \frac{16}{3}\pi G$.

Thus in view of equ.(16) and (17), equ.(18) gives

$$A^2 \dot{A} = \left(\frac{n+1}{n-1} \right) c_1 e^{\int k \eta dt} \quad (19)$$

where c_1 is constant of integration. Further, it is considered three cases to get deterministic solutions and for the discussion of the behaviour of models.

3.1 Solution for $\eta = \alpha$

Substituting $\eta = \alpha$ (where α is a constant) in (19) and integrating, we obtain

$$A = (c_2 + c_3 e^{k\alpha t})^{\frac{1}{3}} \quad (20)$$

where c_2 , c_3 are arbitrary constants and

$$c_3 = 3 \frac{c_1}{k\alpha} \left(\frac{n+1}{n-1} \right) \quad (21)$$

In view of equ.(16),(17) and (20) the other metric coefficients

$$B = (c_2 + c_3 e^{k\alpha t})^{\frac{2}{3(n+1)}}, \quad C = (c_2 + c_3 e^{k\alpha t})^{\frac{2n}{3(n+1)}} \quad (22)$$

Thus in view of (20 - 22) the metric(2) for conharmonically flatness is

$$ds^2 = -dt^2 + (c_2 + c_3 e^{k\alpha t})^{\frac{2}{3}} dx^2 + e^{2mx} [(c_2 + c_3 e^{k\alpha t})^{\frac{4}{3(n+1)}} dy^2 + (c_2 + c_3 e^{k\alpha t})^{\frac{4n}{3(n+1)}} dz^2] \quad (23)$$

The scalar expansion (θ), energy density (ρ) and pressure (p) for the model (23) are given by

$$\theta = k\alpha c_3 e^{k\alpha t} (c_2 + c_3 e^{k\alpha t})^{-1} \quad (24)$$

$$8\pi G\rho = k^2\alpha^2 (c_2 + c_3 e^{k\alpha t})^{-2} [3c_2 c_3 + (c_3^2 + 6\frac{c_1^2}{k^2\alpha^2}) e^{k\alpha t}] e^{k\alpha t} \quad (25)$$

$$8\pi Gp = 6m^2 (c_2 + c_3 e^{k\alpha t})^{\frac{-2}{3}} + k\alpha c_3 (8\pi G\xi - k\alpha) e^{k\alpha t} (c_2 + c_3 e^{k\alpha t})^{-1} \quad (26)$$

A numbers of authors such as Santos[14], Pavon[10], maartens[8], Zimadahl[22] have investigated that the bulk viscosity is the simple power function of energy density.

$$\xi(t) = \xi_0 \rho^r \quad (27)$$

where ξ_0 and r are constants. The realistic models are consistent with $0 \leq r \leq 1$. For low density, r may be equal to unity as used in Murphy's work [9] and may corresponding to the radiative fluid (Weinberg [20]). Further, $0 \leq r \leq \frac{1}{2}$ is a more suitable assumption to obtain realistic cosmological models near the big bang [4]. Therefore, for simplicity and to obtain realistic models for physical importance, we adopt the following three cases as $r = 0, \frac{1}{2}, 1$.

$\xi = \xi_0$:

When $r = 0$, equ.(27) reduces to $\xi = \xi_0$. Hence, in this case equ.(26) leads to

$$8\pi Gp = 6m^2 (c_2 + c_3 e^{k\alpha t})^{\frac{-2}{3}} + k\alpha c_3 (8\pi G\xi_0 - k\alpha) e^{k\alpha t} (c_2 + c_3 e^{k\alpha t})^{-1} \quad (28)$$

$\xi = \xi_0 \rho^{\frac{1}{2}}$:

When $r = \frac{1}{2}$, equ.(27) reduces to $\xi = \xi_0 \rho^{\frac{1}{2}}$. Hence, in this case equ.(26) leads to

$$\begin{aligned} 8\pi Gp = 6m^2 (c_2 + c_3 e^{k\alpha t})^{\frac{-2}{3}} + k^2\alpha^2 c_3 \left[\frac{\xi_0 \sqrt{8\pi G}}{(c_2 + c_3 e^{k\alpha t})} \right. \\ \left. \times \left\{ 3c_2 c_3 e^{k\alpha t} + (c_3^2 + \frac{6c_1^2}{k^2\alpha^2}) e^{2k\alpha t} \right\}^{\frac{1}{2}} e^{k\alpha t} - 1 \right] \frac{e^{k\alpha t}}{(c_2 + c_3 e^{k\alpha t})} \end{aligned} \quad (29)$$

$\xi = \xi_0 \rho$:

When $r = 1$, equ.(27) reduces to $\xi = \xi_0 \rho$. Hence, in this case equ.(26) leads to the form

$$8\pi Gp = 6m^2(c_2 + c_3e^{k\alpha t})^{\frac{-2}{3}} + k^2\alpha^2c_3 \left[\frac{k\alpha\xi_0}{(c_2 + c_3e^{k\alpha t})^2} \right. \\ \left. \times \left\{ 3c_2c_3e^{k\alpha t} + (c_3^2 + \frac{6c_1^2}{k^2\alpha^2})e^{2k\alpha t} \right\} e^{k\alpha t} - 1 \right] \frac{e^{k\alpha t}}{(c_2 + c_3e^{k\alpha t})} \quad (30)$$

Some features of the model

The value of shear scalar (σ) is given by

$$\sigma^2 = c_1e^{2k\alpha t}(c_2 + c_3e^{k\alpha t})^{-2} \quad (31)$$

Thus, we have

$$\frac{\sigma^2}{\theta^2} = \frac{c_1}{k^2\alpha^2c_3^2} = \text{constant} \quad (32)$$

The average scale factor (R) and Hubble's parameter (H) are given by

$$R = (c_2 + c_3e^{k\alpha t})^{\frac{1}{3}} \quad (33)$$

$$H = \frac{1}{3}c_3k\alpha e^{k\alpha t}(c_2 + c_3e^{k\alpha t})^{-1} \quad (34)$$

An observational quantity is the deceleration parameter q given by

$$q = -\frac{\ddot{R}H^{-2}}{R} = -(1 + 3\frac{c_2}{c_1}e^{-k\alpha t}) \quad (35)$$

From (35), it is observe that the deceleration parameter is time dependent. The sign of q indicate whether the model is accelerating or not. From equ. (35), we observe that $q < 0$ for all time, thus our model of universe is in accelerating phase which consistent with the recent observations (Gasperini et al. [5], Vishwakarma [19]). As $t \rightarrow \infty$, $q = -1$ which characterizes the de-sitter solution (Weinberg [20]).

3.2 Solution for $\eta \propto \theta$.

Suppose that the shear viscosity is proportional to scalar expansion i.e.;

$$\eta = \beta\theta \quad (36)$$

where β is proportionality constant. Thus using (36) into (16 - 19), we obtain

$$A = (k_2 + k_3t)^\mu \quad (37)$$

$$B = (k_2 + k_3t)^{\frac{2\mu}{n+1}}, \quad C = (k_2 + k_3t)^{\frac{2n\mu}{n+1}} \quad (38)$$

where k_2, k_3 are constant and the constants

$$\mu = (3 - 3k\beta)^{-1}, \quad k_3 = \frac{c_1}{\mu} \left(\frac{n+1}{n-1} \right)$$

Thus in view of (37 - 38) the metric(2) for conharmonically flatness takes the form

$$ds^2 = -dt^2 + (k_2 + k_3 t)^{2\mu} dx^2 + e^{2mx} [(k_2 + k_3 t)^{\frac{4\mu}{n+1}} dy^2 + (k_2 + k_3 t)^{\frac{4n\mu}{n+1}} dz^2] \quad (39)$$

The scalar expansion (θ), energy density (ρ) and pressure (p) for the model (39) are respectively given by

$$\theta = 3\mu k_3 (k_2 + k_3 t)^{-1} \quad (40)$$

$$8\pi G\rho = 3\alpha_1 \mu k_3^2 (k_2 + k_3 t)^{-2} \quad (41)$$

$$\frac{8}{3}\pi Gp = 2m^2 (k_2 + k_3 t)^{-2\mu} + 8\pi G\mu k_3 \xi (k_2 + k_3 t)^{-1} - 3\mu(\mu-1)k_3^2 (k_2 + k_3 t)^{-2} \quad (42)$$

where the constant $\alpha_1 = \frac{(5n+1)\mu-3(n+1)}{n+1}$.

$\xi = \xi_0$:

In this case equ.(42) leads to

$$8\pi Gp = 6m^2 (k_2 + k_3 t)^{-2\mu} + 24\pi G\mu k_3 \xi_0 (k_2 + k_3 t)^{-1} - 9\mu(\mu-1)k_3^2 (k_2 + k_3 t)^{-2} \quad (43)$$

$\xi = \xi_0 \rho^{\frac{1}{2}}$:

In this case equ.(42) leads to

$$8\pi Gp = 6m^2 (k_2 + k_3 t)^{-2\mu} + 6k_3^2 \mu \xi_0 \sqrt{6\pi G\alpha\mu} (k_2 + k_3 t)^{-2} - 9\mu(\mu-1)k_3^2 (k_2 + k_3 t)^{-2} \quad (44)$$

$\xi = \xi_0 \rho$:

In this case equ.(42) leads to

$$8\pi Gp = 6m^2 (k_2 + k_3 t)^{-2\mu} + 9\alpha\mu^2 k_3^3 \xi_0 (k_2 + k_3 t)^{-3} - 9\mu(\mu-1)k_3^2 (k_2 + k_3 t)^{-2} \quad (45)$$

Some features of the model

The value of shear scalar (σ) is given by

$$\sigma^2 = c_1 (k_2 + k_3 t)^{-2} \quad (46)$$

Thus, we have

$$\frac{\sigma^2}{\theta^2} = \frac{c_1}{9k_3^2 \mu^2} = \text{constant} \quad (47)$$

The average scale factor and Hubble's factor are respectively given by

$$R = (k_2 + k_3 t)^\mu \quad (48)$$

$$H = \mu k_3 (k_2 + k_3 t)^{-1} \quad (49)$$

The deceleration parameter q is

$$q = \frac{1}{\mu} - 1 \quad (50)$$

From (50) it is seen that the deceleration parameter is constant. The model gives accelerating universe for $\mu > 1$ and decelerating for $\mu < 1$. It may be noted that the current observation of Supernova favour the accelerating universe, but they do not all together rule out the decelerating ones which are also consistent with the observation (Vishwakarma [19]).

3.3 Case(3): Solution in absence of viscosity

For $\eta \rightarrow 0$, the equations (16- 19) gives

$$A = (a_2 + a_3 t)^{\frac{1}{3}} \quad (51)$$

$$B = (a_2 + a_3 t)^{\frac{2}{3(n+1)}} \quad C = (a_2 + a_3 t)^{\frac{2n}{3(n+1)}} \quad (52)$$

where c_1, a_2, a_3 are constants and $a_3 = 3c_1(\frac{n+1}{n-1})$. Therefore, the metric (2) reduces to the form

$$ds^2 = -dt^2 + (a_2 + a_3 t)^{\frac{2}{3}} dx^2 + e^{2mx} [(a_2 + a_3 t)^{\frac{4}{3(n+1)}} dy^2 + (a_2 + a_3 t)^{\frac{4n}{3(n+1)}} dz^2] \quad (53)$$

The expressions for other physical and kinematical parameters for the model(53) are

$$\theta = a_3(a_2 + a_3 t)^{-1} \quad (54)$$

$$8\pi G\rho = M(a_2 + a_3 t)^{-2}, \quad \text{where } M \text{ is a constant.} \quad (55)$$

$$8\pi Gp = 6m^2(a_2 + a_3 t)^{\frac{-2}{3}} + 8\pi G a_3 \xi (a_2 + a_3 t)^{-1} \quad (56)$$

$\xi = \xi_0$:

The equation (56) leads to

$$8\pi Gp = 6m^2(a_2 + a_3 t)^{\frac{-2}{3}} + 8\pi G a_3 \xi_0 (a_2 + a_3 t)^{-1} \quad (57)$$

$\xi = \xi_0 \rho^{\frac{1}{2}}$:

The equ. (56) leads to

$$8\pi Gp = 6m^2(a_2 + a_3 t)^{\frac{-2}{3}} + 2a_3 \xi_0 \sqrt{2\pi G M} (a_2 + a_3 t)^{-2} \quad (58)$$

$\xi = \xi_0 \rho$:

In this case equ. (56) becomes

$$8\pi Gp = 6m^2(a_2 + a_3t)^{-\frac{2}{3}} + Ma_3\xi_0(a_2 + a_3t)^{-3} \quad (59)$$

Some features of the model

The value of shear scalar (σ) is given by

$$\sigma^2 = 3c_1^2(a_2 + a_3t)^{-2} \quad (60)$$

Thus, we have

$$\frac{\sigma^2}{\theta^2} = \frac{3c_1^2}{a_3^2} = \text{constant} \quad (61)$$

The average scale factor and Hubble's factor are respectively given by

$$R = (a_2 + a_3t)^{\frac{1}{3}} \quad (62)$$

$$H = \frac{a_3}{3}(a_2 + a_3t)^{-1} \quad (63)$$

The deceleration parameter q is

$$q = \frac{2a_3^2}{81(a_2 + a_3t)^4} \quad (64)$$

It is observed that the deceleration parameter is a decreasing function of time and $q \rightarrow 0$ when $t \rightarrow \infty$. The model describe the decelerating phase of universe.

4. Discussion and the concluding Remarks

In this paper we have studied a spatially homogeneous and anisotropic Bianchi type-V universe with conharmonically flatness when the source of gravitation is the viscous fluid. The exact solution of Einstein's field equations have been obtained by considering a relation between metric coefficient(16) together with the three different cases - (i) $\eta = \text{constant}$, (ii) $\eta \propto \theta$ and (iii) $\eta = 0$.

In subsection (3.1), we have presented a new class of anisotropic Bianchi type-V cosmological model (23) for constant value of shear viscosity. It has been observed that the model has no initial singularity and all the physical parameters such as R , θ , σ , ρ and p are of constant value at the big-bang epoch ($t = 0$). The scalar expansion of model increases with cosmic time t and it is proportional to the shear scalar (σ). Also, we see that $\frac{\sigma}{\theta} = \text{constant}$ which shows that the model does not approach isotropy at any time. As $t \rightarrow \infty$ the energy density and pressure for derived model become zero, which shows that the proposed model yields empty universe at late time. From (35), we observe

that the deceleration parameter ‘ q ’ is negative for all times which shows that our presented model of universe is in accelerating phase.

In subsection (3.2), we have investigated model (39) for the situation that shear viscosity (η) is proportional to scalar expansion (θ). In this case, the model has a initial singularity at $t = t_0$, where $t_0 = -\frac{k_2}{k_3}$. The Hubble’s parameter, scalar expansion, shear scalar, energy density and pressure become infinite at this epoch. The scalar expansion decreases as t increases. Also we have $\frac{\sigma}{\theta} = \text{constant}$, which shows that the model does not approach isotropic at any time. It has been observed that the energy density and pressure tend to zero as $t \rightarrow \infty$ which exhibits that the model gives empty universe at late time. The deceleration parameter ‘ q ’ is found to be a constant so that model is accelerating when $\mu > 1$ and decelerating when $\mu < 1$.

In subsection (3.3), the cosmological model (53) has initial singularity at $t = t_1$, where $t_1 = -\frac{a_2}{a_3}$. This model starts from $t = t_1$ and goes on expanding until $t = \infty$. The energy density, pressure, expansion scalar and shear scalar tend to infinity at $t = t_1$. The energy density and pressure become zero as $t \rightarrow \infty$ (empty universe). The expansion scalar decreases as t increases and expansion stop at $t = \infty$. Also, we have $\frac{\sigma}{\theta} = \text{constant}$, which shows that the model does not approach isotropy at any time. It has been found that the deceleration parameter is a decreasing function of time and $q \rightarrow 0$ when $t \rightarrow \infty$. The model describe the decelerating phase of universe ($q > 0$). It may be noted that the current observation of Supernova favour the accelerating universe, but they do not all together rule out the decelerating ones which are also consistent with the observation (Vishwakarma [19]).

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